KIRISH

Shavkat Mirziyoev: - «Bizni hamisha o‘ylantirib keladigan muhim masala – bu yoshlarimizning odob-axloqi, yurish-turishi, bir so‘z bilan aytganda, dunyoqarashi bilan bog‘liq. Bugun zamon shiddat bilan o‘zgaryapti. Bu o‘zgarishlarni hammadan ham ko‘proq his etadigan kim – yoshlar. Mayli, yoshlar o‘z davrining talablari bilan uyg‘un bo‘lsin. Lekin ayni paytda o‘zligini ham unutmasin. Biz kimmiz, qanday ulug‘ zotlarning avlodimiz, degan da’vat ularning qalbida doimo aks-sado berib, o‘zligiga sodiq qolishga undab tursin. Bunga nimaning hisobidan erishamiz? Tarbiya, tarbiya va faqat tarbiya hisobidan», deya ta’kidladi Prezidentimiz.

Shavkat Mirziyoev: - «Matematika fani hamma aniq fanlarga asos. Bu fanni yaxshi bilgan bola aqlli, keng tafakkurli bo‘lib o‘sadi, istalgan sohada muvaffaqiyatli ishlab ketadi», - deydi Prezident.

Mavzuning dolzarbligi. Aralash turdagi

$$T(u) = y u_{xx} + u_{yy} = 0 \tag{1}$$

tenglama uchun birinchi fundamental tadqiqotlarni italyan matematigi Francheskо Trikomi bajargan. U hozirgi vaqtda uning nomi bilan ataluvchi quyidagi Trikomi masalasini ta’riflagan va yechgan: $z = x + iy$ kompleks tekisligining $y > 0$ yarim tekisligida, uchlar $A(0,0)$ va $B(1,0)$ nuqtalarda bo’lgan Γ silliq Jordan chizig’i bilan, $y < 0$ yarim tekislikda esa (1) tenglamaning, AC va BC xarakteristikalari bilan chegaralangan bir bog’lamli Ω sohada (1) tenglamaning ushbu

$$u(x,y) = \varphi(x,y) \; , \; (x,y) \in \Gamma \; , \tag{2}$$

$$u(x,y) = \psi(x) \; , \; (x,y) \in AC \; , \tag{3}$$

$$\lim_{y \to -0} u_y = \lim_{y \to +0} u_y \tag{4}$$

shartlarni qanoatlantiruvchi regulyar yechimi, $u(x,y)$ topilsin. $u(x,y)$ funktsiya regulyar yechim deyiladi agarda u ushbu shartlarni qanoatlantirsa:

1) $u(x,y)$ $\overline{\Omega}$ sohada uzluksiz;

2) birinchi tartibli hosilalar A va B nuqtalardan tashqari barcha $\overline{\Omega}$ sohada uzluksiz va bu nuqtalarda birdan kichik tartibda cheksizlikka aylanishi mumkin;

3) ikkinchi tartibli hosilalar Ω sohaning buzilish chizig'idan tashqari barcha nuqtalarida uzluksiz, bu hosilalar buzilish chizig'ida mavjud bo'lmasligi ham mumkin;

4) $u(x,y)$ funktsiya $\Omega \setminus AB$ sohaning barcha nuqtalarida (1) tenglamani qanoatlantiradi.

Bu ishlardan keyin buziluvchan va aralash turdagi tenglamlar uchun chegaraviy masalalar nazariyasi ko'p yo'nalishlarda o'rganildi va rivojlantirildi. Xususan, Trikomi masalasi umumiyroq aralash turdagi tenglamalar uchun ishlarda, Trikomi masalasining har xil modefikatsiyasi ishlarda spektral masalalar o'rganildi. Eng muhim natijalar va adabiyotlar ro'yxati A. V. Bitsadze [5], M. S. Salahiddinov[20], T. D. Djuraev, A. M. Naxushev[19] , Ye. I. Moiseev, A. P. Soldatov, A.I.Kojanov monografiyalarida keltirilgan.

Tadqiqotning maqsadi va vazifasi. Ushbu magistrlik dissertatsiyasi bo'yicha asosan singulyar koeffitsientli

$$(signy)|y|^m u_{xx} + u_{yy} + (\beta_0 / y)u_y = 0 \qquad (1.1)$$

Gellerstedt tenglamasi o'rganiladi. (1.1) tenglama $z = x + iy$, kompleks tekisligining $lmz > 0$ yuqori yarim tekisligida uchlari $A(-1,0)$ va $B(1,0)$ nuqtalarda va yuqori yarim tekislikda joylashgan $\Gamma : y = f(x)$ chizig'i bilan, $lmz < 0$ pastki yarim tekislikda esa (1.1) tenglamaning AC va BC xarakteristikalari bilan chegaralangan bir bog'lamli D sohada o'rganiladi.

Dissertatsiya (1.1) tenglama uchun F. Trikomi, V.I. Jegalov[13], A. M. Naxushev[14], masalalar shartlarini barchasini o'zida birlashtirib yaxlit bir masala sifatida ta'riflangan masalaning korrekt ekanligi isbotlash maqsad qilib qo'yilgan.

(1.1) tenglama uchun Bitsadze-Samarskiy masalasining sharti noma'lum funktsiyaning kasr tartibli hosilalarining qiymatlarini parallel xarakteristikalarda berilgan masalaning korrektligi o'rganiladi.

Tadqiqotning ob’ekti va predmeti. Tadqiqot ob’ekti sifatida singulyar koeffitsientli Gellerstedt tenglamasi uchun xarakteristikada va buzilish chizig’ida nolokal shartga ega bo’lgan nolokal masala qaraladi.

Tadqiqotning uslubiyati va uslublari. Ta’riflangan masalaning yagonaligi ekstremum printsipi yordamida, mavjudligi esa integral tenglamalar usulida isbotlanadi. Integral tenglamalardan, singulyar integral tenglamalar, Viner-Xopf integral tenglamasi, Fredgolmning II-tur integral tenglamalar nazariyalaridan foydalaniladi.

Tadqiqot natijalaring ilmiy jihatdan yangilik darajasi va tadbiqi. Tadqiqot xarakteristikada va buzilish chizig’ida nolokal shartga ega bo’lgan nolokal masalani yechimini topishni o’rganibgina qolmay, balki fizik, mexanik, texnik, bioligik va boshqa jarayonlarni o’rganish bilan bog’liq.

Tadqiqot natijalarining amaliy ahamiyati va tadbiqi. Tadqiqot zamonaviy xususiy hosilali differentsial tenglamalar nazariyasida asosiy yo’nalishlardan biri hisoblanadi va u muhim amaliy masalalarni yechishda qo’llaniladi.

Tadqiqotning tuzilishi va tarkibi. tadqiqot ishi kirish, 3ta bob, 13 paragraf, xulosalar, foydalanilgan adabiyotlar va ilovalar ro‘yxatidan iborat. 73 bet.

Tadqiqot mavzusi bo‘yicha adabiyotlar sharxi.

[30] adabiyotda Pulkin S.P. tomonidan

$$(sign y)|y|^m u_{xx} + u_{yy} + (\beta_0 / y)u_y = 0.$$

$u(x, y)$ funksiya $\Omega^- \setminus (OC_0 \cup OC_1)$ sohada yuqoridagi tenglamaning R_1 sinfga tegishli bo‘lgan umumlashgan echimi ifodalangan. [5] adabiyotda Bitsadzening ekstrimum prinspiga doir teorema va uning isboti keltirilib berilgan. [17] adabiyotda S.G.Mixlinning

$$A(x)\rho(x) - \lambda \int_{-1}^{c} \left(\frac{1}{t-x} - \frac{1}{1-xt} \right) \rho(t)dt = g(x), \ -1 \le x \le c.$$

tenglamaga takomillashtirilgan Karlemanning regulyarlashtirilgan usuli qo’llanilgan. [20] adabiyotda yechimning mavjudligini keltirilayotganda Bols prinspidan foydalanilgan. [22] adabiyotda M.Mirsaburov elliptik tipdagi tenglamalar uchun yangi turdagi chegaraviy masalalarni o‘rgangan.

Bajarilgan ishning asosiy natijalari. Singulyar integral tenglamalarning o’ng tomonlari nofredgolm operatorlaridan iborat ekanligini hisobga olib ular maxsus almashtirishlar yordamida Viner-Xopf integral tenglamasiga olib kelindi.

Viner-Xopf integral tenglamasi ham singulyar integral tenglamalar sinfiga kiradi va u Fur’e almashtirish yordamida tasvirga nisbatan analitik funktsiyalar nazariyasining Riman masalasiga olib kelinadi va kvadraturalarida yechiladi.

Xulosa va takliflarning qisqacha umumlashgan ifodasi. Aralash sohaning elliptik qismida singulyar koeffitsientli Gellerstedt tenglamasi uchun asosiy chegaraviy masalalar Direxle va shakli o’zgargan Xolmgren masalalarining yechimlarini beruvchi yechimlarining integral formasidan foydalanilgan.

I-BOB. ELLIPTIK TIPDAGI TENGLAMALARNING BIR SINFI UCHUN DIRIXLE VA SHAKLI O'ZGARGAN XOLMGREN MASALALARI

1.1-§. Dirixle masalasining qo'yilishi va yechimning yagonaligi.

$z = x + iy$ kompleks tekisligining yuqori $\operatorname{Im} z > 0$ yarim tekisligida

$$y^m u_{xx} + u_{yy} + (\beta_0 / y) u_y = 0, \tag{1.1}$$

tenglamani o'rganamiz, bu yerda m, β_0- o'zgarmas sonlar bo'lib, $m > 0$, $-(m/2) < \beta_0 < 1$ shartlarni qanoatlantiradi.

Ω – chekli bir bog'lamli soha bo'lib, uchlari $A(-a,0)$ va $B(a,0)$, nuqtalarda bo'lgan va $y > 0$ yarim tekislikda yotuvchi silliq Jordan chizig'i $\Gamma : x = x(s), y = y(s)$ bu yerda s parametr $\overset{\cup}{MB}$ yoy uzunligi, hamda $y = 0$ o'qining AB kesmasi bilan chegaralangan bo'lsin. (1.1) tenglama uchun Ω sohada Dirixle va shakli o'zgargan Xolmgren[22] masalalarini o'rganamiz.

Dirixle masalasi. Ω sohada (1.1) tenglamaning ushbu

$$u\big|_{\Gamma} = \varphi(s) \quad 0 \le s \le l; \qquad u(x,0) = \tau(x),\ x \in I\ , \tag{1.2}$$

shartlarni qanoatlantiruvchi regulyar yechimi $u(x,y) \in C(\overline{\Omega}) \cap C^2(\Omega)$ topilsin, bu yerda S - Γ chiziqning MB yoyi uzunligi, l–butun Γ chiziq yoyi uzunligi: $\varphi(s)$ va $\tau(x)$–berilgan uzluksiz funksiyalar, shu bilan birga $\tau(-a) = \varphi(l), \tau(a) = \varphi(0)$, $I = (-a, a), y = 0$ o'qining intervali.

Ekstremum prinsipi: Ω sohada (1.1) tenglamaning $u(x,y)$ regulyar yechimi hech bir $(x,y) \in \Omega$ nuqtada o'zining musbat maksimumiga va manfiy minimumiga erishmaydi.

Isboti. Ushbu

$$\upsilon(x,y) = u(x,y) / A(y) \tag{1.3}$$

funksiyani qaraymiz, bu yerda

$$A(y) = e^{d^{1-\beta_0}} - \varepsilon e^{y^{1-\beta_0}},$$

$d-$bu Ω soha diametri, $0<\varepsilon<1$. Bevosita hisoblashlar yordamida

$$E(u) = A(y)E_1(\upsilon)$$

tenglikni to‘g‘riligiga ishonch hosil qilish qiyin emas, bu yerda

$$E_1(\upsilon) = y^m \upsilon_{xx} + \upsilon_{yy} + \frac{1}{y}\left(\beta_0 + 2yA_y\right)\upsilon_y + \frac{1}{A}\left(\frac{\beta_0}{y}A_y + A_{yy}\right)\upsilon, \qquad (1.4)$$

$$A_y = -\varepsilon\left(1-\beta_0\right)e^{y^{1-\beta_0}}y^{-\beta_0},$$

$$A_{yy} = -\varepsilon\left(1-\beta_0\right)^2 e^{y^{1-\beta_0}}y^{-2\beta_0} + \varepsilon\beta_0\left(1-\beta_0\right)e^{y^{1-\beta_0}}y^{-\beta_0-1},$$

$$\frac{1}{A}\left(\frac{\beta_0}{y}A_y + A_{yy}\right) = -\frac{\varepsilon}{A}\left(1-\beta_0\right)^2 e^{y^{1-\beta_0}}y^{-2\beta_0} < 0 \qquad (1.5)$$

(1.5) tengsizlikka asosan, (1.4) tenglama yechimi $\vartheta(x,y)$ Ω soha ichidagi hech bir (x_0, y_0) nuqtada o‘zining musbat maksimumiga erishmaydi. Haqiqatdan ham, teskarisini faraz qilaylik, (x_0, y_0) nuqtada $\vartheta(x,y)$ funksiya o‘zining musbat maksimumiga erishsin, u holda bu nuqtada

$$\frac{\partial\vartheta(x_0,y_0)}{\partial x} = 0, \frac{\partial\vartheta(x_0,y_0)}{\partial y} = 0, \quad \frac{\partial^2\vartheta(x_0,y_0)}{\partial x^2} \le 0, \quad \frac{\partial^2\vartheta(x_0,y_0)}{\partial y^2} \le 0$$

bo‘lgani uchun, (1.4) dan $E_1(\vartheta) < 0$. Bu esa $E_1(\vartheta) = 0$ tenglikka ziddir. Aynan shu mulohazalarni takrorlab, $\vartheta(x,y)$ funksiya Ω sohaning hech bir ichki nuqtasida o‘zining manfiy minimumiga erishmasligini ko‘rsatish mumkin.

Shunday qilib, (1.3) ga asosan, (1.1) tenglamaning regulyar yechimi $u(x,y)$ o‘zining musbat maksimumi va manfiy minimumini Ω sohaning ichki nuqtalarida qabul qilmaydi.

1.1-teorema. Ω sohada (1.1) tenglama uchun qo‘yilgan Dirixle masalasining yechimi mavjud bo‘lsa, u yagonadir.

Isboti. Faraz qilaylik, qo‘yilgan masala ikkita u_1 va u_2 yechimlarga ega bo‘lsin, u holda berilgan tenglama va chegaraviy shartlar chiziqli bo‘lgani uchun $w = u_1 - u_2$ funksiya (1.1) tenglamani va bir jinsli

$$w\big|_{\Gamma} = (u_1 - u_2)\big|_{\Gamma} = 0 \ ; \ w(x,0) = u_1(x,0) - u_2(x,0) = 0 \qquad (1.6)$$

shartlarni qanoatlantiradi. Ekstremum prinsipiga ko‘ra, $\overline{\Omega}$ sohada uzluksiz $w(x,y)$ funksiya o‘zining ekstremumlarini faqat $\partial\overline{\Omega} = \Gamma \cup AB$ da qabul qiladi, ya’ni

$$0 = \min_{(x,y)\in\partial\overline{\Omega}} w(x,y) \le \underset{(x,y)\in\overline{\Omega}}{w}(x,y) \le \max_{(x,y)\in\partial\overline{\Omega}} w(x,y) = 0 .$$

Bundan esa $w(x,y) \equiv 0$, $(x,y) \in \overline{\Omega}$. 1.1-teorema isbot bo‘ldi.

1.2-§. Shakli o’zgargan Xolmgren masalasining qo’yilishi va yechimning yagonaligi

Shakli o‘zgargan Xolmgren masalasi. Ω sohada (1.1) tenglamaning ushbu

$$u\big|_{\Gamma} = \varphi(s) \quad 0 \le s \le l \ ; \quad \lim_{y\to+0} y^{\beta_0} \frac{\partial u}{\partial y} = \nu(x) \ , \ x \in I \ , \qquad (1.7)$$

shartlarni qanoatlantiruvchi yechimi $u(x,y) \in C(\overline{\Omega}) \cap C^2(\Omega)$ topilsin, bu yerda $\varphi(s)$ funksiya $0 \le s \le l$ da uzluksiz, $\nu(x)$ funksiya esa I intervalda uzluksiz bo‘lib, bu intervalning chegaraviy nuqtalarida $1-2\beta$ dan kichik tartibda cheksizlikka intilishi mumkin.

1.1-lemma. Agar Ω sohada $u(x,y)$ funksiya:

1) $u(x,y)\in C(\overline{\Omega})\cap C^2(\Omega)$ $E(u)\geq 0(\leq 0)$ shartlarni qanoatlantirsa va Ω sohada o'zining eng katta musbat (eng kichik manfiy) qiymatini $(x_0,0)$, $x_0\in I$ nuqtada qabul qilsa,

2) $u(x,y)$ ning Γ chiziqdagi qiymati $u(x_0,0)$ qiymatdan kichik (katta) bo'lsa, u holda

$$\lim_{y\to+0} y^{\beta_0}\frac{\partial u}{\partial y}<0, \quad (>0), \tag{1.8}$$

tengsizlik, (bu limitni mavjud bo'lishi sharti bilan) o'rinlidir.

Isboti. Musbat maksimum holini o'rganib chiqamiz. Bevosita hosila ta'rifidan,

$$\lim_{y\to+0} y^{\beta_0}\frac{\partial u(x_0,y)}{\partial y}>0$$

tengsizlikning bajarilishi mumkin emas. Faraz qilaylik,

$$\lim_{y\to+0} y^{\beta_0}\frac{\partial u(x_0,y)}{\partial y}=0 \tag{1.9}$$

bo'lsin. d orqali Ω sohaning diametrini belgilaymiz. Umumiylikni buzmasdan, $u(x_0,0)=1$ deb olishimiz mumkin. Lemma shartiga ko'ra, $\max\limits_{(x,y)\in\Gamma} u(x,y)\leq 1-\varepsilon$ bu yerda $0<\varepsilon<1$. Ushbu

$$\upsilon(x,y)=u(x,y)/A(y) \ ,$$

funksiyani kiritamiz, bu yerda

$$A(y)=e^{d^{1-\beta_0}}-\varepsilon^{y^{1-\beta_0}}.$$

Γ chiziqda

$$\upsilon(x,y)\leq\frac{1-\varepsilon}{e^{d^{1-\beta_0}}-\varepsilon e^{y^{1-\beta_0}}}\leq\frac{1-\varepsilon}{e^{d^{1-\beta_0}}(1-\varepsilon)}<\frac{1}{e^{d^{1-\beta_0}}-\varepsilon},$$

$[-1,1]$ kesmada esa

$$\upsilon(x,0)\le\frac{1}{e^{d^{1-\beta_0}}-\varepsilon}\ ,\ \ \upsilon(x_0,0)=\frac{1}{e^{d^{1-\beta_0}}-\varepsilon}\ .$$

Ushbu

$$E(u)=A(y)E_1(\upsilon)\ , \tag{1.10}$$

tenglikning to‘g‘riligiga ishonch hosil qilish qiyin emas, bu yerda $E_1(\vartheta)$ (1.4) tenglik bilan aniqlanuvchi operator. Lemma shartiga ko‘ra, (1.10) tenglikdan $E_1(\vartheta)\ge 0$ tengsizlik kelib chiqadi. Bundan tashqari, (1.9) tenglikka ko‘ra,

$$\lim_{y\to+0} y^{\beta_0}\frac{\partial\upsilon(x_0,y)}{\partial y}=\frac{\varepsilon(1-\beta_0)}{\left(e^{d^{1-\beta_0}}-\varepsilon\right)^2}>0\ .$$

Bundan esa $(x_0,0)$ nuqtaning shunday kichik atrofi borki $(y>0)$, bu atrofda $\dfrac{\partial\upsilon}{\partial y}>0$ ya’ni $\upsilon(x,y)$ funksiya bu atrofda $x=x_0$ chizig‘ida o‘suvchi bo‘ladi va o‘zining eng katta qiymatini soha ichida qabul qiladi, buning esa (1.11) tengsizlikka ko‘ra bo‘lishi mumkin emas. Yuqoridagi mulohazalarni takrorlab, manfiy minimum holini ham o‘rganish mumkin. 1.1-lemma isbot bo‘ldi.

1.2-teorema. Shakli o‘zgargan Xolmgren masalasi (1.7) shartlarga mos bir jinsli shartlarda faqat aynan nolga teng bo‘lgan yechimga ega.

Isboti. (1.1) tenglama uchun ekstremum prinsipi va 1.1-lemmaga ko‘ra, shakli o‘zgargan bir jinsli Xolmgren masalasining $u(x,y)\in C(\overline{\Omega})$ yechimi o‘zining ekstremumlarini Γ da qabul qiladi, ya’ni

$$0=\min_{(x,y)\in\Gamma}u(x,y)\le u(x,y)\le\max_{(x,y)\in\Gamma}u(x,y)=0\,.$$

Bu yerdan

$$u(x,y)\equiv 0\ ,\ (x,y)\in\overline{\Omega}\,.$$

1.2-teorema isbot bo‘ldi.

