MAGISTRLIK DISSERTATSIYA ISHI

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TERMIZ DAVLAT UNIVERSITETI

MAGISTRATURA BO‘LIMI

Qo‘lyozma huquqida

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ISMOILOV MUHRIDDIN MAMATQOBIL O‘G‘LI

SONLAR NAZARIYASI USULLARIDAN FOYDALANIB 2-TUR VOLTERRA INTEGRAL TENGLAMASINI TAQRIBIY YECHISH.

70540101- Matematika (algebra va funksional analiz)

Magistr akademik darajasini olish uchun yozilgan

DISSERTATSIYA

Ilmiy rahbar: f.m.f.n. I.Abirayev

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Ilmiy rahbar: ____________ **f-m.f.n. I.Abirayev**

Kafedra mudiri: ____________ **f-m.f.f.d,(phD) S.Choriyeva**

Magistratura bo'limi boshlig'i: ____________ **f-m.f.f.d,(phD) A.Narbayev**

Temiz Davlat Universiteti fizika-matematika fakulteti «(Algebra va funksional analiz» kafedrasi 2-kurs magistranti Ismoilov Muhriddin Mamatqobil o'g'lining "Sonlar nazaryasi usullaridan foydalanib 2-tur Volterra integral tenglamasini taqribiy yechish" Matematika 70540101 (yo'nalishi bo'yicha) magistr darajasini olish uchun yozilgan dissertatsiyasi

ANNOTASIYASI

Tayanch so'zlar: Interpolyatsion ko'phadlar, Lagranj interpolyatsion formulasi, Interpolyatsiyalash xatoligi, kub splayn, Singulyar integral tenglamalar.

Tadqiqot obektlari: 2-tur Volterra integral tenglamasini taqribiy yechish.

Ishning maqsadi: Sonlar nazariyasi usullari yordamida 2-tur Volterra integral tenglamasini taqribiy yechish

Tadqiqot metodlari: Gellerstedt tenglamasi uchun qo'yilgan Bitsadze-Samarskiy masalasi yechimining yagonaligi A.V.Bitsadzening ekstremum prinsipi asosida, yechimning mavjudligi esa integral tenglamalar usulida isbotlangan. Bu erda Gaussning gipergeometrik funksiyalar nazariyasidan, integro- differensial operatorlar nazariyasidan, Singulyar integral nazariyasidan, hamda Fredgolm teoremalaridan keng foydalanilgan.

Olingan natijalar va ularning yangiligi:

- Fredgolm II tur integral tenglamasini taqribiy yechish o'rganilgan.
- Kvadratur va kubatur formula qurish o'rganilgan.

Dissertatsiya ishining nazariy ahamiyati: olingan natijalar nazariy ahamiyatga ega bo'lib, ulardan Volterra II tur integral tenglamasini taqribiy yechishda keng foydalanish mumkin.

Ishning amaliy ahamiyati: Magistrlik dissertatsiya ishida to'plangan materiallardan Fredgolm II tur integral tenglamasini taqribiy yechish haqida ma'lumotlar olish mumkin. Bundan tashqari kvadratur va kubatur formula qurishda foydalanish mumkin.

"Approximate solution of Volterra integral equation of the 2nd type using the methods of number theory" by Ismailov Muhriddin Mamatqabil, 2nd-year master's student of the Faculty of Physics and Mathematics of Temiz State University, Department "Algebra and Functional Analysis" Mathematics 70540101 (in the direction) dissertation written for obtaining a master's degree

ANNOTATION

Key words: Interpolation polynomials, Lagrange interpolation formula, Interpolation error, cubic spline, Singular integral equations.

Research objects: approximate solution of Volterra integral equation of the 2nd type.

Research methods: the uniqueness of the solution of the Bitsadze-Samarsky problem for the Gellerstedt equation was proved based on the extremum principle of A. V. Bitsadze, and the existence of the solution was proved by the method of integral equations. Gauss's theory of hypergeometric functions, theory of integro-differential operators, theory of singular integral, and Fredholm's theorems are widely used here.

The results obtained and their novelty:

- The approximate solution of the Fredholm integral equation of type II is studied.
- Construction of quadrature and cubature formula was learned.

Theoretical importance of the thesis work: the obtained results are of theoretical importance and can be widely used in the approximate solution of the integral equation of Volterra II type.

Practical importance of the work: from the materials collected in the Master's dissertation, information can be obtained about the approximate solution of the Fredholm integral equation of type II. It can also be used to construct quadrature and cubature formulas.

MUNDARIJA

KIRISH

Matematika hamma aniq fanlarga asos. Bu fanni yaxshi bilgan bola aqlli, keng tafakkurli bo'lib o'sadi, istalgan sohada muvaffaqiyatli ishlab ketadi.

Sh. Mirziyoyev

Hozirgi kunda ilm-fanga Prezidentimiz tomonidan alohida e'tibor berilmoqda. Ayniqsa, 2023 yilning Prezidentimiz tomonidan "Insonga e'tibor va sifatli ta'lim yili" deb e'lon qilinishi hamda bu yilda matematika, kimyo, biologiya va geologiya fanlarini rivojlantirishga alohida e'tibor berilishi biz yosh matematiklar uchun katta imkoniyatlar yaratdi.

Jamiyat ijtimoiy sohasining eng muhim tarkibiy qismlaridan biri ta'lim tarbiya sohasi bo'lib, uning rivojisi yosiy-huquqiy, iqtisodiy va ma'naviy sohalarga bevosita ta'sir etadi hamda ijtimoiy sohalar me'yoriy mohiyatini, kamolot darajasini belgilab beradi.

O'zbekistonda ta'lim tizimining isloh qilishning dasturiy hujjatlarida ta'kidlanganidek, mamlakatimiz ta'lim tizimi xodimlari oldida raqobat bardosh kadrlar tayyorlash, ta'lim tarbiya jarayonini jahon andozalar darajasiga etkazishni ta'minlash asosiy vazifa qilib qo'yilgan.

Shu ma'noda olib qaraganda, yoshlarning yangi avlodi istiqbol masalalarini kun tartibiga dadil qo'yadigan va uni echa oladigan, fikr yuritishning yuksak madaniyatini egallagan, siyosiy hamda ijtimoiy iqtisodiy hayotda o'ziga mustaqil yo'l topa oladigan qobiliyatga ega bo'lishi kerak. Ushbu magistrlik dissertatsiyasi mavzusi ana shu talab va vazifalardan kelibchiqib tanlandi.