1.3-§. Tenglamaning fundamental yechimlari

(1.1) tenglamaning yechimini

$$u=\left(r_1^2\right)^{-\beta}\omega(\sigma) \qquad (1.12)$$

ko'rinishda izlaymiz, bu yerda

$$\sigma=\frac{r^2}{r_1^2}\quad , \quad \beta=\frac{m+2\beta_0}{2(m+2)},$$

$$\left.\begin{matrix} r^2 \\ r_1^2 \end{matrix}\right\}=\left(x-x_0\right)^2+\frac{4}{(m+2)^2}\left(y^{\frac{m+2}{2}}\mp y_0^{\frac{m+2}{2}}\right)^2 \quad ,$$

$\omega(\sigma)$ -noma'lum funksiya (1.12) ni (1.1) tenglamaga qo'yib va $\omega(\sigma)$ ga nisbatan ba'zi bir hisoblashlarni bajarib, ushbu

$$E\left[\left(r_1^2\right)^{-\beta}\omega(\sigma)\right]=\sigma(1-\sigma)\frac{d^2\omega}{d\sigma^2}+\left[1-(1+2\beta)\sigma\right]\frac{d\omega}{d\sigma}-\beta^2\omega=0$$

Gauss tenglamasiga kelamiz.

Bu tenglama $\sigma=1$ nuqta atrofida quyidagi ikkita chiziqli erkli yechimga ega:

$$\omega_1(\sigma)=F(\beta,\beta,2\beta;1-\sigma)$$
$$\omega_2(\sigma)=(1-\sigma)^{1-2\beta}F(1-\beta,1-\beta,\,2-2\beta;1-\sigma) \qquad (1.13)$$

(1.13) ni (1.12) tenglikka qo'yib, ushbu:

$$q_1(x, y; x_0, y_0) = k_1 (r_1^2)^{-\beta} F(\beta, \beta, 2\beta; 1-\sigma)$$

$$q_2(x, y, x_0, y_0) = k_2 \left(\frac{4}{m+2}\right)^{4\beta-2} (r_1^2)^{-\beta} (1-\sigma)^{1-2\beta} \times F(1-\beta, 1-\beta, 2-2\beta; 1-\sigma)$$

(1.14)

yechimlarga kelamiz, bu yerda

$$k_1 = \frac{1}{4\pi}\left(\frac{4}{m+2}\right)^{2\beta} \frac{\Gamma^2(\beta)}{\Gamma(2\beta)}, \quad k_2 = \frac{1}{4\pi}\left(\frac{4}{m+2}\right)^{2-2\beta} \frac{\Gamma^2(1-\beta)}{\Gamma(2-2\beta)} \qquad (1.15)$$

I bob yuzasidan xulosa.

Ushbu bobda $z = x + iy$ kompleks tekisligining yuqori $\operatorname{Im} z > 0$ yarim tekisligida

$$y^m u_{xx} + u_{yy} + (\beta_0 / y) u_y = 0,$$

tenglama o‘rganilgan, bu yerda m, β_0- o‘zgarmas sonlar bo‘lib, $m > 0$, $-(m/2) < \beta_0 < 1$ shartlarni qanoatlantiradi.

Ω – chekli bir bog‘lamli soha bo‘lib, uchlari $A(-a,0)$ va $B(a,0)$, nuqtalarda bo‘lgan va $y > 0$ yarim tekislikda yotuvchi silliq Jordan chizig‘i $\Gamma : x = x(s), y = y(s)$, bu yerda s parametr AB yoy uzunligi, hamda $y = 0$ o‘qining AB kesmasi bilan chegaralangan bo‘lsin. Yuqoridagi tenglama uchun Ω sohada Dirixle va shakli o‘zgargan Xolmgren masalalarini o‘rganilgan. Qo’yilgan masala yechimi yagonaligi ekstremum prinsipi yordamida isbotlangan. Xolmgren masalasi yechimini isbotlashda asosiy lemma keltirilgan va u isbotlangan. Masala yechimi yagonaligi isbotlangan.

II-BOB. ARALASH SOHA XARAKTERISTIKASIDA LOKAL VA NOLOKAL SHARTLI MASALALAR.

2.1-§. Aralash tipdagi tenglama uchun shakli o’zgargan Koshi masalasi.

Buziluvchan giperbolik va aralash tipdagi tenglamalar nazariyasining rivojlanish tarixi G. Darbu[19], F. Trikomi YE. Xolmgren[22] va S.Gellerstedtlarning [20,21] mos ravishda 1894, 1923, 1927 va 1935 yillarda chop etilgan fundamental ishlari bilan bog‘liq.

Aralash tipdagi tenglamalar uchun chegaraviy masalalar bo‘yicha dastlabki fundamental tadqiqotlar 1920 yili italyan matematigi Franchesko Trikomi tomonidan olib borilgan. Bu ishdan keyin aralash tipdagi tenglamalar uchun chegaraviy masalalar nazariyasi asosan uchta yo‘nalish bo‘yicha rivojlana boshladi: birinchi yo‘nalish - Trikomi masalasini umumiyroq aralash tipdagi tenglamalar uchun o‘rganish bo‘lib, ularga S. Gellerstedt; A.V.Bitsadze; K.I.Babenko; L. I. Karol; S.P. Pulkin va boshqalarning ishlari bag‘ishlangan; ikkinchi yo‘nalish - Trikomi masalasining har xil modifikatsiyalariga bag‘ishlangan; uchinchi yo‘nalish esa aralash tipdagi tenglamalar uchun spektral masalalarni tadqiq etishdan iborat.

Aralash tipdagi tenglamalar uchun chegaraviy masalalarning rivojlanishida shved matematigi Sven Gellerstedt tomonidan ishlab chiqilgan potensiallar nazariyasi muhim o‘rin egallaydi. S. Gellerstedt yaratgan usul yordamida buziluvchan elliptik tipdagi tenglama uchun Dirixle va Xolmgren masalalarining yechimini qulay integral shaklda yozish mumkin va aralash tipdagi tenglama uchun chegaraviy masalani tadqiq etish juda qulay bo‘ladi. Shuningdek aralash tipdagi tenglama uchun chegaraviy masalalar nazariyasining rivojlanishiga A.V.Bitsadzening ekstremum prinsipi katta turtki bergan. Bu prinsip masala yechimining yagonaligini isbotlashda juda keng qo‘llaniladi. Aralash tipdagi tenglamalar uchun chegaraviy masalalar nazariyasining rivojlanishida muhim o‘rin tutuvchi yana bir natijalardan biri bu S.G. Mixlin[12] tomonidan ishlab chiqilgan Karlemanning Trikomi singulyar integral tenglamasini regulyarlashtirish usuli hisoblanadi va bu usul F.Trikomi integral tenglamasini yechishda qo‘llanilgan.

Quyidagi singulyar koeffitsiyentli buziluvchan giperbolik tipdagi tenglamani $z = x + iy$, $\operatorname{Im} z < 0$ kompleks yarim tekislikda o‘rganamiz

$$-(-y)^m u_{xx} + u_{yy} + \alpha_0 (-y)^{m/2-1} u_x + \beta_0 y^{-1} u_y = 0, \qquad (2.1)$$

bu yerda m, α_0 va β_0 - haqiqiy sonlar hamda ular ushbu

$$-m/2 \le \beta_0 \le (m+4)/2, \; |\alpha_0| \le (m+2)/2,$$

shartlarni qanoatlantiradi D_0 soha $z = x + iy$ komplekis tekislikning bir bog‘lamli sohasi bo‘lib, u (2.1) tenglamaning

$$AC: \quad x - \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = -1,$$

$$BC: \quad x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 1$$

xarakteristikalari hamda $y = 0$ o‘qining AB kesmasi bilan chegaralangan bir bog‘lamli sohasi bo‘lsin.

(2.1) tenglama shu narsa bilan e’tiborliki birinchidan bu tenglamaning kichik hadlari oldidagi koeffitsiyentlari singulyar maxsuslikka ega, ikkinchidan bu yerda

$$K(y)h(x,y)u_{xx} + u_{yy} + a(x,y)u_x + b(x,y)u_y + c(x,y)u = f(x,y) \qquad (2.2)$$

buziluvchan umumiy giperbolik tipdagi tenglama uchun Koshi masalasini normal yechilishining

$$\lim_{y \to -0} \frac{ya(x,y)}{\sqrt{-K(y)}} = 0, \qquad (2.3)$$

Protter sharti buziladi, bu yerda $h(x,y) > 0$, $K(0) \equiv 0$, $K(y) < 0$, $y < 0$ da. (2.3) shart bajarilmasligiga qaramasdan, agar $|\alpha_0| \le m/2$, $\beta_0 = 0$ bo‘lsa (2.1) tenglama uchun Koshi masalasi korrekt qo‘yilgan.

Bundan (2.1) tenglama uchun Koshi masalasini normal yechilishida (2.3) shart zaruriy shart emasligi kelib chiqadi. Endi (2.1) tenglamada $\beta_0 = 0$, $\alpha_0 = -m/2$ bo‘lsin:

$$-(-y)^m u_{xx} + u_{yy} - (m/2)(-y)^{m/2-1} u_x = 0, \qquad (2.4)$$

(2.4) tenglama uchun Darbu masalasini ta’riflaymiz.

Darbuning ikkinchi masalasi: D_0 sohada (2.4) tenglamaning ushbu

$$u_y(x,0) = \nu(x)\ ,\ x \in I\ :\ u\big|_{BC} = \psi(x)\ ,\ x \in [0,1], \qquad (2.5)$$

shartlarni qanoatlantiruvchi regulyar $u(x,y) \in C(\overline{D}_0) \cap C^2(D_0)$ yechimi topilsin, bu yerda $\nu(x) \in C^2(I)$, $\psi(x) \in C^1(\bar{I}) \cap C^2(I)$, $I = (-1,1)$- $y = 0$ o‘qining intervali.

2.1-teorema. Darbuning ikkinchi masalasiga mos bir jinsli masala cheksiz ko‘p chiziqli bog‘liq bo‘lmagan yechimlarga ega, bir jinsli bo‘lmagan masala esa faqat va faqat,

$$\nu(2x-1) = ((m+2)/2)^{\beta}(1-x)^{\beta}\psi'(x)\ ,\ x \in (0,1),$$

shart bo‘lgandagina yechimga ega bo‘ladi, bu yerda $\beta = m/(m+2)$.

Bir jinsli Darbuning ikkinchi masalasining barcha notrivial yechimlar

$$u(x,y) = \tau_0\left(x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}}\right) - \tau_0(1)\ ,$$

formula bilan beriladi, bu yerda $\tau_0(x) \in C(\bar{I}) \cap C^2(I)$ sinfdagi ixtiyoriy funksiya. Endi (2.4) tenglama uchun (2.5) Darbu shartlarini ushbu

$$u_y(x,0) = \nu(x)\ ,\ x \in I\ ;\ u\big|_{AC} = \psi(x),\ x \in [-1,0] \qquad (2.6)$$

shaklda beramiz.

2.2-teorema. (2.4), (2.6) masala yagona yechimga ega.

2.1-teorema va 2.2-teoremalardan ushbu xulosa kelib chiqadi: qat’iy giperbolik tenglamalar uchun qo‘yilgan Koshi masalasining korrektligidan Darbu masalasining korrektligi kelib chiqadi, buziluvchan giperbolik tenglamalarda esa umuman olganda Koshi masalasi korrektligidan Darbu masalasining korrektligi kelib chiqmaydi. Buning ustiga (2.4) buziluvchan giperbolik tenglama uchun umuman olganda xarakteristikalar, chegaraviy shartlarning ularda qo‘yilishi ma’nosida teng huquqli emas.

(2.1) tenglamada $\alpha_0 = 0$ bo‘lsin:

$$-(-y)^m u_{xx} + u_{yy} + (\beta_0/y)u_y = 0 \tag{2.7}$$

bu tenglama juda ko‘p matematiklar tomonidan o‘rganilgan. Umuman olganda, (2.7) tenglama uchun oddiy Koshi masalasi korrekt bo‘lmasligi mumkin. A. V. Bitsadze (2.7) tenglama uchun boshlang‘ich shartlari bir jinsli bo‘lgan:

$$u(x,0) = 0, \ x \in \overline{I} \ ; \quad \lim_{y \to -0} \frac{\partial u}{\partial y} = 0, \ x \in I \ ;$$

Koshi masalasi $\beta_0 = -m/2$ bo‘lganda Ushbu

$$u_0(x,y) = \tau_0\left[x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}}\right] - \tau_0\left[x - \frac{2}{m+2}(-y)^{\frac{m+2}{2}}\right],$$

ko‘rinishdagi notrival yechimlarga ega ekanligini ko‘rsatgan, bu yerda $\tau_0(x)$ ikki marta uzluksiz hosilaga ega bo‘lgan ixtiyoriy funksiya. Shu holatdan kelib chiqib A. V. Bitsadze boshlang‘ich shartlari

$$u(x,0) = \tau(x), \ x \in \overline{I} \ ; \quad \lim_{y \to -0} (-y)^{\beta_0} \frac{\partial u}{\partial y} = \nu(x), \ x \in I \ , \tag{2.8}$$

ko‘rinishda bo‘lgan shakli o‘zgargan Koshi masalasini o‘rgangan va uni korrekt ekanligini ko‘rsatgan, bu yerda $-(m/2) \le \beta_0 < 1$.

Agar $\beta_0 \ge 1$ bo‘lsa, (2.7) tenglamaning yechimlari buzilish chizig‘i atrofida chegaralangan bo‘lmaydi. Haqiqatdan ham ushbu

$$u_0(x,y) = \begin{cases} (-y)^{1-\beta_0} \ , & \text{агар} \ \ \beta_0 \ne 1 \ \ \text{булса}, \\ \ln(-y) \ , & \text{агар} \ \ \beta_0 = 1 \ \ \text{булса} \end{cases}$$

xususiy yechimlar yuqoridagi fikrimizni tasdiqlaydi.

$\beta_0 > 1$ bo‘lganda Koshi masalasi korrekt bo‘lishi uchun boshlang‘ich shartlar

$$\lim_{y\to -0}(-y)^{\beta_0 - 1}u(x,y) = \tau(x); \quad \lim_{y\to -0}(-y)^{2-\beta_0}\frac{\partial}{\partial y}\left((-y)^{\beta_0 - 1}u(x,y)\right)$$

ko‘rinishda bo‘lishi kerak; $\beta_0 = 1$ bo‘lganda esa Koshi masalasi korrekt bo‘lishi uchun boshlang‘ich shartlar

$$\lim_{y\to -0}\frac{u(x,y)}{\ln(-y)^{(m+2)/2}} = \tau(x),$$

$$\lim_{y\to -0}(-y)\ln^2(-y)^{(m+2)/2}\frac{\partial}{\partial y}\left[\frac{u(x,y) - A(x,y)}{\ln(-y)^{(m+2)/2}}\right] = \nu(x),$$

ko‘rinishda bo‘lishi kerak, bu yerda $A(x,y)-$aniq ko‘rinishga ega bo‘lgan maxsus kiritilgan funksiya.

Shunday qilib, (2.1) tenglama yechiminning tuzilishi va differensial xossalari uning kichik hadlari oldidagi koeffitsiyentlar α_0 va β_0 ga bog‘liqdir. (2.1) tenglama uchun masalalar α_0 va β_0 parametrik tekislikda $P\ (\alpha_0, \beta_0)$ nuqtaning o‘zgarishiga qarab qo‘yiladi.

$y > 0$ yarim tekislikda

$$y^m u_{xx} + u_{yy} + (\beta_0 / y)u_y = 0 \tag{2.9}$$

tenglamani o‘rganamiz.

(2.9) tenglama shu bilan xarakterliki uning uchun oddiy N masalasi korrekt emas. Haqiqatdan ham Ω_0 - yuqori $y > 0$ yarim tekislikda yotuvchi va uchlar $A(-1,0)$, $B(1,0)$ nuqtada bo‘lgan (2.9) tenglamaning normal chizig‘i $\sigma_0 : x^2 + 4(m+2)^{-2}y^{m+2} = 1$ chizig‘i hamda $y = 0$ o‘qining AB kesmasi bilan chegaralangan bir bog‘lamli bo‘lsin. Ushbu masalani ta’riflaymiz.

N **masalasi**. Ω_0 sohada (2.9) tenglamaning ushbu

$$u\big|_{\sigma_0} = \varphi_0(x,y)\ ,\ (x,y)\in\sigma_0\ ,$$

$$\frac{\partial u}{\partial y}\Big|_{y=0} = \nu(x)\ ,\ x\in I=(-1,1)\ ,$$

shartlarni qanoatlantiruvchi regulyar yechimi $u(x,y)\in C(\overline{\Omega}_0)\cap C^2(\Omega_0)$ topilsin. Bevosita tekshirish yordamida ko‘rsatish mumkinki ushbu

$$u(x,y) = \frac{1 - x^2 - \dfrac{4}{(m+2)^2}\,y^{m+2}}{\left(1 - x + \dfrac{2}{m+2}\,y^{\frac{m+2}{2}}\right)^2 + \left(1 + x + \dfrac{2}{m+2}\,y^{\frac{m+2}{2}}\right)^2}$$

funksiya bir jinsli N masalaning notrivial yechimi bo‘ladi, ya’ni (2.9) tenglama uchun N masalasi korrekt emas. Shu munosabat bilan A.V.Bitsadze (2.9) tenglama uchun ushbu shakli o‘zgargan N masalasini o‘rgangan: Ω_0 sohada (2.9) tenglamaning ushbu

$$u\big|_{\sigma_0} = \varphi_0(x,y)\ ,\quad (x,y)\in\sigma_0\,,$$

$$\lim_{y\to+0} y^{\beta_0}\frac{\partial u}{\partial y} = \nu(x),\quad x\in I=(-1,1)$$

shartlarni qanoatlantiruvchi regulyar yechimi topilsin.

Shakli o‘zgargan N masalasi korrekt qo‘yilgan. Ushbu qo‘llanmada asosan singulyar koeffitsiyentli

$$(\mathrm{sign}y)\,|y|^m u_{xx} + u_{yy} + \alpha_0|y|^{m/2-1}u_x + (\beta_0/y)u_y = 0 \qquad (2.10)$$

tenglama ham o‘rganilgan. (2.10) tenglama $z = x + iy$, kompleks tekisligining $\mathrm{Im}\,z > 0$ yuqori yarim tekisligida uchlari $A(-1,0)$ va $B(1,0)$ nuqtalarda va yuqori yarim tekislikda joylashgan $\Gamma:\ y = f(x)$ chizig‘i bilan, $\mathrm{Im}\,z < 0$ pastki yarim tekislikda esa (2.10) tenglamaning AC va BC xarakteristikalari bilan chegaralangan bir bog‘lamli D sohada o‘rganildi.

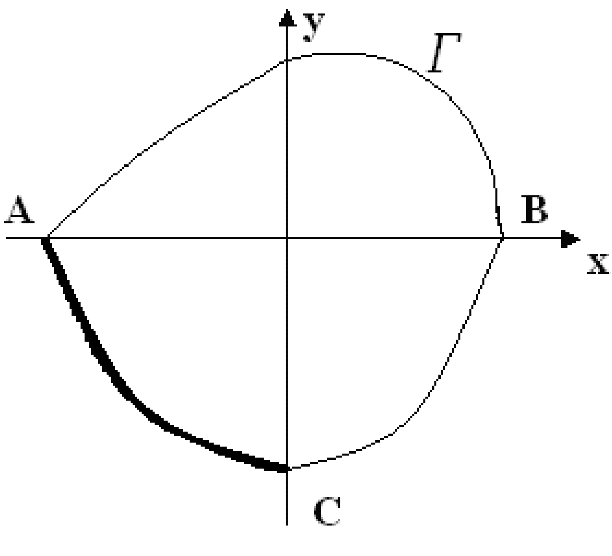
Asosiy e’tibor (2.6) tenglama uchun $D^- = D \cap \{y<0\}$ sohada shakli o‘zgargan Koshi masalasini o‘rganishga, $D^+ = D \cap \{y>0\}$ sohada Dirixle va shakli o‘zgargan N masalasini, aralash D sohada esa Trikomi masalasini hamda Frankl turidagi nolokal masalalarni o‘rganishga qaratilgan.

Ushbu singulyar koeffitsiyentli

$$(signy)|y|^m u_{xx} + u_{yy} + (\beta_0 / y)u_y = 0 \qquad (2.11)$$

tenglama qaraymiz, bu yerda $m>0, -\frac{m}{2} \le \beta_0 \le 1$.

(2.11) tenglama $z = x + iy$, kompleks tekisligining $Imz > 0$ yuqori yarim tekisligida uchlari $A(-1,0)$ va $B(1,0)$ nuqtalarda va yuqori yarim tekislikda joylashgan $\Gamma : y = f(x)$ chizig‘i bilan, $Imz < 0$ pastki yarim tekislikda esa (2.11) tenglamaning AC va BC xarakteristikalari bilan chegaralangan bir bog‘lamli D sohada o‘rganiladi.



2.1-chizma

(2.11) tenglama uchun F. Trikomi, V.I. Jegalov, A. M. Naxushev, masalalarini shartlarini barchasini o‘zida birlashtirib yaxlit bir masala sifatida ta’riflangan masalaning korrekt ekanligi isbotlash maqsad qilib qo‘yilgan. Masalaning ta’rifi

β_0 parametrni o‘zgarishga qarab qo‘yiladi. D^+ va D^- orqali D sohaning mos ravishda $y > 0$ va $y < 0$ yarim tekislikda yotuvchi qismlarini belgilaymiz, C_0 va C_1 orqali esa $E(c,0)$ nuqtadan chiquvchi xarakteristikalarning mos ravishda AC va BC

xarakteristikalar bilan kesishish nuqtasini belgilaymiz, bu yerda $c \in I = (-1,1) - y = 0$ o‘qining intervali.

2.2-§. $\beta_0 \in (-m/2,1)$. bo‘lgan hol.

F. Trikomi masalasi chegaraviy shartida AC va BC xarakteristikalar teng xuquqli ishtirok etmaydi, A. M. Naxushevning siljishli masalasida chegaraviy shartlarda xarakteristikalar teng xuquqli bo‘lib ularning barcha nuqtalarida chegaraviy shartlar berilgan.

Ushbu masalada AC va BC xarakteristikalarning mos ravishda C_0C va C_1C qismlari siljishli chegaraviy shartlardan ozod etiladi va bu yetishmaydigan A. M. Naxushev sharti AB xarakteristikada F.I.Frankl sharti bilan ekvivalent almashtirilgan masalaning korrekt ekanligi o‘rganiladi.

BS **- masalasi.** D sohada ushbu shartlarni qanoatlantiruvchi $u(x,y) \in C(\overline{D})$ funksiya topilsin.

1. $u(x,y) \in C^2(D^+)$ va bu sohada (2.11) tenglamani qanoatlantiradi.

2. D-sohada $u(x,y)$ funksiya (2.11) tenglamaning R_1 sinfga tegishli yechimi.

3. AB intervalda ushbu ulanish shartlari bajariladi

$$\lim_{y \to -0} (-y)^{\beta_0} \frac{\partial u}{\partial y} = \lim_{y \to +0} y^{\beta_0} \frac{\partial u}{\partial y} \quad x \in I / \{c\},$$

shu bilan birga bu limitlar $x = \pm 1,\ x = c$ nuqtalarda $1 - 2\beta$ dan katta bo‘lmagan tartibda cheksizlikka aylanishi mumkin;

4. Ushbu chegaraviy shartlar bajariladi

$$u(x,y)\Big|_{\sigma_0} = \varphi(x), \quad -1 \le x \le 1,$$
$$u[\theta_0(x)] + \mu u[\theta_1(p(x))] = \psi(x), \quad -1 \le x \le c,$$
$$u(x,0) - \mu u(p(x),0) = f(x), \quad -1 \le x \le c,$$

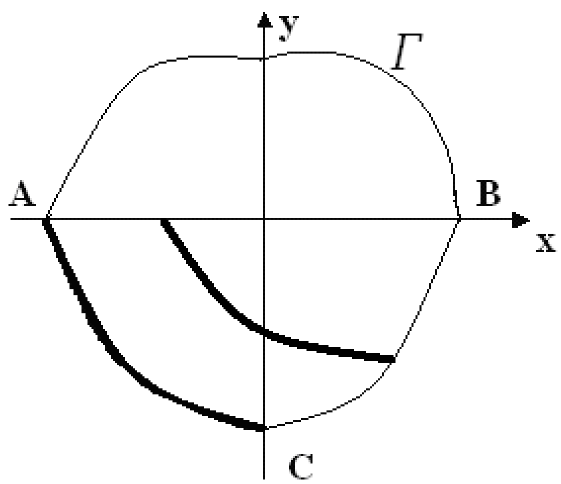
bu yerda $\theta(x_0)$ va $\theta_1(x_0)$ - mos ravishda AC va BC xarakteristikalarning $(x_0,0)$ $x_0 \in I$ nuqtadan chiqqan xarakteristikalar bilan kesishish nuqtasining affiksidir. $\varphi(x), \psi(x), f(x)$- funksiyalar o‘zining berilish sohasining yopig‘ida uzluksiz funksiyalardir.

2.3-§. Bitsadze-Samarskiy masalasi.

Singulyar koeffitsiyentli

$$(signy)|y|^m u_{xx} + u_{yy} + (\beta_0 / y)u_y = 0$$

aralash tipdagi tenglamani $z = x + iy$, kompleks tekisligining $l\,m\,z > 0$ yuqori yarim tekisligida uchlari $A(-1,0)$ va $B(1,0)$ nuqtalarda va yuqori yarim tekislikda joylashgan $\Gamma: y = f(x)$ chizig‘i bilan, $l\,m\,z < 0$ pastki yarim tekislikda esa (2.11) tenglamaning AC va BC xarakteristikalari bilan chegaralangan bir bog‘lamli D sohada o‘rganamiz.



2.2-chizma

(2.11) tenglama uchun Bitsadze-Samarskiy masalasining shartlarini parallel xarakteristikalardagi qiymatlarining kasr tartibli xosilalarini o‘zida birlashtirgan masalaning korrektligi o‘rganilgan. Ta’riflangan masalaning yagonaligi ekstremum prinsipi yordamida, mavjudligi isbotlashda singulyar integral tenglamalar, Viner-Xopf integral tenglamasi, Fredgolmning II-tur integral tenglamalar nazariyalaridan foydalanilgan.

D^+ va D^- orqali D sohaning mos ravishda $y>0$ va $y<0$ yarim tekislikda yotuvchi qismlarini belgilaymiz, C_0 va C_1 orqali esa $E(c,0)$ nuqtadan chiquvchi xarakteristikalarning mos ravishda AC va BC xarakteristikalar bilan kesishish nuqtasini belgilaymiz, bu yerda $c \in I = (-1,1) - y = 0$ o‘qining intervali.

$q(x)$ orqali $[c,1]$ kesmani $[-1,c]$ kesmaga akslantiruvchi funksiyani kiritamiz. Bu yerda $q'(x)<0,\ q(1)=-1, q(c)=c$. Bu xossalarga ega bo‘lgan funksiya sifatida ushbu chiziqli funksiyani keltiramiz $q(x)=\rho-kx,$ bu yerda $k=(1+c)/(1-c), \rho = 2c/(1-c).$

1. $\beta_0 \in (-m/2,1)$**. bo‘lgan hol.**

BS **-masalasi.** D sohada ushbu shartlarni qanoatlantiruvchi $u(x,y)$ funksiya topilsin:

1. $u(x,y) \in C(\overline{D})$;

2. $u(x,y) \in C^2(D^+)$ va bu sohada (2.11) tenglamani qanoatlantiradi;

3. $u(x,y)$ funksiya D^- sohada (2.11) tenglamaning D^- sohada R_1 sinfga tegishli yechimi;

4. I intervalda ushbu ulanish sharti bajariladi

$$\lim_{y\to -0}(-y)^{\beta_0}\frac{\partial u}{\partial y} = \lim_{y\to +0} y^{\beta_0}\frac{\partial u}{\partial y}\ , \quad x \in I \setminus \{c\}\ , \qquad (2.12)$$

shu bilan birga bu limitlar $x=\pm 1, x=c$ nuqtalarda $1-2\beta$ kichik tartibdagi maxsuslikka ega bo‘lishi mumkin. Bu yerda $\beta = (m+2\beta_0)/2(m+2)$ ushbu shartlar bajariladi

5. $$u(x,y)\big|_{\sigma} = \varphi(x), \quad -1 \le x \le 1 \qquad (2.13)$$

$$a_0(x)D^{1-\beta}_{-1,x}u\left[\theta(q(x))\right]+b_0(x)D^{1-\beta}_{c,x}u\left[\theta^*(x)\right]=c_0(x),\quad x\in[c,1]\ c\le x\le 1. \tag{2.14}$$

$$u(q(x),0)=\mu u(x,0)+f(x),\quad x\in[c,1] \tag{2.15}$$

Bu yerda $D^{1-\beta}_{c,x}$, $D^{1-\beta}_{-1,x}$ – kasr tartibli differensial operatorlar $\theta_0(x),\theta_1(x)$ AC va BC xarakteristikalarni $M(x_0,0)$, $x_0\in[c,1]$ nuqtadan chiquvchi xarakteristikalar bilan kesishish nuqtasining affiksi

$$\theta(x_0)=\frac{x_0-1}{2}-i\left(\frac{m+2}{4}(1+x_0)\right)^{\frac{2}{m+2}},\ \theta^*(x_0)=\frac{x_0-c}{2}-i\left(\frac{m+2}{4}(x_0+c)\right)^{\frac{2}{m+2}}. \tag{2.16}$$

$\varphi(x)$, $\psi(x)$, $a_0(x)$, $b_0(x)$, $c_0(x)$ o‘zining aniqlanish sohasi yopig‘ida uzluksiz differensiallanuvchi funksiyalar bo‘lib ular uchun ushbu shartlar bajariladi

$$a_0^2(x)+b_0^2(x)\ne 0 \tag{2.17}$$

$\varphi(x)$ funksiya esa ushbu ko‘rinishda ifodalanadi

$$\varphi(x)=(1-x^2)\tilde{\varphi}(x) \tag{2.18}$$

bu yerda $\tilde{\varphi}(x)\in C^1(\bar{I})$, $\psi(-1)=0.$

2.3-teorema. BS -masalasi ushbu $\mu(x)<0,\ 0<\mu_0<1$ shartlar bajarilganda bir qiymatli yechimga ega.

2.4-§. Chegaraviy xarakteristikalarda lokal va nolokal shartli masalalar

BS - **masalasining qo‘yilishi.**

Ushbu

$$(signy)|y|^m u_{xx}+u_{yy}+(\beta_0/y)\,u_y=0, \tag{2.19}$$

tenglamani o‘rganamiz, bu yerda $m>0,\ -m/2<\beta_0<1.$ x, y erkli o‘zgaruvchilar tekisligida chekli bir bog‘lamli D soha bo‘lib u, $y>0$ yarim tekislikda uchlari $A=A(-1,0)$ va $B=B(1,0)$ nuqtalarda bo‘lgan $\sigma_0:\ x^2+4(m+2)^{-2}y^{m+2}=1$ normal chiziq bilan, $y<0$ yarim tekislikda esa (2.19) tenglamaning AC va BC xarakteristikalari bilan chegaralangan soha bo‘lsin.

D^+ va D^- orqali mos ravishda D sohaning, $y > 0$ va $y < 0$, yarim tekisliklarda yotgan qismini belgilaymiz. C_0 va C_1, orqali esa mos ravishda AC va BC xarakteristikalarning $E(c,0)$, nuqtadan chiquvchi xarakteristikalar bilan kesishish nuqtasini belgilaymiz, bu yerda $c \in I = (-1,1)$ - $y = 0$ o‘qining intervali.

BS **- masalasi.** D sohada ushbu shartlarni qanoatlantiruvchi $u(x,y)$, funksiya topilsin:

1. $u(x,y) \in C(\overline{D})$;
2. $u(x,y) \in C^2(D^+)$ va ushbu sohada (2.9) tenglamani qanoatlantiradi;
3. $u(x,y)$ funksiya D^- sohada umumlashgan yechim, ya'ni R_1 sinfga tegishli;
4. I intervalda ushbu ulanish sharti bajariladi

$$\lim_{y \to -0} (-y)^{\beta_0} \frac{\partial u}{\partial y} = \lim_{y \to +0} y^{\beta_0} \frac{\partial u}{\partial y} \;, \quad x \in I \setminus \{c\} \;, \tag{2.20}$$

shu bilan birga bu limitlar $x = \pm 1, x = c$ nuqtalarda $1 - 2\beta$ dan kichik tartibli maxsuslikka ega bo‘lishi mumkin, Bu yerda $\beta = (m + 2\beta_0)/2(m+2)$;

5. Ushbu shartlar bajariladi

$$u(x,y)|_{\sigma_0} = \varphi(x) \;, \quad -1 \le x \le 1 \;, \tag{2.21}$$

$$u|_{AC_0} = \psi(x) \;, \quad -1 \le x \le (c-1)/2 \;, \tag{2.22}$$

$$a_0(x)(1+x)^{\beta} D^{1-\beta}_{c,x} u\left[\theta_0(x)\right] + b_0(x)(1-x)^{\beta} D^{1-\beta}_{x,1} u\left[\theta_1(x)\right] = c_0(x), \tag{2.23}$$

bu yerda $D^{1-\beta}_{c,x}$, $D^{1-\beta}_{x,1}$ $-$operatorlar $1-\beta$ kasr tartibli Liuvill ma'nosidagi hosilalardir. $\theta_0(x)$ va $\theta_1(x)$ esa mos ravishda AC va BC xarakteristikalarning $M(x_0,0)$ nuqtadan chiquvchi xarakteristikalar bilan kesishish nuqtalarining affikslaridir, bu yerda $x_0 \in [c,1]$:

$$\theta_0(x_0) = \frac{x_0 - 1}{2} - i\left(\frac{m+2}{4}(1+x_0)\right)^{\frac{2}{m+2}}, \qquad \theta_1(x_0) = \frac{x_0 + 1}{2} - i\left(\frac{m+2}{4}(1-x_0)\right)^{\frac{2}{m+2}}. \tag{2.24}$$

Berilgan $\psi(x)$, $a_0(x)$, $b_0(x)$, $c_0(x)$ funksiyalar o‘zlarining aniqlanish sohasining yopig‘ida uzluksiz differensiallanuvchi funksiyalardir, shu bilan birga

$$a_0^2(x)+b_0^2(x)\neq 0,\ \ b_0(c)=b_0(1)=0$$

$$d(x)=a_0(x)+b_0(x)>0 \quad ,\ x\in(c,1) \tag{2.25}$$

$$d(c)+\lambda\pi\, ctg 3\alpha\pi\,(a_0(c)-b_0(c)\neq 0$$

$\varphi(x)$ funksiyani ushbu ko‘rinishda tasvirlash mumkin.

$$\varphi(x)=(1-x^2)^{1-2\beta}\tilde{\varphi}(x) \tag{2.26}$$

bunda $\tilde{\varphi}(x)\in C^1(\bar{I})$, $\psi(-1)=0$.

Shuni ta’kidlab o‘tamizki, BS - masalasi F.Trikomi va A.M.Naxushev masalalarining umumlashmasidan iboratdir, ya’ni $c=-1$ TH masalasidan A.M.Naxushev masalasi, $c=1$ esa ushbu qo‘shimcha shart bajarilganda:

$$a_0(x)(1+x)^\beta D_{c,x}^{1-\beta}\,\psi(0)\Big|_{\substack{x=1\\ c=1}}+b_0(x)(1-x)^\beta D_{x,1}^{1-\beta}\,\varphi(1)\Big|_{x=1}=c_0(1)\,, \tag{2.27}$$

F.Trikomi masalasi kelib chiqadi.

Ushbu tenglikka asosan

$$D_{c,x}^{1-\beta}u[\theta_0(x)]=D_{-1,x}^{1-\beta}u[\theta_0(x)]-\frac{1}{\Gamma(\beta)}\frac{d}{dx}\int_{-1}^{c}\frac{u[\theta_0(t)]\,dt}{(x-t)^{1-\beta}} \quad ,\ \ x\in(c,1)$$

(2.25) chegaraviy shartni ushbu ko‘rinishda yozib olamiz

$$a_0(x)(1+x)^\beta D_{-1,x}^{1-\beta}u\left[\theta_0(x)\right]+b_0(x)(1-x)^\beta D_{x,1}^{1-\beta}u\left[\theta_1(x)\right]=c_1(x)\ ,\ \ x\in(c,1) \tag{2.28}$$

$$c_1(x)=c_0(x)+\frac{\alpha_0(x)}{\Gamma(\beta)}\frac{d}{dx}\int_{-1}^{c}\frac{\psi((t-1)/2)\,dt}{(x-t)^{1-\beta}}\,.$$

2.4-teorema. BS - masalasi bir qiymatli yechimga egadir.

BS - masalasi yechimini isbotlashda ekstremum prinsipidan foydalanilgan masla yechimini mavjudligini isbotlashda esa integral tenglamalar usuli qo‘llanilgan.

2.5-§. Parametr $\beta_0=-m/2$ bo‘lgan xolda singulyar koeffitsiyentli Gellerstedt tenglamasini o‘rganish.

Singulyar koeffitsiyentli Gellerstedt tenglamasini $\beta_0 = -m/2$ bo‘lgan xolda o‘rganamiz

$$(signy)\,|y|^m u_{xx} + u_{yy} - (m/2y)u_y = 0\,. \qquad (2.29)$$

BS_1**- masalasi.** D sohada ushbu shartlarni qanoatlantiruvchi $u(x,y)$, funksiya topilsin:

1. $u(x,y) \in C(\overline{D})$;

2. $u(x,y) \in C^2(D^+)$ va bu sohada (2.29) tenglamani qanoatlantiradi.

3. D^- sohada $u(x,y)$ $R_1(\tau'(x),\nu(x) \in H)$ sinfga tegishli umumlashgan yechim.

4. I intervalda ushbu ulanish sharti bajariladi

$$\lim_{y\to -0}(-y)^{-m/2}\frac{\partial u}{\partial y} = \lim_{y\to +0} y^{-m/2}\frac{\partial u}{\partial y}\;, \quad x \in I \setminus \{c\}\;, \qquad (2.30)$$

shu bilan birga bu limitlar $x = \pm 1,\ x = c$ nuktalarda birdan kichik maxsuslikka ega bo‘lishi mumkin.

5. Ushbu shartlar bajariladi

$$u(x,y) = \mu(x)u(x,0) + \rho(x), \ -1 \le x \le 1\;, \qquad (2.31)$$

$$u\,|_{AC_0} = \psi(x), \quad -1 \le x \le (c-1)/2\;, \qquad (2.32)$$

$$a_0(x)\frac{d}{dx}u[\theta_0(x)] - b_0(x)\frac{d}{dx}u[\theta_1(x)] = c_0(x),\ \ c \le x \le 1. \qquad (2.33)$$

Bu masala xam F.Trikomi, A. M. Naxushev va Bitsadze-Samarskiy masalalarini o‘zida birlashtirgan masaladir.

Trikomi masalasida AC xarakteristikaning barcha nuqtalarida $u(x,y)$ funksiya qiymati beriladi $u(x,y)|_{AC} = \psi(x)$. Bu masalada esa xarakteristika ixtiyoriy ravishda ikkiga bo‘linib, bir qismida chegaraviy shart berilgan.

Qo‘yilgan masala yechimining yagonaligi va mavjudligi ham xuddi yuqoridagi masala yechimi kabi isbotlanadi.

Bu funksiyalar (x,y) o‘zgaruvchilarga nisbatan (1.1) tenglamaning yechimidan iborat, shu bilan birga yaxshi ma’lum

$$F(a,b,a+b;1-\sigma)=-\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}F(a,b,1;\sigma)\ln\sigma+$$

$$+\frac{\Gamma(a+b)}{\Gamma^2(a)\Gamma^2(b)}\sum_{k=0}^{\infty}\frac{\Gamma(a+k)\Gamma(b+k)}{(k!)^2}\left[2\frac{\Gamma'(1+k)}{\Gamma(1+k)}-\right. \tag{2.34}$$

$$\left.-\frac{\Gamma'(a+k)}{\Gamma(a+k)}-\frac{\Gamma'(b+k)}{\Gamma(b+k)}\right]\sigma^k$$

formulaga ko‘ra bu yechimlar $r\to 0$ ga, ya’ni $\sigma\to 0$ ga intilganda logarifmik maxsuslikka ega. Demak, (2.32) yechimlar (2.1) tenglamaning fundamental yechimlari ekan.

Bevosita hisoblashlar yordamida yuqoridagi formulalardan

$$\lim_{y\to 0} y^{\beta_0}\frac{\partial q_1(x,y;x_0,y_0)}{\partial y}=0 \tag{2.35}$$

$$q_2(x,0;x_0,y_0)=0 \tag{2.36}$$

tengliklarning to‘g‘ri ekanligini ko‘rsatish mumkin.

II bob yuzasidan xulosa.

Dissertatsiyaning ushbu bobida singulyar koeffitsiyentli tenglama uchun shakli o'zgargan Koshi masalasi o'rganilagan.

Qat'iy giperbolik tenglamalar uchun qo'yilgan Koshi masalasining korrektligidan Darbu masalasining korrektligi kelib chiqadi, buziluvchan giperbolik tenglamalarda esa umuman olganda Koshi masalasi korrektligidan Darbu masalasining korrektligi kelib chiqmaydi.

Buziluvchan umumiy giperbolik tipdagi tenglama uchun Koshi masalasini normal yechilishining

$$\lim_{y\to -0}\frac{ya(x,y)}{\sqrt{-K(y)}}=0$$

Protter sharti buzilishi izohlangan, bunda $h(x,y)>0$, $K(0)\equiv 0$, $K(y)<0$, $y<0$.

Aralash tipdagi tenglama uchun shakli o'zgargan *N* masalasi ta'riflangan. Singulyar koeffisiyentli aralash tipdagi tenglamalar uchun tenglama uchun Bitsadze-Samarskiy masalasining shartlarini parallel xarakteristikalardagi qiymatlarining kasr tartibli hosilalarini o'zida birlashtirgan nolokal shartli masalalar qo'yilgan.