Mavzuning dolzarbligi. Fizika, biologiya, mexanika, ekonomika, sotsiologiya va hakozalarning murakkab xodisalarining matematik modellari, xususan qarshiliklar nazariyasida signallarni tiklash, mikro ob'ektlarni kuzatish reduksiyasi, spektroskopmasalalari, uchlamchi yulduzlar konfiguratsiyasining haqiqiy bo'linish funksiyasini aniqlash, tutilgan yorug'lik sistemasining egri

Volterra integral tenglamalariga va ularning sistemasiga keltiriladi ([7] qarang).

Tadqiqot obyekti va predmeti. Fredgolm yoki Volterra integral tenglamalar va ularning sistemasini echishning ko‘pgina analitik va taqribiy usullari mavjud. Analitik usulda echish Laplas, Fure, Mellin va boshqalarning almashtirishlariga asoslangan bo‘lib, qo‘llanilishida tabiiy tusiqlar mavjud, ular ma’lum doiradagi masalarga qo‘llaniladi va EHM da hisoblash qiyin. Integral tenglamalarini echishda kvadratur va kubatur formulalar, iteratsiya usuli, proeksion usullar (momentlar usuli, Galerkin-Petrov, kollakatsiyava boshqalar) ([1], [3], [14], [7] qarang).

Ikkinchi tarafdan aerodinamika, gidrodenimika, egiluvchanlik nazariyasi, elektrodinamika, matematik fizikaning chegaraviy masalasi, analitik funksiyalar chegaraviy masalasini echish singulyar integrallarga keltiriladi ([6], [9], [20], [21] qarang). Bunday tenglamalar nazariyasi yaxshi rivojlangan.

Fredgolm va Volterr integral tenglamalarini taqribiy echish usullari L.V.Kantorovich va V.I.Kri’lov [14], N.S.Baxvalova [1] , I.V. Berezina i I.P.Jidkova [3], A. F. Verlenya va A. S. Sizikova [7] va xokazolarning monografiyalarida bayon etilgan.

Koshi va Gil’berta yadrali singulyar integral tenglamalarni taqribiy echish V. V .Ivanova [11] , B. G. Gabdulxaeva [8], I. V. Boykova [4] , S. M. Belotserkovskogo i I. K. Lifanova [2], Ye. Ye. Тыrтыshnikova [19] , V. A. Zolotorevskogo [10] va boshqalarning monografiya va maqolalarida yaxshi yoritilgan.

Shuning uchun, Fredgolm va Volterr yadroli bir o‘lchovli va ko‘p o‘lchovli integral tenglamalarni sonli echishning effektiv algoritmlarini qurish hisoblash matematikasining aktual masalalaridan hisoblanadi.

Ushbu dissertatsiyada biz integral tenglamalarni echishda sonlar nazariyasig usullariga asoslangan kubatur formulalar quramiz.

Oxirgi yillarda regulyar va singulyar integrallar uchun kvadratur va kubatur formulalar qurish nazariyasiga qiziqish ortganligi sababli sonlar nazariyasi usullarini qullash sezilarli darajada oshdi.

Hozirgi vaqtda sonlar-nazariyasi usullari hisoblash matematikasining turli masalarini echishda keng qo'llanilayapdi. Ayniqsa regulyar integrallarga kubatur formulalar qurish, ko'p o'zgaruvchili funksiyalarni interpolyatsiyalash, ko'p o'lchovli Fredgol'm 2-tur integral tenglamalarini echish masalalarida qo'llaniladi [15-19], [22], [23-24], [25], [26].

Sonlar-nazariyasi usullari xuddi shunday Gil'bert yadroli singulyar integrallar uchun kubatur formula qurish masalasiga keng qo'llanildi.

Shuni aytish kerakki sonlar-nazariyasining turli bo'limlari S. L. Sobolevaning (sm. [12-13]) invariant kubatur formulalar nazariyasiga ham qo'llanilgan.

Tadqiqot maqsad va vazifasi. Sonlar nazariyasi usullaridan foydalanib, N. M. Korobov va Ye. Hlawkalar biritnchi bo'lib, karrali integrallarni taqribiy hisoblashda integrallash to'rlarining maxsus sinflarini yaratdi. N. M. Korobov ularni "parallelepipedal to'r" deb atadi, g'arb adabiyotlarida esa ular " Good lattice points " degan nomni oldi.

K. K. Frolova [23] ning ishlarida parallelepipedal to'r yordamida integrallashni umumlashtirishga qadam qo'yilgan va keyin Sloan va Kachoyan[29] lar tamonidan rivojlantirilgan. Keyinchalik taqribiy integrallashning bu yo'nalishiga V. A. Выkovskim [5] , Lyness [28] , Niederreiter [27] , Sloan [27] va boshqalar katta xissa qo'shdilar.

Ilmiy yangiligi. Sonlar nazariyasi usullaridan foydalanib qurilgan kubatur formullar , integralning karraligiga bog'liq emas (Monte – Karlo usuliga o'xshash), shuning uchun bunday formulalar integral tenglamalarni iteratsiya usuli bilan taqribiy echishga qulay bo'ladi.

Tadqiqot asosiy masalalari va farazlari. Dissertatsiya ishining kafedra tadqiqot ishlari bilan bog'liqligi. Dissertatsiyaning mavzusi Termiz davlat universiteti ilmiy kengashi tomonidan tasdiqlangan va Termiz davlat universiteti Algebra va geometriya kafedrasida olib borilayotgan ilmiy tadqiqot ishlari bilan bevosita bog'liq.

Tadqiqot mavzusi bo'yicha adabiyotlar sharhi.

Baxvalov N.S. Chislennie metodi. – M . : Nauka. T.1. 1975. 631 s.
Belotserkovskiy S.M . Lifanov I.K. Chislennie metodi v aerodinamike , teorii uprugosti , elektrodinamika . M . : Nauka. 1985. 256 s.
Berezin I.S . ,Jidkov N.P. Metodi vichisleniy. T.1. M . : Nauka. 1966. 466 s.
Boykov I.V. Optimalnie potochnosti algoritmi vichisleniya singulyarnix integralov. – Saratov.Izd-vo Saratovskogo universiteta. 1983. 210 s.

Tadqiqotda qo'llanilgan metodikaning tavsifi. Kubatur formulalar va sonlar nazariyasi orasidagi bog'liqlikka qisqacha to'xtalamiz. $G_n = \{ 0 \le x_i \le 1$, i=1 , 2 ,… , $n\}$ birlik giperkubda
$f(x) = f(x_1, x_2, \ldots, x_n)$ funksiya aniqlangan, uzluksiz bo'lsin va har bir x_i o'zgaruvchi bo'yicha davriy bo'lsin.

Faraz qilaylik, $f(x)$ ni G_n da Fur'e qatoriga yoyish mumkin bo'lsin

$$f(x) = \sum_{m_1, \ldots, m_n = -\infty}^{\infty} C(m) e^{2\pi i (m,x)} ,$$

bunda

C(m)=C($m_1, \ldots, m_n$); $m_1x_1 + \ldots + m_nx_n = (m, x)$.

ma'lumki

$$C(0) = \int_{G_n} f(x)dx ,$$

u holda

$$\int_{G_n} f(x)dx \cong \frac{1}{N}\sum_{k=1}^{N} f(x^{(k)}) \qquad (1)$$

kubatur formulaning xatoligini

$$R_N(f) = \frac{1}{N} \sum_{m_1, \ldots, m_n = -\infty}^{\infty}{}' C(m)S(m)$$

ko'rinishda yozish mumkin. Bunda summadagi shtrix $(m_1, \dots, m_n) \neq (0, \dots, 0)$ ekanligini bildiradi. Aynan mana shu tenglik sonlar nazariyasi bilan kubatur formullar orasida bog'liklik o'rnatadi

$$S(m) = \sum_{k=1}^{N} e^{2\pi i(m\,, x^{(k)})}$$

ichki summa trigonometrik summa bo'ladi.

Xususiy xolda $x^{(k)}$ sifatida teng taqsimlanmagan $x^{(k)} = \left(\left\{\frac{k}{N}\right\}, \ \dots \ , \left\{\frac{k^n}{N}\right\}\right)$ to'rni olsak, u holda $S(m)$ sonlar nazariyasida ma'lum bo'lgan ratsional trigonometrik yig'indiga aylanadi, bunda N – natural son, $\{y\}$ - y sonning kasr qismi.

$$S(m) = \sum_{k=1}^{N} e^{2\pi i \frac{m_1 k + \cdots + m_n k^n}{N}} .$$

N . M. Korbov [39] har bir x_i o'zgaruvchisi bo'yicha davriy bo'lgan va Fur'e koeffitsienti

$$|S(m)| \leq C(K(\overline{m}))^{-\alpha} \ , \qquad K(\overline{m}) = \prod_{j=1}^{n} max\left(1\,, |m_j|\right).$$

Tengsizlikni qanoatlantiruvchi $f(x) = f(x_1, x_2, \dots, x_n)$ funksiyalarning $E_n^{\alpha}(C)$ sinfini kiritdi. Bunda $\alpha > 1$ va S o'zgarmas $m_1, \dots, m_n$ larga bog'liq emas. Keyin sonlar nazariyasi usullarini qo'llab, u parallelepipedal to'r deb ataluvchi

$$x^{(k)} = M_{kn} = \left(\left\{\frac{k}{N}\right\}, \quad \dots \ , \left\{\frac{k^n}{N}\right\}\right) \qquad (2)$$

to'rni qurdi, bunda $a_1, \dots, a_n$ maxsus tanlangan natural sonlar ((sm. [39], c. 98)) va (2) to'r uchun (1) kubatur formula

$$R_N(f) = 0\left(\frac{log^{\alpha n} N}{N^{\alpha}}\right)$$

diyarli optimal yaqinlashishiga ega bo'ladi.(2)

Parallelepipedal to‘r qurilgan $a_1, ..., a_s$ optimal koeffitsientlar diofant yaqinlashishlar nazariyasi bilan yaqin bog‘liq, xususan kasr ulushlarning teng taqsimlanishi masalalari bilan.

$\gamma_1, ..., \gamma_s$ lar [0,1] intervaldan olingan ixtiyoriy xaqiqiy sonlar bo‘lsin.

$0 \leq x_1 \leq \gamma_1$, ..., $0 \leq x_s \leq \gamma_s$ (3)

shartni qanoatlantiruvchi sohani qaraylik va (2) to‘rning shu sohaga yotuvchi nuqtalar sonini $T_N(\gamma_1, ..., \gamma_s)$ orqali belgilaylik.

[39] da ko‘rsatilganidek, $a_1, ..., a_s$ butun sonlarning optimal koeffitsient bo‘lishligining zaruriy va etarlilik sharti

$$T_N(\gamma_1, ..., \gamma_s) = \gamma_1 ... \gamma_s \ N + R_O ,$$

(4) bajarilishi bo‘ladi, bu erda $R_O = 0(log^{\beta} N)$ va β lar N ga bog‘liq emas.

Tadqiqot natijalarining nazariy va amaliy ahamiyati. Shunday qilib, parallelepipedal to‘r tugunlari shunday joylashganki, ularning (3) ko‘rinishdagi ixtiyoriy soha bilan ustma-ust tushishlar soni soha hajmini to‘rning barcha nuqtalari soniga ko‘paytirilganiga asimtotik ravishda teng bo‘ladi.

M_{ks} to‘r nuqtalarining birlik kubda teng taqsimlanishi (4) dagi qoldiq hadning yaxshi baholanishiga bog‘liq. To‘r qanchalik teng taqsimlangan bo‘lsa, bu to‘r bilan tuzilgan kubatur formula shunchalik aniqroq bo‘ladi.$a_1, ..., a_s$ butun sonlar xar qanday tanlanganda ham (4) tenglikni qoldiq qismini $\log N$ ning biror darajasidan yaxshiroq baholab bo‘lmaydi.

Bundan ko‘rinadiki, optimal koeffitsientlarda qoldiq xadi eng yaxshi bahoga erishadi va shu bilan parallelepipedal to‘rda qurilgan kubatur formularning optimalligini ta’minlaydi.

Parallelepipedal to‘rda qurilgan kubatur formullarning o‘ziga xos xususiyati ularning chekli trigonometrik polinomlar

$$P(x_1, ... , x_s) = \sum_{\bar{m}_1 ... \bar{m}_s \leq C_0 N^{1-\varepsilon}} C(m_1 , ..., m_s) e^{2\pi i(m_1 x_1 + \cdots + m_s x_s)} , \qquad (5)$$

ni aniq hisoblashida, ya’ni

$$\int_0^1 \cdots \int_0^1 P(x_1, \dots, x_s) dx_1 \dots dx_s = \frac{1}{N} \sum_{k=1}^{N} P\left(\left\{\frac{k}{N}\right\}, \quad \dots, \left\{\frac{k^n}{N}\right\}\right)$$

tenglik har qanday (5) ko'rinishdagi palinom uchun bajariladi.