III-BOB. SOHA ICHIDA BUZILADIGAN SINGULYAR KOEFFITSIYENTLI GIPERBOLIK VA ARALASH TIPDAGI TENGLAMALAR UCHUN BITSADZE-SAMARSKIY SHARTLI MASALA.

3.1-§. Singulyar koeffitsentli Gallerstend tenglamasini Bitsadze-Samarskiy ko'rinishdagi va sohaning ichida maxsuslikka ega bo'lgan chegaraviy masalani G masala deb ataymiz.

G masalaning qo'yilishi. Quyidagi tenglamani qaraymiz

$$-|y|^m u_{xx} + u_{yy} + \alpha_0 |y|^{m/2-1} u_x + (\beta_0 / y) u_y = 0 \tag{3.1}$$

(3.1) tenglamadagi $m > 0$, α_0 va β_0 o'zgarmaslar $-(m/2) \le \beta_0 < 1,\ 0 \le \alpha_0 < (m+2)/2.$ shartni qanoatlantirsin.

(3.1) Tenglama uchun qo'yiladigan chegaraviy masalalarning korrekt bo'lishi, kichik hadlarning α_0 va β_0 parametrlarga bog'liq

$\alpha_0 o \beta_0$ parametrlar teksligida

$$S_0B_0 : \beta_0 = 1, B_0C_0 : \beta_0 - \alpha_0 = -(m/2), C_0S_0 : \alpha_0 = 0.$$

To'g'ri chiziqlar bilan chegaralangan $S_0B_0C_0$ uchburchakni qaraymiz

(3.1) tenglamaning

$$\left.\begin{matrix} AC_1 \\ BC_1 \end{matrix}\right\}: \ x \mp \frac{2}{m+2} y^{\frac{m+2}{2}} = \mp 1, \quad y > 0,$$

$$\left.\begin{matrix} AC_2 \\ BC_2 \end{matrix}\right\}: \ x \mp \frac{2}{m+2} (-y)^{\frac{m+2}{2}} = \mp 1, \quad y < 0$$

xarakteristikalar bilan chegaralangan Ω chekli bir bog'lamli, x,y o'zgaruvchilar sohasi bo'lsin.

$P(\alpha_0, \beta_0) \in \Delta S_0B_0C_0$, т.е. $\alpha \ge 0,\ \beta < 1,\ 0 \le \alpha + \beta < 1,\ \beta < \alpha,$ bunda

$$\left.\begin{matrix} \alpha \\ \beta \end{matrix}\right\} = \frac{m + 2(\beta_0 \pm \alpha_0)}{2(m+2)}. \text{ bo'lsin.}$$

Keyinchalik ishlatadigan integrallash va differensiallash kasr operatorlarning ta’rifini keltiramiz: L(a,b) sinfga tegishli bo’lgan f(x) funksya berilgan bo’lsin,bunda $L(a,b),\ a<b<\infty.$ Quyidagi belgilashlarni kiritamiz

$$D_{a,x}^{l} f(x)=\begin{cases}\dfrac{1}{\Gamma(-l)}\displaystyle\int_a^x \frac{f(t)dt}{(x-t)^{1+l}},\ npu\ l<0,\\ \dfrac{d^{n+1}}{dx^{n+1}} D_{a,x}^{l-(n+1)} f(x),\ npu\ l>0,\end{cases}$$

$$D_{x,b}^{l} f(x)=\begin{cases}\dfrac{1}{\Gamma(-l)}\displaystyle\int_x^b \frac{f(t)dt}{(t-x)^{1+l}},\ npu\ l<0,\\ (-1)^{n+1}\dfrac{d^{n+1}}{dx^{n+1}} D_{x,b}^{l-(n+1)} f(x),\ npu\ l>0,\end{cases}$$

bunda $D_{a,x}^{l}$ va $D_{x,b}^{l}$ l-tartibli kasr integrallash operatorlar. l<0 bo’lganda,l tartibli Liuvill ma’nosidagi umumlashtirilgan hosilalar, $l>0$ bo’lganida $n=[l]-\ l$ sonning butun qismi.

G masala. Ω sohada (3.1) tenglamaning $C(\bar{\Omega}_1\cup\bar{\Omega}_2)\cap C^2(\Omega\setminus AB)$ sinfga tegishli va quyidagi chegaraviy shartlarni qanoatlantiruvchi

$$u_j\left[\theta^{(j)}(x)\right]=\mu_1 u_j\left[\theta_{k_1}^{(j)}(x)\right]+\mu_2 u_j\left[\theta_{k_2}^{(j)}(x)\right]+\delta_j(x),\ \forall x\in I=AB,\ j=1,2 \qquad (3.2)$$

Regulyar yechimi topilsin.

$$u(x,y)=\begin{cases}u_1(x,y),\ (x,\ y)\in\Omega_1=\Omega\cap\{y>0\},\\ u_2(x,\ y),\ (x,\ y)\in\Omega_2=\Omega\cap\{y<0\},\end{cases}$$

bunda $j=1$ $\Omega_1,$ sohaga tegishli ,a $j=2$ Ω_2 sohaga tegishli, va

$$\lim_{y\to+0} u_1(x,y)=c\lim_{y\to-0} u_2(x,y),\forall x\in\bar{I}, \qquad (3.3)$$

Qo’shma shartlari

$$\lim_{y\to+0} y^{\beta_0}\frac{\partial u_1}{\partial y}=\rho(x)\lim_{y\to-0}(-y)^{\beta_0}\frac{\partial u_2}{\partial y}+\lambda(x),\ \forall x\in I, \qquad (3.4)$$

bunda $\theta^{(j)}(x)-$ $M(x_0,0)\in I$ nuqtadan chiqgan va BC_j xarakteristikalar kesishgan nuqtaning affiksi, , $\theta^{(j)}_{k_1}(x),\theta^{(j)}_{k_2}(x)-$lar $x+\left[2k_j/(m+2)\right]|y|^{(m+2)/2}=1$ egri chiziq bilan $M(x_0,0)\in I$ nuqtadan chiquvchi xarakteristikalar kesishgan nuqtaning affikslari.

$$\theta^{(j)}(x_0)=\frac{1+x_0}{2}+(-1)^{j-1}i\left[\frac{(m+2)(1-x_0)}{4}\right]^{\frac{2}{m+2}},$$

$$\theta^{(j)}_{k_1}(x_0)=\frac{1+k_1x_0}{1+k_1}+(-1)^{j-1}i\left[\frac{(m+2)(1-x_0)}{2(k_1+1)}\right]^{\frac{2}{m+2}},$$

$$\theta^{(j)}_{k_2}(x_0)=\frac{1+k_2x_0}{1+k_2}+(-1)^{j-1}i\left[\frac{(m+2)(1-x_0)}{2(k_2+1)}\right]^{\frac{2}{m+2}},$$

$c=const\neq 0;$ $\mu_1,\mu_2=const;$ $\delta_j(x),\ \rho(x),\ \lambda(x)$ lar $C^2(\bar{I})\cap C^3(I),$ sinfga tegishli bo'lgan ,berilgan funksiyalar,bunda $\rho(x)-c\neq 0,$ $k_1>k_2>1,$ $\delta^{(n)}_j(1)=0,$ $\lambda^{(n)}(1)=0,$ $n=0,1,2.$

1. Ω_1 $(y>0)$ soha uchun (3.2) chegaraviy shartni qaraymiz.

$$u_1(x,+0)=\tau_1(x),\ \ x\in\bar{I},\quad \lim_{y\to+0}y^{\beta_0}\frac{\partial u_1}{\partial y}=\nu_1(x),x\in I,\quad (3.5)$$

Ko'rinishni o'zgartirgan Koshi shartlarni qanoatlantiruvchi Ω_1, sohadagi (3.1) tenglamaning yechimi:

$$\begin{aligned}u_1(x,y)=\gamma_1\int_{-1}^{1}\tau_1\left[x+\frac{2t}{m+2}y^{\frac{m+2}{2}}\right](1+t)^{\beta-1}(1-t)^{\alpha-1}dt-\\ -\gamma_2y^{1-\beta_0}\int_{-1}^{1}\nu_1\left[x+\frac{2t}{m+2}y^{\frac{m+2}{2}}\right](1+t)^{-\alpha}(1-t)^{-\beta}\,dt.\end{aligned}\quad(3.6)$$

Darbu formula bilan beriladi.

bunda

$$\gamma_1=\frac{2^{1-\alpha-\beta}\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)},\ \gamma_2=-\frac{2^{\alpha+\beta-1}\Gamma(2-\alpha-\beta)}{(1-\beta_0)\Gamma(1-\alpha)\Gamma(1-\beta)}.$$

Bundan,osonlikcha hisoblash mumkin

$$u_1[\theta^{(1)}(x)]=\gamma_1 2^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}\int_x^1\frac{\tau_1(t)dt}{(t-x)^{1-\beta}(1-t)^{1-\alpha}}+ \\ +\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^1\frac{\nu_1(t)dt}{(t-x)^{\alpha}(1-t)^{\beta}}, \tag{3.7}$$

$$u_1[\theta_{k_1}^{(1)}(x)]=\gamma_1(1-x)^{1-\alpha-\beta}\left(\frac{a_1}{2}\right)^{1-\alpha-\beta}\int_x^{a_1+b_1x}\frac{\tau_1(t)dt}{(a_1+b_1x-t)^{1-\alpha}(t-x)^{1-\beta}}+ \\ +\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^{a_1+b_1x}\frac{\nu_1(t)dt}{(a_1+b_1x-t)^{\beta}(t-x)^{\alpha}} \tag{3.8}$$

va

$$u_1[\theta_{k_2}^{(1)}(x)]=\gamma_1(1-x)^{1-\alpha-\beta}\left(\frac{a_2}{2}\right)^{1-\alpha-\beta}\int_x^{a_2+b_2x}\frac{\tau_1(t)dt}{(a_2+b_2x-t)^{1-\alpha}(t-x)^{1-\beta}}+ \\ +\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^{a_2+b_2x}\frac{\nu_1(t)dt}{(a_2+b_2x-t)^{\beta}(t-x)^{\alpha}}, \tag{3.9}$$

Bunda $a_i=\frac{2}{k_i+1},\ b_i=\frac{k_i-1}{k_i+1}=1-a_i,\ i=1,2.$

Endi (3.7)-(3.9), ifodalarni (3.8), chegaraviy shartga qo'yamiz,va quyidagi natijaga erishamiz

$$\gamma_1 2^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}\int_x^1\frac{\tau_1(t)dt}{(t-x)^{1-\beta}(1-t)^{1-\alpha}}+\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^1\frac{\nu_1(t)dt}{(t-x)^{\alpha}(1-t)^{\beta}}=$$

$$=\mu_1\gamma_1(1-x)^{1-\alpha-\beta}\left(\frac{a_1}{2}\right)^{1-\alpha-\beta}\int_x^{a_1+b_1x}\frac{\tau_1(t)dt}{(a_1+b_1x-t)^{1-\alpha}(t-x)^{1-\beta}}+$$

$$+\mu_1\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^{a_1+b_1x}\frac{\nu_1(t)dt}{(a_1+b_1x-t)^{\beta}(t-x)^{\alpha}}+$$

$$+\mu_2\gamma_1(1-x)^{1-\alpha-\beta}\left(\frac{a_2}{2}\right)^{1-\alpha-\beta}\int\limits_x^{a_2+b_2x}\frac{\tau_1(t)dt}{(a_2+b_2x-t)^{1-\alpha}(t-x)^{1-\beta}}+$$

$$+\mu_2\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int\limits_x^{a_2+b_2x}\frac{\nu_1(t)dt}{(a_2+b_2x-t)^{\beta}(t-x)^{\alpha}}+\delta_1(x) \tag{3.10}$$

Olingan munosabat uchun $D_{x,1}^{1-\alpha}$, kasr differensiallash operatorni qo'llaymiz va

$$D_{x,b}^{\alpha}\,(b-x)^{2\alpha-1}D_{x,b}^{\alpha-1}(b-x)^{-\alpha}\Phi(x)=(b-x)^{\alpha-1}D_{x,b}^{2\alpha-1}\Phi(x), \tag{3.11}$$

$$D_{x,b}^{-\alpha}\,D_{x,b}^{\alpha}(b-x)^{-\alpha}\Phi(x)=(b-x)^{-\alpha}\Phi(x), \tag{3.12}$$

ayniyatlarni inobatga olgan holga,quyidagi natijaga erishamiz.

$$\gamma_1\Gamma(\beta)2^{\alpha+\beta-1}(1-x)^{-\beta}D_{x,1}^{-\beta}\tau_1(x)+\gamma_2((m+2)/2)^{1-\alpha-\beta}\Gamma(1-\alpha)(1-x)^{-\beta}\nu_1(x)=$$

$$=\mu_1\gamma_1\left(\frac{a_1}{2}\right)^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int\limits_x^{a_1+b_1x}\frac{\tau_1(s)ds}{(a_1+b_1x-s)^{1-\alpha}(s-x)^{1-\beta}}+$$

$$+\mu_1\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int\limits_x^{a_1+b_1x}\frac{\nu_1(s)ds}{(a_1+b_1x-s)^{\beta}(s-x)^{\alpha}}+$$

$$+\mu_2\gamma_1\left(\frac{a_2}{2}\right)^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int\limits_x^{a_2+b_2x}\frac{\tau_1(s)ds}{(a_2+b_2x-s)^{1-\alpha}(s-x)^{1-\beta}}+$$

$$+\mu_2\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int\limits_x^{a_2+b_2x}\frac{\nu_1(s)ds}{(a_2+b_2x-s)^{\beta}(s-x)^{\alpha}}+D_{x,1}^{1-\alpha}\delta_1(x) \tag{3.13}$$

(3.13) munosabat Ω_1. sohadan I ga o'tkazilgan $\tau_1(x)$ va $\nu_1(x)$ nomalum funksiyalar orasidagi birinchi fundamental munosabatdir.

Endi Ω_2 $(y<0)$.soha uchun (3.12) chegaraviy shartni qaraymiz.

$$u_2(x,-0)=\tau_2(x), x\in\bar{I},\quad \lim_{y\to-0}(-y)^{\beta_0}\frac{\partial u_2}{\partial y}=\nu_2(x), x\in I, \tag{3.14}$$

Ko'rinishni o'zgartirgan Koshi shartlarni qanoatlantiruvchi Ω_2,sohadagi (3.1) tenglamaning yechimi:

$$u_2(x,y)=\gamma_1\int_{-1}^{1}\tau_2\left[x+\frac{2t}{m+2}(-y)^{\frac{m+2}{2}}\right](1+t)^{\beta-1}(1-t)^{\alpha-1}dt-$$
$$-\gamma_2(-y)^{1-\beta_0}\int_{-1}^{1}\nu_2\left[x+\frac{2t}{m+2}(-y)^{\frac{m+2}{2}}\right](1+t)^{-\alpha}(1-t)^{-\beta}\,dt, \quad (3.15)$$

Darbu formula bilan beriladi.

Bunda

$$\gamma_1=\frac{2^{1-\alpha-\beta}\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)},\ \gamma_2=-\frac{2^{\alpha+\beta-1}\Gamma(2-\alpha-\beta)}{(1-\beta_0)\Gamma(1-\alpha)\Gamma(1-\beta)}.$$

bundan, osonlikcha hisoblash mumkin

$$u_2[\theta^{(2)}(x)]=\gamma_1 2^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}\int_x^1\frac{\tau_2(t)dt}{(t-x)^{1-\beta}(1-t)^{1-\alpha}}+$$
$$+\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^1\frac{\nu_2(t)dt}{(t-x)^{\alpha}(1-t)^{\beta}}, \quad (3.16)$$

$$u_2[\theta_{k_1}^{(2)}(x)]=\gamma_1(1-x)^{1-\alpha-\beta}\left(\frac{a_1}{2}\right)^{1-\alpha-\beta}\int_x^{a_1+b_1x}\frac{\tau_2(t)dt}{(a_1+b_1x-t)^{1-\alpha}(t-x)^{1-\beta}}+$$
$$+\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^{a_1+b_1x}\frac{\nu_2(t)dt}{(a_1+b_1x-t)^{\beta}(t-x)^{\alpha}} \quad (3.17)$$

va

$$u_2[\theta_{k_2}^{(2)}(x)]=\gamma_1(1-x)^{1-\alpha-\beta}\left(\frac{a_2}{2}\right)^{1-\alpha-\beta}\int_x^{a_2+b_2x}\frac{\tau_2(t)dt}{(a_2+b_2x-t)^{1-\alpha}(t-x)^{1-\beta}}+$$
$$+\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^{a_2+b_2x}\frac{\nu_2(t)dt}{(a_2+b_2x-t)^{\beta}(t-x)^{\alpha}} \quad (3.18)$$

Endi (3.16) – (3.18) ifodalarni (3.2), chegaraviy shartga qo’yamiz

$$\gamma_1 2^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}\int_x^1\frac{\tau_2(t)dt}{(t-x)^{1-\beta}(1-t)^{1-\alpha}}+\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^1\frac{\nu_2(t)dt}{(t-x)^{\alpha}(1-t)^{\beta}}=$$
$$=\mu_1\gamma_1(1-x)^{1-\alpha-\beta}\left(\frac{a_1}{2}\right)^{1-\alpha-\beta}\int_x^{a_1+b_1x}\frac{\tau_2(t)dt}{(a_1+b_1x-t)^{1-\alpha}(t-x)^{1-\beta}}+$$

$$+\mu_1\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^{a_1+b_1x}\frac{\nu_2(t)dt}{\left(a_1+b_1x-t\right)^{\beta}\left(t-x\right)^{\alpha}}+$$

$$+\mu_2\gamma_1\left(1-x\right)^{1-\alpha-\beta}\left(\frac{a_2}{2}\right)^{1-\alpha-\beta}\int_x^{a_2+b_2x}\frac{\tau_2(t)dt}{\left(a_2+b_2x-t\right)^{1-\alpha}\left(t-x\right)^{1-\beta}}+$$

$$+\mu_2\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^{a_2+b_2x}\frac{\nu_2(t)dt}{\left(a_2+b_2x-t\right)^{\beta}\left(t-x\right)^{\alpha}}+\delta_1(x) \qquad (3.19)$$

Olingan natijaga $D_{x,1}^{1-\alpha}$ kasr differensiallash operatorni qo'llaymiz va (3.11) va (3.13), ayniyatni inobatga olib,quyidagi natijaga erishamiz

$$\gamma_1\Gamma(\beta)2^{\alpha+\beta-1}(1-x)^{-\beta}D_{x,1}^{-\beta}\tau_2(x)+\gamma_2((m+2)/2)^{1-\alpha-\beta}\Gamma(1-\alpha)(1-x)^{-\beta}\nu_2(x)=$$

$$=\mu_1\gamma_1\left(\frac{a_1}{2}\right)^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int_x^{a_1+b_1x}\frac{\tau_2(s)ds}{\left(a_1+b_1x-s\right)^{1-\alpha}\left(s-x\right)^{1-\beta}}+$$

$$+\mu_1\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int_x^{a_1+b_1x}\frac{\nu_2(s)ds}{\left(a_1+b_1x-s\right)^{\beta}\left(s-x\right)^{\alpha}}+$$

$$+\mu_2\gamma_1\left(\frac{a_2}{2}\right)^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int_x^{a_2+b_2x}\frac{\tau_2(s)ds}{\left(a_2+b_2x-s\right)^{1-\alpha}\left(s-x\right)^{1-\beta}}+$$

$$+\mu_2\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int_x^{a_2+b_2x}\frac{\nu_2(s)ds}{\left(a_2+b_2x-s\right)^{\beta}\left(s-x\right)^{\alpha}}+D_{x,1}^{1-\alpha}\delta_2(x) \qquad (3.20)$$

(3.20) munosabat Ω_2.sohadan I ga o'tkazilgan $\tau_2(x)$ va $\nu_2(x)$, nomalum funksiyalar orasidagi ikkinchi fundamental munosabatdir.

Endi (3.13) ifodani (3.13), (3.14) qo'shma shartlarga ko'ra, yani : $\tau_1(x)=c\tau_2(x),\ \nu_1(x)=\rho(x)\nu_2(x)+\lambda(x)$ inobatga olib quyidagi ko'rinishga olib kelamiz

$$\gamma_1\Gamma(\beta)2^{\alpha+\beta-1}(1-x)^{-\beta}D_{x,1}^{-\beta}c\tau_2(x)+\gamma_2((m+2)/2)^{1-\alpha-\beta}\times$$
$$\times\Gamma(1-\alpha)(1-x)^{-\beta}(\rho(x)\nu_2(x)+\lambda(x))=$$

$$=\mu_1\gamma_1\left(\frac{a_1}{2}\right)^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int_x^{a_1+b_1x}\frac{c\tau_2(s)ds}{\left(a_1+b_1x-s\right)^{1-\alpha}\left(s-x\right)^{1-\beta}}+$$

$$+\mu_1\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta} D_{x,1}^{1-\alpha}\int_x^{a_1+b_1x}\frac{(\rho(s)\nu_2(s)+\lambda(s))ds}{(a_1+b_1x-s)^{\beta}(s-x)^{\alpha}}+$$

$$+\mu_2\gamma_1\left(\frac{a_2}{2}\right)^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int_x^{a_2+b_2x}\frac{c\tau_2(s)ds}{(a_2+b_2x-s)^{1-\alpha}(s-x)^{1-\beta}}+$$

$$+\mu_2\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta} D_{x,1}^{1-\alpha}\int_x^{a_2+b_2x}\frac{(\rho(s)\nu_2(s)+\lambda(s))ds}{(a_2+b_2x-s)^{\beta}(s-x)^{\alpha}}+D_{x,1}^{1-\alpha}\delta_1(x) \qquad (3.21)$$

(3.20) va (3.21) dan $\tau_2(x)$ ni yo'qotsak $\nu_2(x)$ noma'lum funksiya orqali quyidagi integral tenglamani hosil qilamiz:

$$\Gamma(1-\alpha)(\rho(x)-c)(1-x)^{-\beta}\nu_2(x)=\mu_1D_{x,1}^{1-\alpha}\int_x^{a_1+b_1x}\frac{(\rho(s)-c)\nu_2(s)ds}{(a_1+b_1x-s)^{\beta}(s-x)^{\alpha}}+$$
$$+\mu_2D_{x,1}^{1-\alpha}\int_x^{a_2+b_2x}\frac{(\rho(s)-c)\nu_2(s)ds}{(a_2+b_2x-s)^{\beta}(s-x)^{\alpha}}+f(x) \qquad (3.22)$$

bunda

$$f(x)=\frac{1}{\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}}D_{x,1}^{1-\alpha}(\delta_1(x)-c\delta_2(x))-\Gamma(1-\alpha)(1-x)^{-\beta}\lambda(x)+$$

$$+\mu_2D_{x,1}^{1-\alpha}\int_x^{a_2+b_2x}\frac{\lambda(s)ds}{(a_2+b_2x-s)^{\beta}(s-x)^{\alpha}}+\mu_1D_{x,1}^{1-\alpha}\int_x^{a_1+b_1x}\frac{\lambda(s)ds}{(a_1+b_1x-s)^{\beta}(s-x)^{\alpha}}.$$

Endi (3.22).tenglamaning o'ng tomonidagi kasrli hosilalarni hisoblaymiz.

$$I_1(x)=D_{x,1}^{1-\alpha}\int_x^{a_1+b_1x}\frac{\nu(s)ds}{(a_1+b_1x-s)^{\beta}(s-x)^{\alpha}}=\frac{d}{dx}D_{x,1}^{-\alpha}\int_x^{a_1+b_1x}\frac{\nu(s)ds}{(a_1+b_1x-s)^{\beta}(s-x)^{\alpha}}=$$

$$=\frac{1}{\Gamma(\alpha)}\frac{d}{dx}\int_x^1\frac{dt}{(t-x)^{1-\alpha}}\int_t^{a_1+b_1t}\frac{\nu(s)ds}{(a_1+b_1t-s)^{\beta}(s-t)^{\alpha}}, \qquad (3.23)$$

$$\nu(x)=(\rho(x)-c)\nu_2(x).$$

Bunda integrallash tartibini o'zgartiramiz

$$I_1(x)=\frac{1}{\Gamma(\alpha)}\frac{d}{dx}\int_x^1dt\int_t^{a_1+b_1t}\frac{\nu(s)ds}{(a_1+b_1t-s)^{\beta}(t-x)^{1-\alpha}(s-t)^{\alpha}}=$$

$$=\frac{1}{\Gamma(\alpha)}\frac{d}{dx}\left[\int_{x}^{a_1+b_1x}\nu(s)ds\int_{x}^{s}\frac{dt}{\left(a_1+b_1t-s\right)^{\beta}\left(t-x\right)^{1-\alpha}\left(s-t\right)^{\alpha}}+\right.$$

$$\left.+\int_{a_1+b_1x}^{1}\nu(s)ds\int_{\frac{s-a_1}{b_1}}^{s}\frac{dt}{\left(a_1+b_1t-s\right)^{\beta}\left(t-x\right)^{1-\alpha}\left(s-t\right)^{\alpha}}\right]=I_1^{*}(x)+I_1^{**}(x). \quad (3.24)$$

(3.24) dagi ichki integralni hisoblaymiz

$$I_1^{*}(x,s)=\int_{x}^{s}\frac{dt}{\left(a_1+b_1t-s\right)^{\beta}\left(t-x\right)^{1-\alpha}\left(s-t\right)^{\alpha}}, \quad (3.25)$$

$$I_1^{**}(x,s)=\int_{\frac{s-a_1}{b_1}}^{s}\frac{dt}{\left(a_1+b_1t-s\right)^{\beta}\left(t-x\right)^{1-\alpha}\left(s-t\right)^{\alpha}}. \quad (3.26)$$

(3.26), integralda $t=s-(s-x)\sigma$, almashtirishni bajaramiz

$$I_1^{*}(x,s)=\int_{0}^{1}\sigma^{-\alpha}(1-\sigma)^{\alpha-1}\left(1-\frac{b_1(s-x)}{a_1(1-s)}\right)^{-\beta}d\sigma. \quad (3.27)$$

Bundan Gaussning gipergeometrik funksiyaning integral ko'rinishi bilan foydalanamiz:

$$\int_{0}^{1}t^{a-1}(1-t)^{c-a-1}(1-xt)^{-b}dt=\frac{\Gamma(a)\Gamma(c-a)}{\Gamma(c)}F(a,b,c;x), \quad (3.28)$$

bundan

$$I_1^{*}(x,s)=\left(a_1(1-s)\right)^{-\beta}\Gamma(1-\alpha)\Gamma(\alpha)F\left(1-\alpha,\beta,1;\frac{b_1(s-x)}{a_1(1-s)}\right). \quad (3.29)$$

(3.26) integralda $t=s-\left(s-\frac{s-a_1}{b_1}\right)\sigma$,almashtirish bajaramiz va quyidagi natijaga erishamiz

$$I_1^{**}(x,s)=\left(a_1(1-s)\right)^{-\beta}\left(\frac{a_1(1-s)}{b_1(s-x)}\right)^{1-\alpha}\int_{0}^{1}\sigma^{-\alpha}(1-\sigma)^{-\beta}\left(1-\frac{a_1(1-s)}{b_1(s-x)}\right)^{\alpha-1}d\sigma \quad (3.30)$$

(3.29) ni inobatga olib (3.30) dan

$$I_1^{**}(x,s)=\left(a_1(1-s)\right)^{-\beta}\left(\frac{a_1(1-s)}{b_1(s-x)}\right)^{1-\alpha}\frac{\Gamma(1-\alpha)\Gamma(1-\beta)}{\Gamma(2-\alpha-\beta)}\times$$
$$\times F\left(1-\alpha,1-\alpha,2-\alpha-\beta;\frac{a_1(1-s)}{b_1(s-x)}\right). \tag{3.31}$$

(3.29) va (3.31) larga ko'ra (3.24) ni quyidagi ko'rinishda yozib olamiz

$$I_1(x)=\frac{1}{\Gamma(\alpha)}\frac{d}{dx}\int_x^{a_1+b_1x}\frac{\nu(s)}{(a_1(1-s))^{\beta}}F\left(1-\alpha,\beta,1;\frac{b_1(s-x)}{a_1(1-s)}\right)+$$
$$+\frac{\Gamma(1-\alpha)\Gamma(1-\beta)}{\Gamma(\alpha)\Gamma(2-\alpha-\beta)}\frac{d}{dx}\int_{a_1+b_1x}^{1}\frac{\nu(s)}{(a_1(1-s))^{\beta}}\left(\frac{a_1(1-s)}{b_1(s-x)}\right)^{1-\alpha}\times$$
$$\times F\left(1-\alpha,1-\alpha,2-\alpha-\beta;\frac{a_1(1-s)}{b_1(s-x)}\right). \tag{3.32}$$

Endi

$$\frac{d}{dx}F\left(a,b,c;x\right)=\frac{ab}{c}F\left(a+1,b+1,c+1;x\right), \tag{3.33}$$

$$\frac{d}{dx}x^{a}F\left(a,b,c;x\right)=ax^{a-1}F\left(a+1,b,c;x\right), \tag{3.34}$$

Formulaga ko'ra , (3.33) da hosilani hisoblaymiz

$$I_1(x)=\Gamma(1-\alpha)\left[\frac{\nu(a_1+b_1x)}{(a_1(1-a_1-b_1x))^{\beta}}F\left(1-\alpha,\beta,1;\frac{b_1(a_1+b_1x-x)}{a_1(1-a_1-b_1x)}\right)b_1-\right.$$
$$-\frac{\nu(x)}{(a_1(1-x))^{\beta}}F\left(1-\alpha,\beta,1;\frac{b_1(x-x)}{a_1(1-x)}\right)+$$
$$\left.+b_1\int_x^{a_1+b_1x}\frac{\nu(s)}{(a_1(1-s))^{\beta}}\frac{\beta(1-\alpha)}{1}F\left(2-\alpha,1+\beta,2;\frac{b_1(s-x)}{a_1(1-s)}\right)\frac{ds}{a_1(1-s)}\right]+$$
$$+\frac{\Gamma(1-\alpha)\Gamma(1-\beta)}{\Gamma(\alpha)\Gamma(2-\alpha-\beta)}\left[-\frac{\nu(a_1+b_1x)}{(a_1(1-a_1-b_1x))^{\beta}}\left(\frac{a_1(1-a_1-b_1x)}{b_1(a_1+b_1x-x)}\right)^{1-\alpha}\times\right.$$
$$\times F\left(1-\alpha,\beta,1;\frac{a_1(1-a_1-b_1x)}{b_1(a_1+b_1x-x)}\right)b_1+\int_{a_1+b_1x}^{1}\frac{(1-\alpha)\nu(s)}{(a_1(1-s))^{\beta}}\left(\frac{a_1(1-s)}{b_1(s-x)}\right)^{1-\alpha}\times$$

$$\times F\left(2-\alpha,1-\alpha,2-\alpha-\beta;\frac{a_1(1-s)}{b_1(s-x)}\right)\frac{1}{s-x}\Bigg].$$

Quyidagi

$$F\left(a,b,c;1\right)=\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}, \tag{3.36}$$

$$F\left(a,b,c;0\right)=1, \tag{3.37}$$

$$F\left(a,b,c;x\right)=\left(1-x\right)^{c-a-b}F\left(c-a,c-b,c;x\right), \tag{3.38}$$

Formulalar yordamida (3.35) formulani quyidagi ko'rinishga keltiramiz

$$I_1(x)=-\Gamma(1-\alpha)(1-x)^{-\beta}\nu(x)-$$

$$-\mu_1\Gamma(1-\alpha)\beta(1-\alpha)\int\limits_x^{a_1+b_1x}\frac{\nu(s)ds}{(a_1(1-s))^{\alpha}}\frac{b_1}{(a_1+b_1x-s)^{1-\alpha+\beta}}F\left(\alpha,1-\beta,2;\frac{b_1(s-x)}{a_1(1-s)}\right)+$$

$$+\frac{\Gamma(1-\alpha)\Gamma(1-\beta)}{\Gamma(2-\alpha-\beta)\Gamma(\alpha)}\beta(1-\alpha)\int\limits_{a_1+b_1x}^{1}\frac{\nu(s)ds}{\left(a_1(1-s)\right)^{\alpha}}\left(\frac{a_1(1-s)}{b_1(s-x)}\right)^{1-\beta}\frac{1}{\left(s-a_1-b_1x\right)^{1+\beta-\alpha}}\times$$

$$\times F\left(-\beta,1-\beta,2-\alpha-\beta;\frac{a_1(1-s)}{b_1(s-x)}\right) \tag{3.39}$$

Shunga o'xshash, ravshanki

$$I_2(x)=D_{x,1}^{1-\alpha}\int\limits_x^{a_2+b_2x}\frac{\nu(s)ds}{\left(a_2+b_2x-s\right)^{\beta}\left(s-x\right)^{\alpha}}=\Gamma\left(1-\alpha\right)\left(a_2\left(1-x\right)\right)^{-1}\nu\left(x\right)+$$

$$+\Gamma(1-\alpha)\beta(1-\alpha)\int\limits_x^{a_2+b_2x}\frac{\nu(s)ds}{(a_2(1-s))^{\alpha}}\frac{b_2}{(a_2+b_2x-s)^{1-\alpha+\beta}}F\left(\alpha,1-\beta,2;\frac{b_2(s-x)}{a_2(1-s)}\right)-$$

$$-\frac{\Gamma(1-\alpha)\Gamma(1-\beta)}{\Gamma(2-\alpha-\beta)\Gamma(\alpha)}\beta(1-\alpha)\int\limits_{a_2+b_2x}^{1}\frac{\nu(s)ds}{\left(a_2(1-s)\right)^{\alpha}}\left(\frac{a_2(1-s)}{b_2(s-x)}\right)^{1-\beta}\frac{1}{\left(s-a_2-b_2x\right)^{1+\beta-\alpha}}\times$$

$$\times F\left(-\beta,1-\beta,2-\alpha-\beta;\frac{a_2(1-s)}{b_2(s-x)}\right). \tag{3.40}$$

Endi $I_1(x)$ va $I_2(x)$ lar uchun, mos ravishda (3.39) va (3.40) dan (3.22) ga qo'ysak quyidagi natijaga erishamiz

$$\Gamma(1-\alpha)(\rho(x)-c)(1-x)^{-\beta}\nu_2(x)=I(x)+f(x), \tag{3.41}$$

bunda

$$I(x)=I_1(x)+I_2(x)=$$

$$=\Gamma(1-\alpha)(\rho(x)-c)(1-x)^{-\beta}\mu_1 a_1^{-\beta}\nu_2(x)+\mu_1\Gamma(1-\alpha)\beta b_1(1-\alpha)\times$$

$$\times\int_x^{a_1+b_1x}\frac{(\rho(s)-c)\nu_2(s)}{(a_1(1-s))^{\alpha}(a_1+b_1x-s)^{1-\alpha+\beta}}F\left(\alpha,1-\beta,2;\frac{b_1(s-x)}{a_1(1-s)}\right)ds-$$

$$-\mu_1\frac{\Gamma(1-\beta)\Gamma(1-\alpha)(1-\alpha)}{\Gamma(2-\alpha-\beta)\Gamma(\alpha)}\int_{a_1+b_1x}^{1}\frac{(\rho(s)-c)\nu_2(s)}{(a_1(1-s))^{\alpha}}\left(\frac{a_1(1-s)}{b_1(s-x)}\right)^{1-\beta}\frac{1}{(s-a_1-b_1x)^{1-\alpha+\beta}}\times$$

$$\times F\left(-\beta,1-\beta,2-\alpha-\beta;\frac{a_1(1-s)}{b_1(s-x)}\right)ds+\Gamma(1-\alpha)(\rho(x)-c)(1-x)^{-\beta}\mu_2 a_2^{-\beta}\nu_2(x)+$$

$$+\mu_2\Gamma(1-\alpha)\beta b_2(1-\alpha)\int_x^{a_2+b_2x}\frac{(\rho(s)-c)\nu_2(s)}{(a_2(1-s))^{\alpha}(a_2+b_2x-s)^{1-\alpha+\beta}}\times$$

$$\times F\left(\alpha,1-\beta,2;\frac{b_2(s-x)}{a_2(1-s)}\right)ds-\mu_2\frac{\Gamma(1-\beta)\Gamma(1-\alpha)(1-\alpha)}{\Gamma(2-\alpha-\beta)\Gamma(\alpha)}\times \tag{3.42}$$

$$\times\int_{a_2+b_2x}^{1}\frac{(\rho(s)-c)\nu_2(s)}{(a_2(1-s))^{\alpha}}\left(\frac{a_2(1-s)}{b_2(s-x)}\right)^{1-\beta}\frac{1}{(s-a_2-b_2x)^{1-\alpha+\beta}}\times$$

$$\times F\left(-\beta,1-\beta,2-\alpha-\beta;\frac{a_2(1-s)}{b_2(s-x)}\right)ds,$$

(3.42) ga asoslanib (3.41) tenglamani quyidagi ko’rinishda yozish mumkin

$$\nu_2(x)=\int_x^1\frac{K_1(x,s)\nu_2(s)ds}{|s-a_1-b_1x|^{\ell}}+\int_x^1\frac{K_2(x,s)\nu_2(s)ds}{|s-a_2-b_2x|^{\ell}}+f(x),\ \ \forall x\in I \tag{3.43}$$

bunda $\ell=1-\alpha+\beta<1,$

$$K_k(x,s)=\begin{cases} \dfrac{\mu_k\beta(1-\alpha)(1-x)^{\beta}b_k(\rho(s)-c)}{(a_k(1-s))^{\alpha}(\rho(x)-c)(1-a_k^{-\beta}\mu_1-a_k^{-\beta}\mu_2)}\times \\ \times F(\alpha,1-\beta,2;b_k(s-x)/a_k(1-s)), \quad x\le s\le a_k+b_kx, \\ -\dfrac{\mu_k\Gamma(1-\beta)(1-\alpha)(a_k(1-s))^{1-\beta-\alpha}(1-x)^{\beta}(\rho(s)-c)}{(b_k(s-x))^{1-\beta}\Gamma(2-\alpha-\beta)\Gamma(\alpha)(\rho(x)-c)(1-a_k^{-\beta}\mu_1-a_k^{-\beta}\mu_2)}\times \\ \times F(-\beta,1-\beta,2-\alpha-\beta;a_k(1-s)/b_k(s-x)), \\ a_k+b_kx\le s\le 1. \end{cases} \quad (3.44)$$

Shu bilan birgalikda

$$K_k(x,\, a_k+b_kx-0)=\frac{\mu_k(1-x)^{\beta-\alpha}\Gamma(1-\alpha+\beta)}{a_k^{\alpha}b_k^{\alpha}\Gamma(1-\alpha)\Gamma(\beta)}, \quad k=1,2, \quad (3.45)$$

$$K_k(x,a_k+b_kx+0)=-\frac{\mu_k(1-x)^{\beta-\alpha}\Gamma(1-\beta)\Gamma(1-\alpha+\beta)}{a_k^{\alpha}b_k^{\alpha}\Gamma^2(1-\alpha)\Gamma(\alpha)}, \quad k=1,2. \quad (3.46)$$

Quyidagi formula o'rinlidir.

3.1-lemma. Funksiya $K_k(x,t)\in C^2(\bar{I})\setminus\gamma_{k,}$ $k=1,2$, bu funksiya $\gamma_k: t=a_k+b_kx,$ egri chiziqda birinchi tur uzilishga ega,(1,1) nuqtadan tashqari,bu nuqtada ular $\alpha-\beta$ tartibdagi maxsuslikga ega. Bu lemmaning isboti (3.45) dan darhol kelib chiqadi.

(3.43) integral tenglamani o'rganish. (3.43) tenglama shu bilan qiziqarliki,u a_1+b_1x va a_2+b_2x chiziqda kichik yadroga ega $x<a_1+b_1x<a_2+b_2x$, bunda $0<a_k$, $b_k<1$, $a_k+b_k=1$, $k=1,2$.

Qulaylik uchun $\nu_2(x)=\nu(x)$ deb olamiz.

(3.43) tenglamani yechish uchun ketma-ket yaqinlashish usuli bilan foydalanamiz [35]. Funksional ketma –ketlikning $\nu_0(x), \nu_1(x), \ldots, \nu_n(x), \ldots$ hadlarni topish uchun quyidagi rekurrent munosabatni tuzamiz

$$\nu_n(x)=\int_x^1\left(\frac{\mu_1K_1(x,t)}{|t-a_1-b_1x|^{\ell}}+\frac{\mu_2K_2(x,t)}{|t-a_2-b_2x|^{\ell}}\right)\nu_{n-1}(t)dt+f(x) \quad (3.47)$$

Nol yaqinlashish sifatida $\vartheta_0(x) = f(x)$ ni olamiz. Bu holda birinchi yaqinlashish, quyidagi ko'rinishga ega

$$\nu_1(x) = \int_x^1 \left(\frac{\mu_1 K_1(x,t)}{|t-a_1-b_1x|^\ell} + \frac{\mu_2 K_2(x,t)}{|t-a_2-b_2x|^\ell} \right) \nu_0(t)dt + f(x) \qquad (3.48)$$

Bundan osonlikcha quyidagi baholashni topamiz

$$|\nu_1(x)-\nu_0(x)| \le |\mu_1| \int_x^1 \frac{|K_1(x,t)||f(x)|dt}{|t-a_1-b_1x|^\ell} + |\mu_2| \int_x^1 \frac{|K_2(x,t)||f(x)|dt}{|t-a_2-b_2x|^\ell} \le$$

$$\le |\mu_1| K_1 M (1-x)^{\beta-\alpha} \int_x^1 \frac{dt}{|t-a_1-b_1x|^\ell} + |\mu_2| K_2 M (1-x)^{\beta-\alpha} \int_x^1 \frac{dt}{|t-a_2-b_2x|^\ell} =$$

$$= |\mu_1| K_1 M (1-x)^{\beta-\alpha} \left(\int_x^{a_1+b_1x} \frac{dt}{(a_1+b_1x-t)^\ell} + \int_{a_1+b_1x}^1 \frac{dt}{(t-a_1-b_1x)^\ell} \right) +$$

$$+ |\mu_2| K_2 M (1-x)^{\beta-\alpha} \left(\int_x^{a_2+b_2x} \frac{dt}{(a_2+b_2x-t)^\ell} + \int_{a_2+b_2x}^1 \frac{dt}{(t-a_2-b_2x)^\ell} \right) =$$

$$= |\mu_1| K_1 M (1-x)^{\beta-\alpha} 2^{1-\ell} \frac{a_1^{1-\ell}+b_1^{1-\ell}}{1-\ell} + |\mu_2| K_2 M (1-x)^{\beta-\alpha} 2^{1-\ell} \frac{a_2^{1-\ell}+b_2^{1-\ell}}{1-\ell},$$

где $\left|(1-x)^{\alpha-\beta} K_k(x,t)\right| \le K_k$, $|f(x)| \le M$.