V. S. Ryaben'kiy [22] sonlar nazariyasi usuli bilan tuzilgan to'rlardan ko'p o'zgaruvchili funksiyalarni interpolyatsiyalash masalasida foydalanish mumkinligini ko'rsatdi. $f(x_1, \dots, x_s) \in E_s^\alpha$ funksiyaning Fur'e koeffitsientiga parallelepipedal to'rda qurilgan kubatur formulani qo'llab

$$f(x_1, \dots, x_s) = \frac{1}{N} \sum_{k=1}^{N} f\left(\left\{\frac{a_1 k}{N}\right\}, \dots, \left\{\frac{a_s k}{N}\right\}\right) \phi_k(x_1, \dots, x_s) + 0\left(\frac{log^{\gamma_1} N}{N^{0.5(\alpha-1)}}\right) \quad (6)$$

interpolyatsion formulani olamiz, bu erda $\phi_k(x_1, \dots, x_s)$ - biror aniq funksiya va γ_1 esa N ga bog'liq emas.

E_s^α sinfda $s > 2$ da (6) formula boshqa formulalardan sezilarli darajada aniqroq bo'lib,

xatolik shu sinfda $\frac{1}{N^{(\alpha-1)/s}}$ tartibga ega.

Matematik fizika masalalarida ko'pincha

$$\frac{\partial^{n_1+\dots+n_s} f}{\partial {x_1}^{n_1} \dots \partial {x_s}^{n_s}} \quad (0 \le n_1 + \dots + n_s \le \alpha s, 0 \le n_\nu \le \alpha)$$

tartibli hosilaga ega bo'lgan funksiyalar bilan ishlashga to'g'ri keladi, bunday funksiyalarning sinfi N_s^α deb belgilanadi.

Haqiqatdan ham, masalan,

$$\varphi(x) = f(x) + \lambda \int_0^1 K(x, y) \varphi(y) dy \, .$$

Fredgolmning 2-tur integral tenglamasini qaraylik.

Faraz qilaylik, $f'(x)$, $\frac{\partial K}{\partial x}$, $\frac{\partial K}{\partial y}$ va $\frac{\partial^2 K}{\partial x \partial y}$ hosilalar mavjud va uzluksiz bo'lsin. Bu tenglamani iteratsiya usuli bilan echganda

$$\int_0^1 \dots \int_0^1 K(x, x_1) \dots K(x_{s-1}, x_s) f(x_s) dx_1 \dots dx_s \, , \quad (7)$$

ko'rinishdagi integrallarni echishga to'g'ri kelar edi, bunda integral ostidagi

$$F(x_1, \dots, x_s) = K(x, x_1) \dots K(x_{s-1}, x_s) f(x_s)$$

funksiyani H_s^α sinfga tegishli deb qarash mumkin.

F funksiyani H_s^α sinfga tegishli deb qarab, biz (7) integral to‘g‘risida to‘liq ma’lumotga ega bo‘lamiz.

Agar (7) integralni hisoblash uchun tugunlari koordinata o‘qlariga parallel bo‘lgan to‘r (reshyotka) hosil qiladigan kubatur formuladan foydalansak, u holda xatolik uchun

$$R = 0\left(\frac{1}{N^{\frac{1}{s}}}\right) \qquad (8)$$

dan yaxshi baho olalmaymiz.

Bu usul bilan olingan natija s ning oshishi bilan yanada yomonlashadi. Shuning uchun (7) integralni hisoblashda kata s lar uchun klassik kubatur formulalar yaroqsiz bo‘lib qoladi.

Sodir bo‘lgan qiyinchiliklarni integral ostidagi funksiya H_s^1 ga tegishli bo‘lgan holdagi kubatur formulalar yordamida engish mumkin.

Bunday kubatur formulalarning to‘ri sonlar nazariyasida ta’riflangan xarakterga ega bo‘lish kerak.

Sonlar nazariyasi xarakteridagi to‘rini qo‘llash ixtiyoriy $\varepsilon > 0$ da va H_s^1 sinfda xatoligi

$$R = 0\left(\frac{1}{N^{1-\varepsilon}}\right). \qquad (9)$$

dan oshmaydigan kubatur formulani hosil qiladi.

Bu baho eng yaxshi baho hisoblanadi, chunki H_s^1 sinfda to‘rni boshqacha xar qanday tanlaganda ham olinadigan baho

$$R = 0\left(\frac{1}{N}\right)$$

dan yaxshi bo‘lmaydi.

Ko‘rinib turibdiki (9) baho H_s^1 sinfda klassik usulda olingan (8) va xuddi shunday Monte – Karlo usulida olingan

$$R = 0\left(\frac{1}{\sqrt{N}}\right).$$

bahodan ancha yaxshi.

Tadqiqot ishining tuzilishi. Ushbu dissertatsiyada sonlar nazariyasiga asoslangan to‘rlar yordamida:

- Parallelepipedal to‘r hisoblangan;
- ko‘p o‘lchovli ikkinchi tur Volterr integral tenglamasi taqribiy echilgan.
- taqribiy echimning xatoligi ko‘rsatilgan.

Dissertatsiya ishi 3 ta bob va adabiyotlar ro‘yxatidan iborat.

1-bob 1.1-§ da sonlar nazariyasining keyinchalik foydalanadigan ta’riflar va ba’zi oldindan ma’lum tasdiqlar keltirilgan.

1.2-§ da sonlar nazariyasining usllaridan foydalanib parallelepipedal to‘r qurilgan.

2-bob 2.1-§ va 2.2-§ da Volterrning integral tenglamasi taqribiy hisoblangan va xatoligi ko‘rsatilgan. Xatolik sonlar nazariyasining usullaridan foydalangan xoldagidan ancha kattaligi ko‘rsatilgan.

Misol ishlab ko‘rsatilgan. Bunda sonlar nazariyasining usllaridan foydalanilmagan.

3-bob 3.1-§ da funksiyalarni davriylashtirish masalasi ko‘rib chiqilgan.

3.2-§ da esa ko‘p o‘lchovli Volterr integral tenglamalarni sonlar nazariyasi usullari yordamida taqribiy echish masalasi ko‘rib chiqilgan.

I BOB. SONLAR NAZARIYASI USULLARIDAN FOYDALANIB 2-TUR VOLTERRA INTEGRAL TENGLAMASINI YECHISH.

1.1-§ Ba'zi yordamchi tasdiqlar va ta'riflar

$[a,b]$ -sonlar o'qining ixtiyoriy chekli va cheksiz kesmasi bo'lsin, $f(x)$ – biror K sinfning ixtiyoriy funksiyasi va $p(x)$ K sinfdagi ixtiyoriy $f(x)$ funksiya bilan ko'paytmasi $[a,b]$ da integrallanuvchi fiksirlangan (tayinlangan) funksiya. Keyin

$$F = \int_a^b p(x)f(x)dx \,. \tag{1.1}$$

aniq integralni topish talab etilsin.

$f(x)$ funksiyani umumlashgan interpolyatsion ko'phad bilan almashtiramiz.

Bunday aproksimatsiyalash parametrlarga nisbatan chiziqli bo'ladi, funksiya biror chiziqli ifoda bilan almashadi, bunda koeffitsient sifatida funksiyaning $x_1, \dots, x_n$ tugun nuqtalardagi qiymatlari keladi:

$$, \qquad f(x) = \sum_{k=1}^{n} f(x_k)\varphi_k(x) + r_n(x) \tag{1.2}$$

bunda, $r_n(x)$ -aproksimatsiyaning qoldiq hadi.

(1.2) ni (1.1) ga qo'yib,

$$F = \sum_{k=1}^{n} f(x_k)C_k + R_n \,, \tag{1.3}$$

sonli integrallash formulasini olamiz, bunda

$$C_k = \int_a^b p(x)\varphi_k(x)dx \,, \qquad\qquad R_n = \int_a^b p(x)r_n(x)dx \,.$$

(1.3) ning o‘ng qismi kvadratur yig‘indi deyiladi, x_k – tugunlar yoki absissalar, S_k – koeffitsientlar, R_n – esa xatolik yoki (1.3) ning qoldiq hadi deyiladi.

Xatolik integralni kvadratur yig‘indi bilan almashtirishda hosil bo‘ladi.

(1.3) formula $2n + 1$ parametrga ega bo‘ladi: n ta x_k absissalar, n ta S_k – koeffitsientlar va tugunlar soni n .

Bu parametrlarni shunday tanlash keraki, biror sinfdan olingan barcha f funksiyalar uchun (1.1) formula “yetarlicha kichik xatolik” bersin.

Kvadratur yig‘indida qo‘shiluvchilar soni qancha ko‘p bo‘lsa, S_k va x_k larni tanlash bilan aniqlikni shunchalik katta qilish mumkin.

Shuning uchun, taqribiy formula qo‘rishda, n ixtiyoriy fiksirlangan natural son, S_k koeffitsientlar va x_k tugunlar esa ixtiyoriy deb hisoblanadi. Ularni quyidagi maqsadda tanlashadi:

1. Aniqlik darajasini oshirish.
2. Xatolikni minimallashtirish.
3. Hisoblashlarni osonlashtirish.

1-ta’rif. Kvadratur formula m algebraik aniqlik darajasiga ega deyiladi, agarda u barcha m –darajali ko‘phadlar uchun aniq va $\mathrm{m} + 1$
darajali ko‘phadlar uchun aniq bo‘lmasa, yoki boshqacha aytganda

$$\int_a^{\partial} p(x)f(x)dx = \sum_{k=1}^{n} f(x_k)\varphi_k(x) + r_n(x)$$

Tenglik $i = 0, 1, \dots, \mathrm{m}$ lar uchun bajarilib, $i = \mathrm{m} + 1$ lar uchun bajarilmaydi.

$f\,(x_1, \dots, x_n)$ funksiya $0 \le x_v \le 1$ $(\mathrm{v} = \overline{1, \mathrm{n}})$ tengsizlik bilan aniqlangan G_n birlik kubda uzluksiz va har bir $x_1, \dots, x_n$ o‘zgaruvchilar bo‘yicha birga teng davrga ega bo‘lsin.

$S_f(m_1, \dots, m_n)$ orqali f funksiyaning Fure koeffitsientini belgilaymiz:

$$(S_f m_1, \dots, m_n) = \int\limits_0^1 \dots \int\limits_0^1 f(x_1, \dots, x_n) e^{2\pi i(m_1x_1 + , \dots , + m_nx_n)} dx_1 dx_n .$$

$v = 1, 2, \dots, n$ lar uchun $\overline{m}_v$ kattalik

$$\overline{m}_v = \begin{cases} 1, & agar\ m_v = 0, \\ |m_v|, & agar\ m_v \neq 0. \end{cases}$$

tenglik bilan aniqlangan bo‘lsin.

2-ta'rif. $f(x_1, \dots, x_n)$ funksiya E_n^α sinfga tegishli deyiladi, agarda $(S_f(m_1, \dots, m_n) = 0 \quad ((\overline{m_1}, \dots, \overline{m_n})^{-\alpha})$

baho bajarilsa, bunda α birdan katta xaqiqiy son va ⟨⟨0⟩⟩ belgidagi o‘zgarmas $m_1, \dots, m_n$ ga bog‘liq emas.

Bu konsttantaning qiymatini ko‘rsatish kerak bo‘lsa E_n^α o‘rniga E_n^α(C) ni yozish kerak va oldingi bahoni

$$|S_f(m_1, \dots, m_n)| \leq \frac{C}{(\overline{m_1}, \dots, \overline{m_n})^{\alpha}}$$

tengsizlik bilan almashtirish kerak.

3-ta'rif. $\alpha > 1$ butun bo‘lsin, $f(x_1, \dots, x_n)$ funksiya H_n^α sinfga tegishli deyiladi, agarda s -o‘lchovli birlik G_s kubda u

$$\frac{\partial^n f}{\partial x_1^{n_1} \cdots \partial x_s^{n_s}} \qquad (0 \leq n \leq \alpha s, \quad 0 \leq n_s \leq \alpha)$$

turdagi uzluksiz hosilaga ega bo‘lsa.

Agar bu hosila butun s -o‘lchovli fazoda uzluksiz va $f(x_1, \dots, x_n)$ funksiya barcha o‘zgaruvchilari bo‘yicha birga teng davrga ega bo‘lsa, u holda $f(x_1, \dots, x_n)$ funksiya H_n^α sinfda davriy funksiya deyiladi.

N > 1 -butun son, $a_\gamma = a_\gamma(N) \quad (\gamma = \overline{1, n})$ –butun sonlar N bilan o‘zaro tub sonlar va $\delta_n(m)$ miqdor m ning N ga bo‘linish yoki bo‘linmasligiga qarab bir yoki nolga teng bo‘lsin.

4-ta'rif. $a_1, \dots, a_n$ butun sonlar N modul bo‘yicha optimal koeffitsientlar deyiladi, agarda shunday $\beta = \beta(n)\ va\ C_0 = C_0(n)$ konstantalar topilsaki, unda N ning biror cheksiz ketma-ketligida

$$\sum_{m_1, \dots m_n=-(N-1)}^{N-1,} \frac{\delta_N(\alpha_1 m_1, \dots \alpha_n m_n)}{\bar{m}_1 \dots \bar{m}_n} \le C_0 \frac{ln^{\beta} N}{N} \qquad (1.4)$$

tengsizlik bajarilsa. Bu erda summadagi shtrix, qo'shish amali barcha $(m_1, \dots m_n) \neq (0, \dots 0)$ larda bajariladi. Bu xolda β o'zgarmas optimal koeffitsientlarning indeksi deyiladi.