Shunday qilib

$$|\nu_1(x)-\nu_0(x)| \le M \left(\frac{|\mu_1| K_1 \left(a_1^{1-\ell}+b_1^{1-\ell}\right) + |\mu_2| K_2 \left(a_2^{1-\ell}+b_2^{1-\ell}\right)}{1-\ell} \right) \qquad (3.49)$$

Endi ikkinchi yaqinlashishni qo'llaymiz , (3.47) dan $n=2$ bo'lganida

$$\nu_2(x) = \int_x^1 \left(\frac{\mu_1 K_1(x,t)}{|t-a_1-b_1x|^\ell} + \frac{\mu_2 K_2(x,t)}{|t-a_2-b_2x|^\ell} \right) \nu_1(t)dt + f(x) \qquad (3.50)$$

(3.47) dan (3.48) ni ayirsak

$$|\nu_2(x)-\nu_1(x)| \le \int_x^1 \left(\frac{|\mu_1||K_1(x,t)|}{|t-a_1-b_1x|^\ell} + \frac{|\mu_2||K_2(x,t)|}{|t-a_2-b_2x|^\ell} \right) |\nu_1(x)-\nu_0(x)| dt.$$

Bundan , (3.49) ni inobatga olsak

$$\left|\nu_2(x)-\nu_1(x)\right|\le M\left(\frac{|\mu_1|K_1\left(a_1^{1-\ell}+b_1^{1-\ell}\right)+|\mu_2|K_2\left(a_2^{1-\ell}+b_2^{1-\ell}\right)}{1-\ell}\right)\times$$

$$\times\left(|\mu_1|K_1M(1-x)^{\beta-\alpha}\int_x^1\frac{dt}{\left|t-a_1-b_1x\right|^{\ell}}+|\mu_2|K_2M(1-x)^{\beta-\alpha}\int_x^1\frac{dt}{\left|t-a_1-b_1x\right|^{\ell}}\right)=$$

$$=M\left(\frac{|\mu_1|K_1\left(a_1^{1-\ell}+b_1^{1-\ell}\right)+|\mu_2|K_2\left(a_2^{1-\ell}+b_2^{1-\ell}\right)}{1-\ell}\right)^2 \qquad (3.51)$$

Bu amalni davom ettirsak

$$\left|\nu_n(x)-\nu_{n-1}(x)\right|\le M\left(\frac{|\mu_1|K_1\left(a_1^{1-\ell}+b_1^{1-\ell}\right)+|\mu_2|K_2\left(a_2^{1-\ell}+b_2^{1-\ell}\right)}{1-\ell}\right)^n \qquad (3.52)$$

Funksional qator

$$\nu_0(x)+(\nu_1(x)-\nu_0(x))+(\nu_2(x)-\nu_1(x))+...+(\nu_n(x)-\nu_{n-1}(x))+... \qquad (3.53)$$

sonli qator bilan mojorlanadi.

$$M\sum_{k=0}^{\infty}\left(\frac{|\mu_1|K_1(a_1^{1-\ell}+b_1^{1-\ell})+|\mu_2|K_2(a_2^{1-\ell}+b_2^{1-\ell})}{1-\ell}\right)^n \qquad (3.54)$$

Shunday qilib

$$\left|\frac{|\mu_1|K_1(a_1^{1-\ell}+b_1^{1-\ell})+|\mu_2|K_2(a_2^{1-\ell}+b_2^{1-\ell})}{1-\ell}\right|<1 \qquad (3.55)$$

shart bajarilganida tekis yaqinlashadi, va uning yig’indisi uzluksiz funksiyadir.

Aytib o’tish kerakki (3.43), bir jinsli integral tenglama notrivial yechimga ega. Haqiqatdan ham

$$\nu(x)=\sum_{k=1}^{2}\int_x^1\frac{K_k(x,t)\nu(t)dt}{\left|t-a_k-b_kx\right|^{1-\alpha+\beta}} \qquad (3.56)$$

Bir jinsli tenglamani qaraylik yoki

$$\nu(x)=\sum_{k=1}^{2}\int_x^{a_k+b_kx}\frac{\mu_k\beta(1-\alpha)(1-x)^{\beta}b_kF(\alpha,1-\beta,2;b_k(t-x)/a_k(1-t))\nu(t)dt}{a_k^{\alpha}(1-t)^{\alpha}(a_k+b_kx-t)^{1-\alpha+\beta}}+$$

$$+\sum_{k=1}^{2}\int_{a_k+b_kx}^{1}\frac{\mu_k\Gamma(1-\beta)(1-\alpha)a_k^{1-\alpha-\beta}(1-t)^{1-\alpha-\beta}(1-x)^{\beta}}{b_k^{1-\beta}(t-x)^{1-\beta}\Gamma(2-\alpha-\beta)\Gamma(\alpha)(t-a_k-b_k)^{1-\alpha+\beta}}\times \tag{3.58}$$

$$\times F(-\beta,1-\beta,2-\alpha-\beta;a_k(1-t)/b_k(t-x))\nu(t)dt.$$

(3.57) ning o'ng tomonidagi birinchi yig'indida $t=x+(a_k+b_kx-x)\sigma=x+a_k(1-x)\sigma$, almashtirish, ikkinchi yig'indiga esa $t=a_k+b_kx+(1-a_k-b_kx)\sigma=a_k+b_kx+b_k(1-x)\sigma.$ almashtirish bajaramiz.

Bunday almashtirish yordamida (3.57) tenglamani quyidagi ko'rinishga olib kelamiz

$$\nu(x)=\sum_{k=1}^{2}\int_{0}^{1}\frac{\mu_k\beta(1-\alpha)F(\alpha,1-\beta,2;b_k\sigma/(1-a_k\sigma))}{a_k^{\beta}(1-a_k\sigma)^{\alpha}(1-\sigma)^{1-\alpha+\beta}}\nu\left[x+a_k(1-x)\sigma\right]d\sigma+$$

$$+\sum_{k=1}^{2}\int_{0}^{1}\frac{\mu_k\Gamma(1-\beta)(1-\alpha)a_k^{1-\alpha-\beta}b_k^{-\beta}\nu\left[a_k+b_kx+b_k(1-x)\sigma\right]d\sigma}{\Gamma(\alpha)\Gamma(2-\alpha-\beta)\sigma^{1-\alpha+\beta}(a_k+b_k\sigma)^{1-\beta}}\times \tag{3.58}$$

$$\times F(-\beta,1-\beta,2-\alpha-\beta;(1-\sigma)/(a_k+b_k\sigma))d\sigma.$$

(3.58) dan ko'rinib turibdiki, $\nu(x)=(1-x)^{\lambda}$, $\lambda>0$ aniq μ_k tenglaganda (3.58) tenglamaning notrivial yechimidir.

Haqiqatdan ham

$$\nu\left[x+a_k(1-x)\sigma\right]=(1-x-a_k(1-x)\sigma)^{\lambda}=(1-x)^{\lambda}(1-a_k\sigma)^{\lambda}$$

$$\nu\left[a_k+b_kx+b_k(1-x)\sigma\right]=(b_k(1-x)(1-\sigma))^{\lambda}.$$

Bu holda (3.58) dan

$$(1-x)^{\lambda}=\sum_{k=1}^{2}\int_{0}^{1}\frac{\mu_k\beta(1-\alpha)F(\alpha,1-\beta,2;b_k\sigma/(1-a_k\sigma))}{a_k^{\beta}(1-\alpha_k\sigma)^{\alpha}(1-\sigma)^{1-\alpha+\beta}}\times$$

$$\times(1-x)^{\lambda}(1-a_k\sigma)^{\lambda}d\sigma+ \tag{3.59}$$

$$+\sum_{k=1}^{2}\int_{0}^{1}\frac{\mu_k\Gamma(1-\beta)(1-\alpha)a_k^{1-\alpha-\beta}b_k^{-\beta}b_k^{\lambda}(1-x)^{\lambda}(1-\sigma)^{\lambda}}{\Gamma(\alpha)\Gamma(2-\alpha-\beta)\sigma^{1-\alpha+\beta}(a_k+b_k\sigma)^{1-\beta}}\times$$

$$\times F(-\beta,1-\beta,2-\alpha-\beta;(1-\sigma)/(a_k+b_k\sigma))d\sigma.$$

Shunday qilib,ihtiyoriy $\lambda>0$ da (3.58) tenglik bajarilishi uchun μ_1 va μ_2 tenglash mumkin.

Bundan esa, (3.59) yechilishi uchun (3.55) shart juda muhim ekan. $\nu_2(x)$ nomalum funksiya topilganidan so'ng (3.21) tenglamani $\tau_2(x)$ noma'lum funksiyaga nisbatan integral tenglama ekan

$$c\gamma_1\Gamma(\beta)2^{\alpha+\beta-1}\tau_2(x)=$$

$$=\mu_1\gamma_1\left(\frac{a_1}{2}\right)^{\alpha+\beta-1}(1-x)^{1-\alpha}D_{x,1}^{1-\alpha+\beta}\int_x^{a_1+b_1x}\frac{c\tau_2(s)ds}{\left(a_1+b_1x-s\right)^{1-\alpha}\left(s-x\right)^{1-\beta}}+$$

$$+\mu_2\gamma_1\left(\frac{a_2}{2}\right)^{\alpha+\beta-1}(1-x)^{1-\alpha}D_{x,1}^{1-\alpha+\beta}\int_x^{a_2+b_2x}\frac{c\tau_2(s)ds}{\left(a_2+b_2x-s\right)^{1-\alpha}\left(s-x\right)^{1-\beta}}+F(x), \quad (3.60)$$

Bunda

$$F(x)=(1-x)^{\beta}D_{x,1}^{1-\alpha+\beta}\delta_1(x)-\gamma_2((m+2)/2)^{1-\alpha-\beta}\times$$

$$\times\Gamma(1-\alpha)D_{x,1}^{\beta}(\rho(x)\nu_2(x)+\lambda(x))+$$

$$+\mu_1\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}(1-x)^{\beta}D_{x,1}^{1-\alpha+\beta}\int_x^{a_1+b_1x}\frac{(\rho(s)\nu_2(s)+\lambda(s))ds}{\left(a_1+b_1x-s\right)^{\beta}\left(s-x\right)^{\alpha}}+$$

$$+\mu_2\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}(1-x)^{\beta}D_{x,1}^{1-\alpha+\beta}\int_x^{a_2+b_2x}\frac{(\rho(s)\nu_2(s)+\lambda(s))ds}{\left(a_2+b_2x-s\right)^{\beta}\left(s-x\right)^{\alpha}}, \quad (3.61)$$

$0<1-\alpha+\beta<1.$

(3.60) tenglamani o'rganish (3.43) tenglamani o'rganish bilan bir xildir.

3.1-teorema. G masala yetarlicha kichik μ_1 *va* μ_2 lar uchun (3.55) yechimga ega bo'ladi.

3.2-§. *BS* (Bitsadze-Samarskiy masalasi) masalasining qo'yilishi.

Ushbu tenglamani o'rganamiz

$$sign y|y|^m u_{xx}+u_{yy}+(\beta_0/y)u_y=0 \quad (3.62)$$

bu yerda m va β_0 -o'zgarmas sonlar bo'lib, ular uchun $m>0, -m/2<\beta_0<1$, tengsizliklar o'rinli. Ω soha $z=x+iy$ kompleks tekisligining chekli bir bog'lamli sohasi bo'lib, u $y>0$ tekisligida joylashgan va uchlari $A=A(-1,0), B=B(1,0)$

nuqtalarda yotuvchi (3.62) tenglamaning normal chizig'i $\sigma_0 : x + \frac{4}{(m+2)^2} y^{m+2} = 1$ bilan, $y < 0$ tekislikda (3.62) tenglamaning AC va BC xarakteristikalari bilan chegaralangan bo'lsin. Ushbu belgilashlarni kiritamiz $\Omega^+ = \Omega \cap \{y > 0\}$, $\Omega^- = \Omega \cap \{y < 0\}$, C_0 va C_1 orqali esa $O(0,0)$ nuqtadan chiquvchi xarakteristikaning AC va BC xarakteristikalar bilan kesishish nuqtasini belgilaymiz. $J = (-1,1)$, $y = 0$ o'qining intervali.

BS **masalasi.** Ω sohada quyidagi shartlarni qanoatlantiruvchi $u(x, y) \in C(\overline{\Omega})$ funktsiya topilsin:

1. $u(x, y) \in C^2(\Omega^+)$ va bu sohada (3.1) tenglamani qanoatlantiradi;

2. $u(x, y)$ funktsiya $\Omega^- \setminus (OC_0 \cup OC_1)$ sohada (3.1) tenglamaning R_1 sinfga tegishli bo'lgan umumlashgan yechimi;

3. $J = AB$ -intervalda ushbu ulanish shartlari bajariladi

$$\lim_{y \to -0} (-y)^{\beta_0} \frac{\partial u}{\partial y} = \lim_{y \to +0} y^{\beta_0} \frac{\partial u}{\partial y} \;, \quad x \in J \setminus \{0\} \tag{3.63}$$

va bu limitlar $x = -1$, $x = 0$, $x = 1$ nuqtalarda $1 - 2\beta$ dan kichik tartibda maxsuslikka ega bo'lishi mumkin, bu yerda $\beta = (m + 2\beta_0)/2(m+2)$;

4. $$u(x, y)\big|_{\sigma_0} = \varphi(x) \;, \; x \in J \tag{3.64}$$

$$a(x) D_{-1,x}^{1-\beta} u[\theta(x)] - b(x) D_{x,1}^{1-\beta} u[\theta(-x)] = \psi(x), x \in J \tag{3.65}$$

$$u(x,0) - u(-x,0) = f(x), \quad x \in \overline{J} \tag{3.66}$$

Bu yerda $\theta(x_0) = \frac{x_0 - 1}{2} - i \left[\frac{m+2}{4}(1 + x_0) \right]^{2/(m+2)}$ $(x_0, 0)$, $x_0 \in J$ nuqtadan chiquvchi xarakteristikaning AC xarakteristika bilan kesishish nuqtasining affiksi. $a(x), b(x), \psi(x), f(x), \varphi(x) \in C(\overline{J}) \cap C^{1,\alpha}(J)$ berilgan funktsiyalar bo'lib, ular uchun $a(-x) = b(x),\ a(x)\cdot b(x) \neq 0,\ \psi(-x) = -\psi(x)$, $f(-x) = -f(x),\ \varphi(\pm 1) = 0,\ f(\pm 1) = 0$ munosabatlar o'rinli.

(3.65) va (3.66) shartlar Frankl shartlari bo'lib, ular mos ravishda AC xarakteristikada va AB buzilish chizig'ida berilgan.

3.3-§. BS masalasi yechimining yagonaligi.

(3.66) tenglamaning shakli o'zgargan Koshi masalasi yechimini beruvchi Darbu formulasidan foydalanib ushbu qiymatlarni hosil qilamiz

$$u[\theta(x)] = \gamma_1\Gamma(\beta)((1+x)/2)^{1-2\beta} D_{-1,x}^{-\beta}(1+x)^{\beta-1}\tau(x) + \\ + \gamma_2\Gamma(1-\beta)((m+2)/2)^{1-2\beta} D_{-1,x}^{\beta-1}(1+x)^{-\beta}\nu(x)\,, \tag{3.67}$$

$$u[\theta(-x] = \gamma_1\Gamma(\beta)((1-x)/2)^{1-2\beta} D_{x,1}^{-\beta}(1-x)^{\beta-1}\tau(-x) + \\ + \gamma_2((m+2)/2)^{1-2\beta}\Gamma(1-\beta) D_{x,1}^{\beta-1}(1-x)^{-\beta}\nu(-x)\,, \tag{3.68}$$

bu yerdan

$$D_{-1,x}^{1-\beta}u[\theta(x] = \gamma_1\Gamma(\beta)2^{2\beta-1}(1+x)^{-\beta} D_{-1,x}^{1-2\beta}\tau(x) + \\ + \gamma_2\Gamma(1-\beta)((m+2)/2)^{1-2\beta}(1+x)^{-\beta}\nu(x)\,, \tag{3.69}$$

$$D_{x,1}^{1-\beta}u[\theta(-x] = \gamma_1\Gamma(\beta)2^{\beta-1}(1-x)^{-\beta} D_{x,1}^{1-2\beta}\tau(-x) + \\ + \gamma_2\Gamma(1-\beta)((m+2)/2)^{1-2\beta}(1-x)^{-\beta}\nu(-x)\,. \tag{3.70}$$

(3.69) va (3.70) tengliklardan (3.65) va (3.66) nolokal shartlarga asosan

$$(1-x)^{\beta}a(x)\nu(x) - (1+x)^{\beta}b(x)\nu(-x) = \\ = \gamma[(1-x)^{\beta}a(x)D_{-1,x}^{1-2\beta}\tau(x) - \\ - (1+x)^{\beta}b(x)D_{x,1}^{1-2\beta}\tau(x)] + \psi_1(x) \tag{3.71}$$

munosabatga kelamiz, bu yerda

$$\psi_1(x)=\frac{(1-x)^{\beta}(1+x)^{\beta}\psi(x)}{\gamma_2\Gamma(1-\beta)((m+2)/2)^{1-2\beta}}+\gamma(1+x)^{\beta}b(x)D_{x,1}^{1-2\beta}f(x)$$
$$\gamma=-\frac{\gamma_1\Gamma(\beta)2^{2\beta-1}}{\gamma_2\Gamma(1-\beta)((m+2)/2)^{1-2\beta}}. \quad (3.72)$$

3.1-teorema. Agar $a(x)$ va $b(x)$ funktsiyalar uchun ushbu

$$a(x)\cdot b(x)<0, \quad x\in\overline{J}, \quad (3.73)$$

tengsizlik o'rinli bo'lib $\varphi(x)\equiv 0,$ $\psi(x)\equiv 0,$ $f(x)\equiv 0,$ bo'lsa u holda *БС* masalasi faqat trivial yechimga ega.

Isbot. Teskari tasdiqni faraz qilamiz, $\overline{\Omega}^{+}$ sohada $u(x,y)\neq 0$ bo'lsin. Xopf printsipiga ko'ra $u(x,y)$ funktsiya o'zining eng katta musbat qiymatini va eng kichik manfiy qiymatini Ω^{+} sohaning ichki nuqtalarida qabul qilmaydi. $u(x,y)\big|_{\sigma_0}=0$ bo'lgani uchun bu qiymatlarga σ_0 nuqtalarida ham erishilmaydi. Faraz qilaylik $u(x,y)$ funktsiya o'zining eng katta musbat va eng kichik manfiy qiymatlarini $AB\setminus\{O\}$ nuqtalarda qabul qilsin, u holda (3.66) shartga mos bir jinsli shartga asosan bu ekstremum qiymatlar $(x_0,0)$ va $(-x_0,0)$ nuqtalarda qabul qilinadi. $\tau(x)$ funktsiyaning musbat maksimumini (manfiy minimumini) qabul qiluvchi x_0 nuqtada kasr tartibli $D_{-1,x_0}^{1-2\beta}\tau(x)$ va $D_{x_0,1}^{1-2\beta}\tau(x)$ hosilalar qiymati qat'iy musbat (qat'iy manfiy) bo'lgani uchun va bu nuqtalarda lemmaga asosan

$$\nu(x_0)=\lim_{y\to+0}y^{\beta_0}\frac{\partial u(x_0,y)}{\partial y}<0 \quad (>0)$$

bo'lgani uchun (3.71) tenglikka mos, bir jinsli ushbu munosabat:

$$(1-x)^{\beta}a(x)\nu(x)-(1+x)^{\beta}b(x)\nu(-x)=$$
$$=\gamma[(1-x)^{\beta}a(x)D_{-1,x}^{1-2\beta}\tau(x)-(1+x)^{\beta}b(x)D_{x,1}^{1-2\beta}\tau(x)] \quad (3.74)$$

$(x_0,0)$ va $(-x_0,0)$ nuqtalarda o'rinli emas, chunki bu nuqtalarda (3.74) tenglikning o'ng va chap tomonlari turli ishorali.

Shunday qilib, $u(x,y)$ yechim o'zining eng katta musbat va eng kichik manfiy qiymatlarini $O(0,0)$ nuqtada erishadi. Bundan esa Ω^+ sohada $u(x,y)=const$, ya'ni o'zgarmas ekanligi kelib chiqadi, lekin $u|_{\sigma_0}=0$ bo'lgani uchun $u(x,y)\equiv 0$ $(x,y)\in\overline{\Omega}^+$ bundan $u(x,y)\equiv 0$ $(x,y)\in\Omega$ ekanligi kelib chiqadi. **3.1-teorema** isbot bo'ldi.