5-ta'rif . N modul bo'yicha $a_1, \dots, a_n$ optimal koeffitsientlardan tuzilgan

$$M_{kn} = \left(\left\{\frac{k}{N}\right\}, \dots, \left\{\frac{k^n}{N}\right\}\right)$$

to'r parallelepipedal to'r deb ataladi. Bunda $\{x\}$ yozuv x sonining kasr ulushini anglatadi.

n butun sonni $(n-1)^s \le N < n^s$ shartdan olamiz va s -o'lchovli birlik kubning xar bir qirrasini n ta teng qismga bo'lamiz. Bu bo'linish nuqtasidan koordinatalar tekisligiga parallel tekisliklar o'tkazib, s -o'lchovli kubni n^s kichik kublarga bo'lamiz.

Bu kublarning uchlarida joylashgan

$$M_{kn} = \left(\frac{l_1}{k}, \dots, \frac{l_s}{k}\right) \qquad (1 \le l_\nu \le n, \nu = 1,2, \dots, s) \qquad (1.6)$$

nuqtalar to'plamini, teng taqsimlangan to'r deb aytamiz.

1-lemma. $\delta_N(m)$ kattalik

$$\delta_n(m) = \begin{cases} 1, \text{agar } m \equiv 0 (\text{mod N}), \\ 0, \text{agar } m \not\equiv 0 (\text{mod N}). \end{cases}$$

tenglik bilan berilgan bo'lsin. U holda

$$\frac{1}{N} \sum_{k=1}^{N} e^{2\pi i \frac{mk}{N}} = \delta_N(m).$$

bo'ladi.

2-lemma. Ixtiyoriy N tub son uchun N bilan o'zaro tub bo'lgan $a_\nu = a_\nu(N)$ $(\nu = \overline{1,n})$ sonlar mavjudki, ular uchun

$$\sum_{m_1, \ ... \ , m_n=-(N-1)}^{N-1 \ \prime} \frac{\delta_N(a_1 m_1 + \ ... + \ a_n m_n)}{\bar{m}_1 \ ... \ \bar{m}_n} < \frac{2(3 + 2lnN)^n}{N}$$

bo'ladi.

3-lemma. Ixtiyoriy $\alpha > 1$ va t > 1 xaqiqiy sonlar uchun

$$\sum_{\bar{m}_1 \ ... \ \bar{m}_n > t} \frac{1}{(\bar{m}_1 \ ... \ \bar{m}_n)^\alpha} \leq \alpha \left(\frac{\alpha}{\alpha - 1}\right)^n \frac{(1 + lnt)^{n-1}}{t^{\alpha-1}}$$

baho o'rinli bo'ladi.

4-lemma.

$$\sum_{\bar{m}_1 \ ... \ \bar{m}_n < t} 1 \leq 3^n t log^{n-1} t \ .$$

tengsizlik o'rinli.

5-lemma. Agar $f(x_1, \ ... \ , x_s)$ funksiya E_n^α sinfga tegishli va $\bar{m}_1 \ ... \ \bar{m}_n < M$ bo'lsa, u xolda

$$\varphi(x_1, \ ... \ , \quad x_n) \equiv f(x_1, \ ... \ , \quad x_n) e^{-2\pi i(m_1 x_1 + \ ... + \ m_n x_n) i}$$

funksiya $E_n^\alpha(ACM^\alpha)$ sinfga tegishli bo'ladi, bu erda A -konstanta α va n larga bog'liq

$$A = \left(2^{\alpha+1}\left(3 + \frac{2}{\alpha - 1}\right)\right)^\alpha.$$

6 -lemma. $f_1(x_1, \ ... \ , x_s)$ *va* $f_2(x_1, \ ... \ , x_s)$ funksiyalar mos ravishda $E_s^\alpha(C_1)$ *i* $E_s^\alpha(C_2)$ sinflarga tegishli bo'lsin. U xolda ixtiyoriy B_1 va B_2 lar uchun $B_1 f_1 + B_2 f_2 \in E_s^\alpha(|B_1|C_1 + |B_2|C_2)$ va $f_1 f_2 \in E_s^\alpha(AC_1 C_2)$,
bunda

$$A \leq \left(2^{\alpha+1}\left(3 + \frac{2}{\alpha - 1}\right)\right)^n.$$

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Eslatma. Agar $(x_1, \ ... \ , x_s, y_1, \ ... \ , y_s) \in E_{2s}^\alpha$ va $f_1(x_1, ..., x_s) = f(x_1, \ ... \ , x_s, x_1, \ ... \ , x_s)$, u holda

$$f(x_1, \ ..., \ x_s) \in E_s^\alpha.$$

7-lemma. E_s^α sinfdan olingan $f(x_1, \ ..., \ x_s)$ funkstya o‘z o‘zgaruvchisining biror bir s' tasining funksiyasi sifatida qaralganda $E_{s'}^\alpha$ sinfga tegishli bo‘ladi, bunda $1 \leq s' \leq s$.

8-lemma. Agar $f_1(x_1, \ ..., \ x_n, y_1, \ ..., \ y_t) \in E_{n+t}^\alpha(C_1)$ va $f_2(x_1, \ ..., \ x_n, z_1, \ ..., \ z_{t'}) \in E_{n+t'}^\alpha(C_1)$ bo‘lsa,

u holda $f_1 f_2 \in E_{n+t'+t}^\alpha(AC_1C_2)$ bo‘ladi,

bu erda $A \leq \left(2^{\alpha+1}\left(3+\frac{2}{\alpha-1}\right)\right)^n$.

9 -lemma. $a_1, \ ..., \ a_n$ N modulli β indeksli optimal koeffitsientlar bo‘lsin. Agar $f(x_1, \ ..., \ x_n)$ funksiya E_n^α sinfga tegishli bo‘lsa, u holda

$$\int_0^1 ... \int_0^1 f(x_1, \ ..., x_n) dx_1 ... dx_n \approx \frac{1}{N}\sum_{k=1}^{N} f\left(\left\{\frac{a_1 k}{N}\right\}, \ ..., \ \left\{\frac{a_n k}{N}\right\}\right)$$

kubatur formulaning $R_N[f]$ qoldiq hadi uchun

$$|R_N[f]| \leq CC'N^{-\alpha} log^{\alpha\beta} N,$$

baho o‘rinli bo‘ladi, bunda

$$C' = C_0^\alpha + nB^n, \quad B3 + \frac{2}{\alpha - 1}.$$

10-lemma. Ixtiyoriy p tub soni uchun shunday $a_1, \ ..., a_n$ optimal koeffitsientlar mavjud bo‘ladiki, unda $\alpha < 1$ qanday bo‘lishidan qat’iy nazar, ixtiyoriy $\varepsilon \in (0,1)$ da

$$\sum_{\bar{m}_1, \ ..., \bar{m}_n=-\infty)}^{\infty} \frac{\delta_N(a_1 m_1 + \ ... + a_n m_n)}{(\bar{m}_1 ... \bar{m}_n)^\alpha} < \frac{\left(6\alpha + \frac{12\alpha^2}{\varepsilon_1}\right)^{2\alpha s}}{p^{\alpha-\varepsilon_1}}.$$

baho o‘rinli bo‘ladi.