3.4-§. *БС* **masalasi yechimining mavjudligi.**

Ω^+ sohada (3.62) tenglama uchun Dirixle va shakli o'zgargan N masalarining yechimlaridan foydalanib $y=0$ o'qida mos ravishda

$$\nu(x)=-k_2(1-\beta_0)\frac{m+2}{2}\left[\int_{-1}^{1}\frac{x-t}{|x-t|^{2-2\beta}}\tau'(t)dt+\right.$$
$$\left.+(1-2\beta)\int_{-1}^{1}\frac{\tau(t)dt}{(1-xt)^{2-2\beta}}\right]+\Phi_2(x), \tag{3.75}$$

$$\tau(x)=-k_1\int_{-1}^{1}\nu(t)[|x-t|^{-2\beta}-(1-xt)^{-2\beta}]dt+\Phi_1(x), \tag{3.76}$$

$$\Phi_2(x)=k_2(1-\beta)(1-\beta_0)(m+2)(1-x^2)\int_{-1}^{1}\varphi(t)(1-2xt+x^2)^{\beta-2}dt, \tag{3.77}$$

$$\Phi_1(x)=k_1\beta(m+2)(1-x^2)\int_{-1}^{1}\varphi(t)\eta^{\beta_0-1}(t)(1-2xt+x^2)^{-\beta-1}dt, \tag{3.78}$$

bu yerda

$$k_1=\frac{1}{4\pi}\left(\frac{4}{m+2}\right)^{2\beta}\frac{\Gamma^2(\beta)}{\Gamma(2\beta)},\; k_2=\frac{1}{4\pi}\left(\frac{4}{m+2}\right)^{2-2\beta}\frac{\Gamma^2(1-\beta)}{\Gamma(2-2\beta)}$$

$$\eta(t)=\left(\left(\frac{m+2}{2}\right)^2(1-t^2)\right)^{1/(m+2)}$$

(3.75) munosabatda x ni $-x$ ga almashtirib va ushbu

$$\tau(x)=\tau(-x)+f(x),\quad \tau'(x)=-\tau'(-x)+f'(x), \tag{3.79}$$

munosabatlarni hisobga olib, quyidagi tenglikni hosil qilamiz

$$\nu(-x)=\nu(x)+F_2(x) \tag{3.80}$$

bu yerda

$$F_2(x)=k_2(1-\beta_0)\frac{m+2}{2}\left[\int_{-1}^{1}\frac{(x-t)f'(t)dt}{|x-t|^{2-2\beta}}+(1-2\beta)\int_{-1}^{1}\frac{f(t)dt}{(1-xt)^{2-2\beta}}\right]+$$
$$+\Phi_2(-x)-\Phi_2(x) \tag{3.81}$$

(3.80) ga asosan (3.81) tenglikdan ushbu

$$\left[(1-x)^\beta a(x)-(1+x)^\beta b(x)\right]\nu(x)=$$
$$\gamma\left[(1-x)^\beta a(x)D_{-1,x}^{1-2\beta}\tau(x)-(1+x)^\beta b(x)D_{x,1}^{1-2\beta}\tau(x)\right]+\psi_2(x) \tag{3.82}$$

munosabatga kelamiz, bu yerda

$$\psi_2(x)=\psi_1(x)+(1+x)^\beta b(x)F_2(x). \tag{3.83}$$

3.5-§. BS masalasini $(1-x)^\beta a(x)-(1+x)^\beta b(x)=0$ bo'lgan holda o'rganish.

BS masalasini (3.84) shart bajarilmagan holda o'rganamiz, ya'ni

$$(1-x)^\beta a(x)-(1+x)^\beta b(x)=0,\; x\in J\,. \tag{3.85}$$

Bu holda (3.75) munosabat ushbu ko'rinishda bo'ladi.

$$D_{-1,x_0}^{1-2\beta}\tau(x) - D_{x,1}^{1-2\beta}\tau(x) = \psi_2(x) \tag{3.86}$$

Dastlab bu holda ham BS masalasi yechimi yagona ekanligini isbotlaymiz. (3.86) tenglikda $\psi_2(x) \equiv 0$ bo'lsin (3.86) ni ushbu ko'rinishda yozib olamiz

$$\frac{d}{dx}\left[D_{-1,x}^{-2\beta}\tau(x) + D_{-1,x}^{-2\beta}\tau(x)\right] = 0$$

yoki

$$\left[D_{-1,x}^{-2\beta}\tau(x) + D_{-1,x}^{-2\beta}\tau(x)\right] = c = const\ . \tag{3.87}$$

(3.87) tenglikka $D_{-1,x}^{2\beta}$ operatorini qo'llab, va

$$D_{-1,x}^{2\beta}D_{x,1}^{-2\beta}\tau(x) = \cos 2\beta\pi\tau(x) + \frac{\sin 2\beta\pi}{\pi}\int_{-1}^{1}\left(\frac{1+t}{1+x}\right)^{2\beta}\frac{\tau(t)dt}{t-x}\ ,$$

$$D_{-1,x}^{2\beta}D_{-1,x}^{-2\beta} = \tau(x)\ , \quad D_{-1,x}^{2\beta}c = \frac{(1+x)^{-2\beta}}{\Gamma(1-2\beta)}c$$

formulalarni hisobga olib, ushbu singulyar integral tenglamani hosil qilamiz

$$\varphi(x) + \frac{tg\beta\pi}{\pi}\int_{-1}^{1}\frac{\varphi(t)dt}{t-x} = f_0(x) \tag{3.88}$$

bu yerda

$\varphi(x) = (1+x)^{2\beta}\tau(x)\ ,\ f_0(x) = \dfrac{c}{2\Gamma(1-2\beta)\cos^2\beta\pi}$. (3.88) integral tenglama yechimini (-1,1) intervalda Gyolder sinfiga tegishli, $x=-1,\ x=1$ nuqtalarda esa uzluksiz bo'lgan funktsiyalar sinfida izlaymiz ya'ni $h(-1,1)$ sinfda. Bu sinfda (3.88) tenglama indeksini hisoblaymiz. (3.88) singulyar integral tenglamada $A=1,\ B=tg\beta\pi$.

Ushbu funksiyani tuzamiz

$$G(t) = \frac{A(t) - iB(t)}{A(t) + iB(t)}$$

bevosita hisoblash yordamida ko'rsatish mumkinki $\alpha_0 + i\beta_0 = \beta - k,$ ya'ni $\alpha_0 = \beta - k_1,\ \beta_0 = 0;\ \alpha_1 + i\beta_1 = -\beta + k,$ ya'ni $\alpha_1 = -\beta + k,\ \beta_1 = 0.$ Endi λ_0 va λ_1 butun sonlarni shunday tanlaymizki $0 < \alpha_k + \lambda_k < 1,\ k = 0,1$ tengsizliklar o'rinli bulsin, bunda $\lambda_0 = k,\ \lambda_1 = 1 - k$

Demak (3.88) tenglama indeksi

$$\chi = -(\lambda_0 + \lambda_1) = -1.$$

Shunday qilib (3.88) tenglama yechimining $h(-1,1)$ sinfdagi indeksi -1 ga teng, kanonik funksiya esa ushbu ko'rinishda bo'ladi

$$X(z) = (1+z)^{\delta+\lambda}(1-z)^{\delta+\lambda} = (1+z)^{\beta}(1-z)^{1-\beta}.$$

Indeks $\chi < 0,$ demak singulyar integral tenglamalar uchun Nyoter nazariyasi, Fredgolm integral tenglamalar nazariyasi bilan ustma-ust tushmaydi. (bu nazariyalar faqat $\chi = 0$ bo'lgandagina ustma-ust tushadi).

$\chi < 0$ bo'lganda (3.88) tenglamasining yagona yechimi. Ushbu [6]

$$\int_{-1}^{1} \frac{t^{\mu} f_0(t)dt}{[a(t) + ib(t)]X^{+}(t)} = 0,\ (\mu = 0,1... - \chi - 1) \qquad (3.89)$$

zaruriy va yetarli shartlar bajarilgandagina o'rinli bo'ladi.

Bizning tenglamamiz uchun $\chi = -1$ demak $\mu = 0,\ f_0(t) = \dfrac{c}{2\Gamma(1-2\beta)\cos^2\beta\pi}$ va (3.89) shart ushbu ko'rinishni oladi.

$$\int_{-1}^{1} \frac{f(t)dt}{(a(t) + ib(t)X^{+}(t))} = \frac{c}{2\cos^2\beta\pi\Gamma(1-2\beta)(1+itg\beta\pi)} \int_{-1}^{1} \frac{dt}{X^{+}(t)} = 0$$

bu yerdan $c = 0$ ekanligi kelib chiqadi.

Shunday qilib, biz

$$\varphi(x) + \frac{tg\beta\pi}{\pi} \int_{-1}^{1} \frac{\varphi(t)dt}{t - x} = 0 \qquad (3.90)$$

tenglamaga kelamiz. Bu tenglama uzluksiz funksiyalar sinfida faqat $\varphi(x)\equiv 0$ trivial yechimga ega bo'ladi, demak $\tau(x)\equiv 0$. Shunday qilib, biz $БC$ masalasi yechimining yagonaligini (3.77) shart buzilgan holda isbotladik.

Endi (3.77) shart buzilgan holda $БC$ masalasi yechimini mavjudligini ko'rsatamiz.

(3.85) shartga asosan

$$a(x)=(1+x)^{\beta}\mu(x), \quad b(x)=(1-x)^{\beta}\mu(x)$$

bu yerda $\mu(x)\in C^{(0,\alpha)}[-1,1]$ va $\mu(x)\neq 0\ \forall x\in[-1,1]$. Bu holda (3.75) tenglamani ushbu ko'rinishda yozib olish mumkin

$$D_{-1,x}^{1-2\beta}\tau(x)+D_{x,1}^{1-2\beta}\tau(x)=-\frac{\psi_2(x)}{\gamma(1-x^2)^{\beta}\mu(x)}$$

yoki

$$D_{-1,x}^{-2\beta}\tau(x)-D_{x,1}^{-2\beta}\tau(x)=\int_{-1}^{x}\psi_4(t)dt+c \tag{3.91}$$

bu yerda

$$\psi_4(x)=-\psi_2(x)/\gamma(1-x^2)^{\beta}\mu(x) \tag{3.92}$$

(3.91) tenglikka $D_{-1,x}^{2\beta}$ operatorni qo'llab ushbu integral tenglamaga kelamiz

$$\varphi(x)+\frac{tg\beta\pi}{\pi}\int_{-1}^{1}\frac{\varphi(t)dt}{t-x}=F(x) \tag{3.93}$$

bu yerda

$$\varphi(x)=(1+x)^{2\beta}\tau(x),$$

$$F(x)=\frac{(1+x)^{2\beta}}{2\Gamma(1-2\beta)\cos^2\beta\pi}\int_{-1}^{x}\frac{\psi_4(t)dt}{(x-t)^{2\beta}}+\frac{c}{2\Gamma(1-2\beta)\cos^2\beta\pi}. \tag{3.94}$$

Yuqorida (3.93) tenglamaning $h(-1,1)$ sinfdagi indeksi $\chi=-1$ ekanligini ko'rsatgan edik. Demak (3.93) tenglamaning yagona yechimi

$$\int_{-1}^{1}\frac{t^{\mu}F(t)dt}{[a(t)+ib(t)]X^{+}(t)}=0,\ (\mu=0,1...-\chi-1) \qquad (3.95)$$

zaruriy va yetarli shart bajarilgandagina mavjud bo'ladi.

Bu yerda $a(t)=1, b(t)=tg\beta\pi,\ X^{+}(t)=(1-t^{2})^{\beta}$

$\chi=-1$ bo'lgani uchun $\mu=0$. (3.94) ga asosan (3.95) tenglikni ushbu ko'rinishda yozib olamiz

$$\int_{-1}^{1}\frac{cdt}{(1+t)^{\beta}(1-t)^{\beta}}+\int_{-1}^{1}\frac{(1+t)^{2\beta}dt}{(1+t)^{\beta}(1-t)^{\beta}}\int_{-1}^{t}\frac{\psi_4(s)ds}{(t-s)^{2\beta}}=0 \qquad (3.96)$$

bu yerda integrallash tartibini o'zgartirib va xosmas intergrallarni hisoblab c uchun quyidagi qiymatni hosil qilamiz

$$c=-\frac{2^{\beta}\Gamma(\beta)\Gamma(1-2\beta)\sin\beta\pi}{\Gamma(1-\beta)}\int_{-1}^{1}\frac{\psi_4(s)F\left(\beta,-\beta;1-\beta;\frac{1-s}{2}\right)}{(1-s)^{\beta}}ds$$

Bu yerda $F(a,b,c;z)-$Gaussning gipergeometrik funksiyasi.

Shunday qilib, (3.96) shart bajarilganda (3.93) tenglamaning yagona yechimi mavjud.

(3.93) tenglamani Karleman usulida yechamiz. Shu maqsadda

$$\Phi(z)=\frac{1}{2\pi i}\int_{-1}^{1}\frac{\varphi(t)dt}{t-z},\qquad \Phi(\infty)=0 \qquad (3.97)$$

funksiyani kiritamiz.

Soxotskiy-Plemel formulalariga asosan

$$\Phi^{+}(x)+\Phi^{-}(x)=\frac{1}{\pi i}\int_{-1}^{1}\frac{\varphi(t)dt}{t-x}, \qquad (3.98)$$

$$\Phi^{+}(x)-\Phi^{-}(x)=\varphi(x)\,. \qquad (3.99)$$

Bu yerda

$$\Phi^{+}(x)=\lim_{y\to+0}\Phi(x+iy),$$
$$\Phi^{-}(x)=\lim_{y\to-0}\Phi(x+iy),$$

(3.98) va (3.99) formulalarga asosan (3.93) tenglama ushbu ko'rinishni oladi

$$\Phi^{+}(x)=G(x)\Phi^{-}(x)+f(x)\,,\qquad(-\infty<x<+\infty)\qquad(3.100)$$

bu yerda

$$G(x)=\begin{cases}\dfrac{1-itg\beta\pi}{1+itg\beta\pi}, & -1\le x\le 1,\\ 1\quad, & x\in(-\infty,-1)\cup(1,+\infty);\end{cases}\qquad(3.101)$$

$$f(x)=\begin{cases}\dfrac{F(x)}{1+itg\beta\pi}, & -1\le x\le 1,\\ 0\quad, & x\in(-\infty,-1)\cup(1,+\infty).\end{cases}\qquad(3.102)$$

Shunday qilib, biz golomorf funksiyalar uchun Riman masalasiga keldik: yuqori va quyi yarim tekisliklarda golomorf, cheksiz uzoqlashgan nuqtada $\Phi(\infty)=0$, haqiqiy o'qda esa (3.100) shartni qanoatlantiruvchi $\Phi(z)$ funksiya topilsin.

Ushbu funksiyani o'rganamiz

$$\Psi(z)=\exp\left\{\frac{1}{2\pi i}\int_{-1}^{1}\frac{\ln G(t)dt}{t-z}\right\}=\exp\{-\beta[\ln(1-z)-\ln(-1-z)]\}=$$
$$=\left(\frac{1-z}{1+z}\right)^{\beta}e^{i\beta\arg(-1-z)}$$

(3.103)

(3.103) tenglikdan ushbu chegaraviy qiymatlarni hosil qilamiz

$$\Psi^{+}(x)=\left(\frac{1-x}{1+x}\right)^{\beta}e^{-i\beta\pi}\,,\ \Psi^{-}(x)=\left(\frac{1-x}{1+x}\right)^{\beta}e^{i\beta\pi}\,.\qquad(3.104)$$

Endi kanonik funktsiyani tuzamiz

$$\Pi(z) = (1+z)^{\lambda_0}(1-z)^{\lambda_1}$$

$$X(z) = e^{\psi(z)}\Pi(z) = \left(\frac{1-z}{1+z}\right)^{\beta} e^{i\beta \arg(-1-z)}(1+z)^0(1-z)^1.$$

Shunday qilib,

$$\begin{aligned} X^+(x) &= (1+x)^{\beta}(1-x)^{1-\beta}e^{-i\beta\pi} \\ X^-(x) &= (1+x)^{\beta}(1-x)^{1-\beta}e^{i\beta\pi} \end{aligned} \tag{3.105}$$

(3.105) chegaraviy qiymatlarga asosan (3.100) tenglamani ushbu ko’rinishda yozib olamiz

$$\frac{\Phi^+(x)}{X^+(x)} = \frac{\Phi^-(x)}{X^-(x)} + \frac{F(x)}{X^+(x)}. \tag{3.106}$$

(3.106) tenglamaning xususiy yechimlaridan, biri ushbu ko’rinishda bo’ladi

$$\Phi(z) = \frac{X(z)}{2\pi i}\int_{-1}^{1}\frac{F(t)dt}{t-z}. \tag{3.107}$$

(3.107) ga asosan $\varphi(x)$ ni topamiz

$$\begin{aligned} \varphi(x) &= \Phi^+(t) - \Phi^-(t) = \cos^2\beta\pi F(x) - \\ &- \frac{\sin 2\beta\pi}{2\pi}\int_{-1}^{1}\left(\frac{1-x}{1-t}\right)^{1-\beta}\left(\frac{1+x}{1+t}\right)^{\beta}\frac{F(t)dt}{t-x} \end{aligned} \tag{3.108}$$

(3.108) formula (3.88) singulyar integral tenglama yechimini beradi.

Endi $F(x)$ ning ifodasini (3.94) formuladan (3.108) yechimga qo’yib ushbu ifodaga ega bo’lamiz

$$\varphi(x)=\frac{c}{2\Gamma(1-2\beta)}+\frac{(1+x)^{2\beta}}{2\Gamma(1-2\beta)}\int_{-1}^{x}\frac{\psi_4(t)dt}{(x-t)^{2\beta}}-$$

$$-\frac{c\cdot tg\beta\pi}{2\pi\Gamma(1-2\beta)}(1+x)^{\beta}(1-x)^{1-\beta}\times$$

$$\times\int_{-1}^{1}\frac{(1+t)^{-\beta}}{(1-t)^{1-\beta}}\frac{dt}{t-x}-\frac{tg\beta\pi}{2\pi\Gamma(1-2\beta)}(1+x)^{\beta}(1-x)^{1-\beta}\times \qquad (3.109)$$

$$\times\int_{-1}^{1}\frac{(1+t)^{\beta}}{(1-t)^{1-\beta}}\frac{dt}{t-x}\int_{-1}^{1}\frac{\psi_4(s)ds}{(t-s)^{2\beta}}.$$

Ushbu tenglikka asosan

$$\int_{-1}^{1}\frac{(1+t)^{-\beta}}{(1-t)^{1-\beta}}\frac{dt}{t-x}=\pi ctg\beta\pi\frac{(1+x)^{-\beta}}{(1-x)^{1-\beta}}$$

(3.109) ifoda ushbu ko’rinishda bo’ladi

$$\varphi(x)=\frac{(1+x)^{2\beta}}{2\Gamma(1-2\beta)}\int_{-1}^{x}\frac{\psi_4(t)dt}{(x-t)^{2\beta}}-\frac{tg\beta\pi}{2\pi\Gamma(1-2\beta)}(1+x)^{\beta}(1-x)^{1-\beta}\times$$

$$\times\int_{-1}^{1}\frac{(1+t)^{\beta}dt}{(1-t)^{2\beta}(t-x)}\int_{-1}^{1}\frac{\psi_4(s)ds}{(t-s)^{2\beta}}$$

(3.110)

(3.110) formulada $\varphi(x)=(1+x)^{2\beta}\tau(x)$ tenglikni hisobga olib uni ushbu ko’rinishda yozamiz

$$\tau(x)=\frac{1}{2\Gamma(1-2\beta)}\int_{-1}^{x}\frac{\varphi_4(t)dt}{(x-t)^{2\beta}}-\frac{tg\beta\pi}{2\pi\Gamma(1-2\beta)}\frac{(1-x)^{1-\beta}}{(1+x)^{\beta}}\times$$

$$\times\int_{-1}^{1}\frac{(1+t)^{\beta}}{(1-t)^{1-\beta}}\frac{dt}{t-x}\int_{-1}^{t}\frac{\psi_4(s)ds}{(t-s)^{2\beta}} \qquad (3.111)$$

bu yerda ushbu

$$\left(\frac{1+t}{1+x}\right)^{\beta}=\left(\frac{1+t}{1+x}\right)^{\beta}\frac{t-x}{1+t}-\left(\frac{1+t}{1+x}\right)^{\beta-1}$$

ayniyatni e’tiborga olib, (3.111) yechimni quyidagi ko’rinishda yozib olamiz

$$\tau(x)=\frac{1}{2\Gamma(1-2\beta)}\int_{-1}^{x}\frac{\psi_4(t)dt}{(x-t)^{2\beta}}-\frac{tg\beta\pi}{2\pi\Gamma(1-2\beta)}(1+x)^{1-\beta}(1-x)^{1-\beta}\times$$

$$\times\int_{-1}^{1}\frac{(1+t)^{\beta-1}}{(1-t)^{1-\beta}}\frac{dt}{t-x}\int_{-1}^{t}\frac{\psi_4(s)ds}{(t-s)^{2\beta}}-\frac{tg\beta\pi}{2\pi\Gamma(1-2\beta)}\frac{(1-x)^{1-\beta}}{(1+x)^{\beta}}\times \qquad (3.112)$$

$$\times\int_{-1}^{1}\frac{(1+t)^{\beta-1}}{(1-t)^{1-\beta}}dt\int_{-1}^{t}\frac{\psi_4(s)ds}{(t-s)^{2\beta}}.$$