11-lemma. Agar $\Phi \in E_s^\alpha(C)$ bo‘lsa, u holda $\varepsilon_1 \in (0, \alpha - 1)$ qanday bo‘lishidan qat’iy nazar $N = p$ da optimal koeffitsientlar yordamida qurilgan

$$\int_0^1 \dots \int_0^1 \Phi(x_1,\dots,x_s)dx_1\dots dx_s = \frac{1}{N}\sum_{k=1}^{N}\Phi\left(\left\{\frac{a_1 k}{N}\right\}, \dots, \left\{\frac{a_s k}{N}\right\}\right) - R,$$

kvadratur formulaning xatosi uchun

$$|R| \le C\frac{\left(6\alpha + \frac{12\alpha^2}{\varepsilon_1}\right)^{2\alpha s}}{p^{\alpha-\varepsilon_1}}$$

o'rinli bo'ladi.

12-lemma. G_{rs} birlik rs -o'lchovli kub, $F(P_1,\dots,P_s)$ esa E_{sr}^{α} sinfga tegishli rs -o'zgaruchovli funksiya bo'lsin, $\varepsilon = \varepsilon(n)$, $\lim_{n\to\infty} \varepsilon = 0$ i M_{kn} shunday tanlangan s -o'lchovli birlik kub,da shunday tanlanganki, r = 1 da

$$\int_{G_{rs}} F(P_1,\dots,P_r)dP_1\dots dP_r = \frac{1}{n^r}\sum_{k_1,\dots,k_r=1}^{n} F(M_{k_1},\dots,M_{k_r}) + O((\varepsilon)) \quad (1.7)$$

kvadratur formula o'rinli bo'lsin. U holda bu formula ixtiyoriy fiksirlangan $r > 1$ da ham o'rinli bo'ladi.

6-ta'rif. $f(x_1,\dots,x_s)$ funksiyani oddiy davriylashtirish deb, quyidagi

1^0. $\varphi(x_1,\dots,x_{\nu-1},1,x_{\nu+1},\dots,x_s) = k\varphi(x_1,\cdots,x_{\nu-1},0,x_{\nu+1},\cdots,x_s)$ $(\nu = (\overline{1,s}))$.

2^0. $\int_0^1 \dots \int_0^1 f(x_1,\dots,x_s)dx_1\dots dx_s = \int_0^1 \dots \int_0^1 \varphi(x_1,\dots,x_s)dx_1\dots dx_s$.

shartlarni bajaruvchi $\varphi(x_1,\dots,x_s) \in H_s^{\alpha}$ funksiyani topishga aytiladi.

Agar bu shartlar bajarilsa, birinchisi na faqat φ funksiya uchun balki, uning x_ν bo'yicha $\alpha - 2$ gacha hosilasi uchun ham bajarilsa, u holda mos φ funksiyani topish f funksiyani to'liq davriylashtirish deyiladi.

14-lemma. $\alpha \ge 2, f(x_1,\dots,x_s) \in H_s^{\alpha}$ va $\varphi(x_1,\dots,x_s)$ funksiya $f(x_1,\dots,x_s)$ funksiyani to'liq yoki oddiy davriylashtirish yo'li bilan olingan bo'lsin. U holda

$$\int_0^1 \dots \int_0^1 f(x_1, . . . , x_s)dx_1 . . . dx_s = \int_0^1 \dots \int_0^1 \varphi(\{x_1\}, . . . , \{x_s\})dx_1 . . . dx_s,$$

bunda $\varphi(\{x_1\}, . . . , \{x_s\})$ mos ravishda E_s^2 yoki E_s^αsinfga tegishli tegishli bo'ladi.

1-14 lemmalarning isboti [18] da bor.

Lemma 15. $\alpha > 1$ deb faraz qilaylik. U holda

$$\sup_{f \in E_s^\alpha(C)} \left| \int_{G_{rs}} f(x_1, . . . , x_s)d\bar{x} - \frac{1}{n}\sum_{l_1=1}^{m-1} \dots \sum_{l_s=1}^{m-1} f\left(\frac{l_1}{k}, \dots, \frac{l_s}{k}\right) \right| \leq$$

$$\leq C(2\zeta(\alpha) + 1)^s n^{-\frac{\alpha}{s}},$$

bunda $n = m^s$ va $\zeta(\alpha) = \sum_{k=1}^{\infty} \frac{1}{k^\alpha}$

15-lemmalarning isboti [18] dan qarang.

Keynchalik qulaylik uchun quyidagicha belgilashlar kiritamiz:

$$\left.\begin{array}{l}
\bar{a} = (a_1, . . ., a_n),\ \bar{l} = (l_1, . . ., l_n),\qquad \bar{m} = (m_1, . . ., m_n), \\
\bar{t} = (t_1, . . ., t_n),\ \overline{dt} = (dt_1, . . ., dt_n),\qquad \bar{p} = (x_1, . . ., x_s), \\
(\bar{a}, \bar{m}) = a_1 m_1 + . . . + a_n m_n,\ K(\bar{m}) = \prod_{i=1}^{n} \max(1, |m_i|) \quad , \\
M_{kn} = \left(\left\{\frac{am}{N}\right\}\right),\ Q_j = \left(y_{(j-1)s+1}, \dots, y_{js}\right),\quad j = 1,2, \dots, n \ , \\
\bar{Q} = (Q_1, \dots, Q_n), dQ = dy_1 dy_2 \dots dy_{sn} = dQ_1 dQ_2 \dots dQ_n, \\
\Phi_{in}(x, \bar{t}) = \sum_{j_1, \dots, j_n}^{q} K_{ij_1}(x, t_1) \prod_{\nu=2}^{n} K_{j_{\nu-1} j_\nu}(t_{\nu-1} t_\nu) f_{j_n}(t_n)
\end{array}\right\} \qquad (1.9)$$

G_{ns} - ns -o'lchovli birlik kub, m_i - butun son.