(3.112) tenglikning oxirgi integralida integrallash tartibini o'zgartirib ushbu tenglikni hosil qilamiz

$$I=\int_{-1}^{1}\frac{(1+t)^{\beta-1}}{(1-t)^{1-\beta}}dt\int_{-1}^{t}\frac{\psi_4(s)ds}{(t-s)^{2\beta}}=\int_{-1}^{1}\psi_4(s)\int_{s}^{1}\frac{(1+t)^{\beta-1}}{(1-t)^{1-\beta}}\frac{ds}{(t-s)^{2\beta}}$$

Endi ichki integralda $t=1+(s-1)\sigma$ almashtirish bajarib va gipergeometrik funksiyaning integral ifodasi hamda uning $F(a,b,c;x)=(1-x)^{-a}$ xossasidan foydalanib ushbu qiymatni hosil qilamiz

$$I=2^{2\beta-1}\frac{\Gamma(\beta)\Gamma(1-2\beta)}{\Gamma(1-\beta)}\int_{-1}^{1}\frac{\varphi(s)ds}{(1-s^2)^{\beta}} \qquad (3.113)$$

(3.113) tenglikka asosan (3.112) yechimni ushbu ko'rinishda yozib olamiz

$$\tau(x)=\frac{1}{2\Gamma(1-2\beta)}\int_{-1}^{x}\frac{\psi_4(t)dt}{(x-t)^{2\beta}}-\frac{tg\beta\pi}{2\pi\Gamma(1-2\beta)}(1+x)^{1-\beta}\times$$

$$\times\int_{-1}^{1}\frac{(1+t^2)^{\beta-1}}{t-x}dt\int_{-1}^{t}\frac{\psi_4(s)ds}{(t-s)^{2\beta}}-\frac{2^{2\beta-2}tg\beta\pi\Gamma(\beta)}{\pi\Gamma(1-\beta)}\times$$

$$\times\frac{(1-x)^{1-\beta}}{(1+x)^{\beta}}\int_{-1}^{1}\frac{\psi_4(s)ds}{(1-s^2)^{2\beta}}.$$

(3.114)

(3.114) yechim uzluksiz bo'lishi uchun

$$\int_{-1}^{1} \frac{\psi_4(s)ds}{(1-s^2)^2} = 0 \qquad (3.115)$$

tenglikning bajarilishi zarur. Bu holda (3.88) singulyar integral tenglamaning uzluksiz yechimi ushbu ko'rinishda bo'ladi

$$\tau(x) = \frac{1}{2\Gamma(1-2\beta)} \int_{-1}^{x} \frac{\psi_4(t)dt}{(x-t)^{2\beta}} -$$

$$- \frac{tg\beta\pi}{2\pi\Gamma(1-2\beta)} \int_{-1}^{1} \frac{(1-t^2)^{\beta-1}}{t-x} ds \int_{-1}^{t} \frac{\psi_4(s)ds}{(t-s)^{2\beta}}. \qquad (3.116)$$

Shunday qilib BS masalasi ushbu Dirixle masalasiga olib kelindi: Ω^+ sohada (3.62) tenglamaning ushbu

$$u|_{\sigma_0} = \varphi(x),\ u(x.0) = \tau(x)\ ,\ x \in \bar{J} \qquad (3.117)$$

shartlarni qanoatlantiruvchi regulyar yechimi $u(x,y) \in C(\overline{\Omega})$ funksiya topilsin. Bu yerda $\tau(x)$ (3.116) formula bilan ifodalanadi va $\varphi(0) = \tau(1), \varphi(l) = \tau(-1)$ shartlari bajariladi. Faraz qilaylik $\psi_4(x)$ ushbu shartlarni qanoatlantirsin:

1. $\psi_4(x) \in C^1[0,1]$;

2. $\int_{0}^{t} (1-t)^{1-2\beta} \psi'_2(t)dt = O(1-t)^{1-\beta+\varepsilon}$ (3.118)

ε -ixtiyoriy yetarli kichik son.

(3.116) formulada bo'laklab integrallash amalini bajarib

$$\tau(x)=\frac{\psi_4(-1)}{2\Gamma(2-2\beta)}(1+x)^{1-2\beta}+\frac{1}{2\Gamma(2-2\beta)}\times$$
$$\times\int_{-1}^{x}\psi'_4(t)(x-t)^{1-2\beta}dt-\frac{tg\beta\pi\psi_4(-1)(1-x^2)^{1-\beta}}{2\pi\Gamma(2-2\beta)}\times$$
$$\times\int_{-1}^{1}\frac{(1+t)^{-\beta}(1-t)^{\beta-1}}{t-x}dt-\frac{tg\beta\pi(1-x^2)^{1-\beta}}{2\pi\Gamma(2-2\beta)}\times \tag{3.119}$$
$$\times\int_{-1}^{1}\frac{(1-t^2)^{\beta-1}}{t-x}dt\int_{-1}^{1}\psi'_4(s)(t-s)^{1-2\beta}ds$$

tenglikni hosil qilamiz.

Ushbu

$$\int_{-1}^{1}\frac{(1+t)^{-\beta}(1-t)^{\beta-1}}{t-x}dt=\frac{\pi ctg\beta\pi}{(1+x)^{\beta}(1-x)^{1-\beta}}$$

tenglikni hisobga olib (3.119) munosabatni ushbu ko’rinishda yozib olamiz

$$\tau(x)=\frac{1}{2\Gamma(2-2\beta)}\int_{-1}^{x}\psi'_4(t)(x-t)^{1-2\beta}dt-\frac{tg\beta\pi(1-x^2)^{1-\beta}}{2\pi\Gamma(2-2\beta)}\int_{-1}^{1}\frac{(1-t^2)^{\beta-1}}{t-x}dt\int_{-1}^{1}\psi'_4(s)(t-s)^{1-2\beta}ds \tag{3.120}$$

Bu yerdan (3.118) ni hisobga olib $\tau(x)\in C[-1,1]\cap C^1(-1,1)$ va $\tau'(x)$ hosila (-1,1) interval chegaralarida β dan katta bo’lmagan tartibda cheksizlikka aylanishini ko’rish qiyin emas, shu bilan birga $\tau(-1)=\tau(1)=0$

Endi $\nu(x)$ ni topamiz. Buning uchun Dirixle masalasini yechamiz. Bu yechimdan u bo’yicha hosila olib ushbu tenglikka kelamiz

$$\frac{\partial u}{\partial y}=k_2(1-\beta_0)\int_{-1}^{1}\tau(t)\frac{\partial}{\partial y}y^{\beta_0}\left\{\left[(x-t)^2+\frac{4}{(m+2)^2}y^{m+2}\right]^{\beta-1}-\right.$$
$$\left.-\left[(1-xt)^2+\frac{4t^2}{(m+2)^2}y^{m+2}\right]^{\beta-1}\right\}dt+k_2(1-\beta_0)(m+2)\int_{-1}^{1}\varphi(t)\times$$

$$\times \frac{\partial}{\partial y}\left\{(1-R^2)y^{1-\beta_0}\left(r_1^2\right)^{\beta-2}F(1-\beta,2-\beta,2-2\beta;1-\sigma)\right\}dt. \qquad (3.121)$$

Ushbu tenglikni to'g'riligini bevosita tekshirib ko'rish mumkin

$$\frac{\partial}{\partial y}\left\{y^{1-\beta_0}\left[(x-t)^2+\frac{4}{(m+2)^2}y^{m+2}\right]^{\beta-1}-y^{1-\beta_0}\left[(1-xt)^2+\frac{4t^2}{(m+2)^2}y^{m+2}\right]^{\beta-1}\right\}=$$

$$=\frac{m+2}{2}y^{-\beta_0}\frac{\partial}{\partial t}\left\{(x-t)\left[(x-t)^2+\frac{4}{(m+2)^2}y^{m+2}\right]^{\beta-1}-\right.$$

$$\left.-\frac{1-xt}{x}\left[(1-xt)^2+\frac{4t^2}{(m+2)^2}y^{m+2}\right]^{\beta-1}\right\}. \qquad (3.122)$$

Endi (3.121) tenglikning o'ng tomonidagi birinchi integralda (3.122) tenglikni e'tiborga olib, bo'laklab integrallash operatsiyasini bajaramiz va hosil bo'lgan yangi tenglikni y^{β_0} ga ko'paytirib, u nolga intilganda limitga o'tib $\nu(x)$ ni topamiz

$$\nu(x)=-k_2(1-\beta_0)\frac{m+2}{2}\left\{\frac{\tau(1)}{(1-x)^{1-2\beta_0}}+\frac{\tau(-1)}{(1+x)^{1-2\beta_0}}+\int_{-1}^{1}\frac{(x-t)\tau'(t)dt}{|x-t|^{2-2\beta_0}}-\right.$$

$$\left.-(2\beta-1)\int_{-1}^{1}\frac{\tau(t)dt}{(1-xt)^{2-2\beta}}\right\}+\Phi(x),$$

(3.123)

bu yerda

$$\Phi(x)=k_2(1-\beta)(1-\beta_0)(m+2)\,(1-x^2)\int_{-1}^{1}(x^2-2xt+1)^{\beta-1}\varphi(t)dt$$

БС masalasida $u(x,y)$ funksiyaning σ_0 normal chiziqdagi qiymati $\varphi(x)$ ni ushbu ko'rinishda ifodalash mumkin deb faraz qilamiz

$$\varphi(x)=y\varphi_1(x),\quad (x,y)\in\sigma_0$$

bu yerda $\varphi_1(x)\in C^{(0,\alpha)}[-1,1]$. U holda normal chiziqning $\sigma_0: x^2+\frac{4}{(m+2)^2}y^{m+2}=1$ tenglamasidan foydalanib $\Phi(x)$ ni quyidagicha tasvirlaymiz

$$\Phi(x)=2k_2(1-\beta)(1-\beta_0)\left(\frac{m+2}{2}\right)^{(m+4)(m+2)}\int_{-1}^{1}\frac{(1-t^2)^{2/m+2}\varphi_1(t)dt}{(x^2-2xt+1)^{2-\beta}}$$

$\Phi(x)$ funksiya ifodasidan ko'rinib turibdiki $\Phi(x)\in C[-1,1]$ va $(-1,1)$ intervalda $\Phi(x)$ ixtiyoriy tartibli hosilasiga ega. (3.123) formuladan ko'rinib turibdiki 3.2-teoremaga asosan $\nu(x)$ funksiyamiz (-1,1) intervalda $\beta-\varepsilon$ ko'rsatkich bilan Gyolder shartini qanoatlantiradi.

$\tau(x)$ va $\nu(x)$ ma'lum bo'lgandan keyin Ω^{-} sohada yechimni shakli o'zgargan Koshi masalasi yechimi sifatida tiklaymiz. Bu yechim R_1 sinfga tegishli.

III bob yuzasidan xulosa.

Ushbu bobda soha ichida buziladigan singulyar koeffitsentli giperbolik tipdagi tenglamalar uchun soha ichida yotuvchi ikkita maxsus chiziqdagi yechimning qiymatini Bitsadze-Samarskiy sharti bilan bog'lovchi masala o'rganilgan. Bunda ikkita nuqtada siljishga ega bo'lgan Volterra tipidagi integral tenglama hosil qilingan. Hosil qilingan integral tenglama ketma-ket yaqinlashish usuli yordamida yechilgan.

Quyidagi

$$sign y|y|^m u_{xx} + u_{yy} + (\beta_0 / y) u_y = 0 \qquad (3.62)$$

aralash tipdagi tenglama uchun Bitsadze-Samarskiy masalasi qo'yilgan, bu yerda m va β_0 -o'zgarmas sonlar bo'lib, ular uchun $m > 0, -m/2 < \beta_0 < 1$ tengsizliklar o'rinli.

Qo'yilgan masala yechimi yagonaligi ekstremum prinsipi yordamida isbotlangan, masala yechimi mavjudligi esa

$$(1-x)^{\beta} a(x) - (1+x)^{\beta} b(x) = 0$$

holida o'rganilgan.

XULOSA

Magistrlik dissertatsiyasida singulyar koeffitsientli Gellerstedt tenglamasi uchun o'zida lokal va nolokal shartlarni birlashtirgan chegaraviy masalalar o'rganildi. Chegaraviy masalalar yechimlarining yagonaligi ekstremum printsipi yordamida giperbolik qismida shakli o'zgargan Koshi masalasining yechimini beruvchi Darbu formulasidan foydalanildi.

Aralash sohaning elliptik qismida singulyar koeffitsientli Gellerstedt tenglamasi uchun asosiy chegaraviy masalalar Direxle va shakli o'zgargan Xolmgren masalalarining yechimlarini beruvchi yechimlarining integral formasidan foydalanildi.

Buziluvchan umumiy giperbolik tipdagi tenglama uchun Koshi masalasini normal yechilishining

$$\lim_{y\to -0}\frac{ya(x,y)}{\sqrt{-K(y)}}=0$$

Protter sharti buzilishi izohlangan, bunda $h(x,y)>0$, $K(0)\equiv 0$, $K(y)<0$, $y<0$.

Soha ichida buziladigan singulyar koeffitsentli giperbolik tipdagi tenglamalar uchun soha ichida yotuvchi ikkita maxsus chiziqdagi yechimning qiymatini Bitsadze-Samarskiy sharti bilan bog'lovchi masala o'rganildi. Bunda ikkita nuqtada siljishga ega bo'lgan Volterra tipidagi integral tenglama hosil qilinib, hosil qilingan integral tenglama ketma-ket yaqinlashish usuli yordamida yechildi.

Quyidagi

$$(signy)|y|^{m}u_{xx}+u_{yy}+(\beta_0/y)u_y=0$$

aralash tipdagi tenglama uchun Bitsadze-Samarskiy masalasi qo'yilgan, bu yerda m va β_0 -o'zgarmas sonlar bo'lib, ular uchun $m>0,-m/2<\beta_0<1$ tengsizliklar o'rinli.

Qo'yilgan masala yechimi yagonaligi ekstremum prinsipi yordamida isbotlangan, masala yechimi mavjudligi esa

$$(1-x)^{\beta}a(x)-(1+x)^{\beta}b(x)=0$$

holida o’rganilgan.

Bu holda tenglama indeksi $\chi < 0,$ demak singulyar integral tenglamalar uchun Nyoter nazariyasi Fredgolm integral tenglamalar nazariyasi bilan ustma-ust tushmaydi (bu nazariyalar faqat $\chi = 0$ bo’lgandagina ustma-ust tushadi).

$\chi < 0$ bo’lganda integral tenglama yagona yechimga ega ekanligining

$$\int_{-1}^{1} \frac{t^{\mu} f_0(t)dt}{[a(t)+ib(t)]X^{+}(t)} = 0, \ (\mu = 0,1...-\chi-1) \qquad (3.35)$$

zaruriy va yetarli sharti bajarilgan, yani tenglama yechimi yagonaligi isbotlangan.

Masala yechimi mavjudligi Karleman usulida isbotlanagan.

FOYDALANILGAN ADABIYOTLAR RO'YXATI

1. O'zbekiston Respublikasining "Ta'lim to'g'risida" gi qonuni 23.09.2020, ORQ-637 son.
2. O'zbekiston Respublikasi oliy talim tizimini 2030-yilgacha rivojlantirish konsepsiyasi. O'zbekiston Respublikasi Prezidentining farmoni 08.10.2019-yildagi PF-5847-son.
3. Mirziyoev Sh. M. "Yangi O'zbekiston taraqqiyot strategiyasi" O'ZBEKISTON-2022 217-bet.
4. O'zbekiston Respublikasining "Kadrlar tayyorlash milliy dasturi". Barkamol avlod – O'zbekiston taraqqiyotining poydevori – Toshkent, Sharq 1997.
5. Бицадзе А.В. Уравнения смешанного типа. М.: Изд-во АН СССР. 1959, -164 с.
6. Бицадзе А.В. Некоторые классы уравнений в частных производных. (Наука, М., 1981). -448 с.
7. Бицадзе А.В., Самарский А.А. О некоторых простейших обобщениях линейных эллиптических краевых задач //ДАН СССР. 1969. 185. №4. С. 739-740.
8. Ф.Д., Черский Ю.Н. Уравнения типа свертки. М.: Наука, 1978, -269 с.
9. Девингталь Ю.В. О существовании и единственности решения одной задачи Ф.И.Франкля//Изв. вузов, Математика. 1958.Т.2.№3.С.39-51.
10. Солдатов А.П К нетеровской теории операторов. Одномерные сингулярное операторы общего вида//Дифференц. уравнения. 1978. Т.14, №4, С.706-718.
11. Дудучава Р.В. Интегральные уравнения типа свертки с разрывными предсимволами. Сингулярные интегральные уравнения с неподвижными особенностями и их приложения к задачам механики. (Мицниереба. Тбилиси 1978).

12. Ильин В.А., Моисеев Е.И. Нелокальная краевая задача для оператора Штурма-Лиувилля в дифференциальной и разностной трактовках//Докл. АН СССР. 1986. Т. 291. № 3. С. 534-539.
13. Жегалов В.И. Краевая задача для уравнения смешанного типа с граничными условиями на переходной линии//Ученые записки Казанского ун-та, 1962.-Т.122. кн.3. С.3-16.
14. Кальменов Т.Ш. Краевые задачи для линейных уравнений в частных производных гиперболического типа. Шымкент.: Гылым. 1993.-327 с.
15. Каратопраклиев Г. Об одном обобщении задачи Трикоми//ДАН СССР. 1964. Т. 158, №2, С. 271-274.
16. Линь Цзянь-бин. О некоторых задачах Франкля//Вестник ЛГУ, сер. Матем, мех., и астр. 1961.Т.3.№13.С.18-39.
17. Михлин С.Г. Об интегральном уравнении F.TRICOMI//ДАН СССР. 1948, Т.59.№6.С.1053-1056.
18. Мусхелишвили Н.И. Сингулярные интегральные уравнения. М.: Наука, 1968, -512 с.
19. Нахушев А.М. О некоторых краевых задачах для гиперболических уравнений и уравнений смешанного типа//Дифференциальные уравнения.1969.Т.5.№1.С.44-59.
20. Салахитдинов М.С., Мирсабуров М. Задачи Бицадзе-Самарского для одного класса вырождающегося гиперболических уравнений//Дифференц.уравнения 2002.Т.38.№ 2.С.271-276.
21. Салахитдинов М.С., Мирсабуров М. Нелокальние задачи для уравнений смешанного типа с сингулярными коэффициентами. Ташкент 2005. “Universitet”. “Yangiyo’l poliyrafservis” -224 с.
22. Мирсабуров М. Задача Бицадзе-Самарского для вырождающегося эллиптического уравнения. / / Докл. АН РУз. 1998 , No2. с. 6-9.

23. Франкль Ф.И. Обтекание профилей газом с местной сверхзвуковой зоной, оканчивающейся прямим скачком уплотнения//Прикладная математика и механика.1956.Т.20.№2.С.196-202 .
24. Darboux G. Theorie generale des surfaces.-Paris, 1894,t.3.
25. Gellerstedt S. Sur un probleme aux limites pour ane equation lineaire aux derives partielles du second ordre de tupy mixte. Thesis, Uppsala,1935.
26. Gellerstedt S. Quelgues problemes mixted pour I'equation $y^m z_{xx} + z_{yy} = 0.$ //Arkiv for matematik, astronomi och fysik. 1938, vand 26A. №3, p.1-32.
27. Holmgren E. Sur un probleme aux limites pour I'equation $y^m z_{xx} + z_{yy} = 0.$ //Arkiv for matematik, astronomi och fysik. 1927, vand 19B.№14.
28. Hopf E. Elementare Betrachtungen ber die Lusungen partiellen Differentialgleichungen zweiter Ordnung vom elliptischen Typus//Situngsb. Preuss. Akad. wis., 1927. Bd 19.p.147-152.
29. Чориева С.Т. Задача Бицадзе-Самарского с условием Франкля на отрезке линии вырождения для уравнения смещанного типа с сингулярным коэффициентом//Известия вузов. Математика. 2013. №5. С.51-60.
30. Пулькин С.П.Задачи Трикоми для обобщенного уравнения Лаврентьева-Бицадзе//Докл. АН СССР.1958. Т.118.№1.С.38-41.
31. "Giperbolik tipdagi tenglamalar uchun Koshi masalasi", Academic Research Journal, 2022.
32. "Buziluvchan giperbolik tipdagi tenglamalar uchun Koshi masalasining qo'yilishi", International scientific-practical conference, Sirdaryo, 2022.