Ma'lumki, Gilbert almashtirish quyidagidagicha bo'ladi:

$$\int_0^1 ctg\pi(x-y)sin2\pi kxdx = cos2\pi ky \qquad (k = 1,2,\dots).$$

$$\int_0^1 ctg\pi(x-y)cos2\pi kxdx = sin2\pi ky$$

Bundan

$$\int_0^1 ctg\pi(x-y)e^{2\pi ikx}dx = -sin2\pi ky + icos2\pi ky = ie^{2\pi iky},$$

$$\int_0^1 ctg\pi(x-y)e^{2\pi ikx}dx =$$

$$= \int_0^1 ctg\pi(x-y)cos2\pi kxdx - i\int_0^1 ctg\pi(x-y)sin2\pi kxdx =$$

$$= -sin2\pi ky - icos2\pi ky = -ie^{-2\pi iky} .$$

oxirgi ikkita tenglikdan,

$$sign\, m = \begin{cases} 1\,, agar \;\; \mathrm{m} > 0, \\ 0\,, \qquad agar\; \mathrm{m} = 0\,, \\ -1\,, \;\; agar \;\; \mathrm{m} < 0. \end{cases}$$

ni hisobga olsak, ixtiyoriy m da

$$\int_0^1 ctg\pi(x-y)e^{2\pi ikx}dx = i\, sign(m)e^{-2\pi iky} \tag{1.10}$$

ga ega bo‘lamiz.

1.2-§. Optimal koeffitsientlarni hisoblash

Korobovning [39] ishlarida optimal koeffitsientlar tushunchasi kiritilgan va ixtiyoriy ko‘po‘lchovli s karrali integralarni echish uchun, ularning qiymatlari ko‘rsatilgan. s o‘lchovli N modulli, bu erda N kvadratur formulaningtugunlari soni, optimal koeffitsienlarni hisoblashning turli xil algoritmlari N.M. Korobovning [39, 42] ishlarida topilgan.

Bu algoritmlarni amalga oshirish uchun $O(N)^2$ yoki $O(N)^{1+\frac{1}{3}}$ ta amal bajarish mumkin.

n, s -lar naturalsonlar va $x_1, x_2, \dots, x_{s-1}$ toq sonlar bo'lsin. $v = 1, 2, \dots, n$ da $h_v(x_1, x_2, \dots, x_{s-1})$ funksiyani quyidagi tenglik yordamida aniqlaymiz

$$h_v(x_1, x_2, \dots, x_{s-1}) = 2^{-v} \sum_{m=1}^{2^{v}{}_{\circ}} (2n - 2v +$$

$$+\|m2^{-v}\|^{-1}) \prod_{j=1}^{s-1} (2n - 2v + \|mx_j 2^{-v}\|^{-1}),$$

bu erda $\|y\|$ – soni ixtiyoriy y xaqiqiy son uchun y dan eng yaqin butun songacha bo'lgan masofani bildiradi , Σ_m^0 esa toq m lar bo'yicha yig'indini bildiradi.

$\varepsilon_{jv} = \pm 1 \quad (j = 1,2, \dots, s-1; v = 1,2, \dots, n-1)$ bo'lsin, va $a_{jv} = \pm 1$ $(j == 1,2, \dots, s-1; v = 1,2, \dots, n)$ shunday bo'lsinki, uning uchun

$$a_{11} = a_{21} = \dots = a_{s-1,1} = 1 \text{ va } a_{jv=} \varepsilon_{jv} a_{j,v+1} (mod 2^v)$$

$$(j = 1,2, \dots, s-1; \; v = 1,2, \dots, n-1)$$

bajarilsin.

$$h_v \le h_v(a_{1v}, \dots, a_{s-1,v}) \qquad (v = 1,2, \dots, n).$$

belgilash kiritamiz.

16-lemma . Agar $h_n \le h_{n-1} \le \dots \le h_1$ zanjir tengsizlik bajarilsa, u holda $1, a_{1n}, a_{2n}, \dots, a_{s-1,n}$ butun sonlar 2^n modul bo'yicha optimal koeffitsientlar bo'ladi.

Lemmaning isboti [8].

Bu lemmani qo'llab, 2^n modul bo'yicha optimal koeffitsientlarni hisoblovchi algoritmlarning qator sinfini yaratish mumkin, bunda amallar soni $O(s^2 N)$ ta bo'ladi.

$\varepsilon_{jv} = \pm 1 (j = 1, \dots, s-1; v = 1, \dots, n-1)$ lar fiksirlangan butun sonlar bo'lsin va $h_{rV}(x_1, x_2, \dots, x_{s-1,})$ funksiya

$$h_v(x_1, x_2, \dots, x_{s-1}) = 2^{-v} \sum_{m=1}^{2^{v_\circ}} (2n - 2v + + \|m2^{-v}\|^{-1}) \times$$

$$\times \prod_{j=1}^{r} (2n - 2v + \|mx_j 2^{-v}\|^{-1}) \prod_{j=r+1}^{s-1} (2n - 2v + \|mx_j 2^{-v}\|^{-1}) \qquad (1.4)$$

tenglik bilan aniqlansin.

$a_{11} = a_{21} = \cdots = a_{s-1,1} = 1$ deb tanlab olamiz.

$r \geq 1, v \geq 2$ va $a_{1,v-1}, \dots, a_{s-1,v-1}, a_{1v}, \dots, a_{r-1,v}$ toq sonlar ma'lum bo'lsin.

U holda $2 \leq v \leq$ n da a_{rv} larni

$$a_{rv} = a_{r,v-1}\big(1 + (\varepsilon_{r,v-1} - 1)z')\big) + z' \cdot 2^{v-1} \qquad (1.5)$$

tenglik yordamida aniqlaymiz, bunda z' miqdor z ning

$h_{rv}(a_{1v}, \dots, a_{r-1,v}, a_{r,v-1}(1 + (\varepsilon_{r,v-1} - 1)z + z) + z2^{v-1}, a_{r+1,v-1}, \cdots, a_{s-1,v-1})$

funksiyani minimumga aylantiradigan qiymati.

Teorema 6. Ixtiyoriy n natural sonlarda (1.4) funksiya va (1.5) tenglik yordamida olingan

$$1, a_1 = a_{1n}, a_2 = a_{2n}, \dots, a_{s-1} = a_{s-1,n}$$

butun sonlar, s- o'lchovli 2^n modul bo'yicha optimal koeffitsient bo'ladi. Teoremaning isboti [8].

Biz tomondan (1.4), (1.5) (optimal koeffitsientlarni hisoblash) algoritmlar uchun Fortran -77 tilida dastur tuzilgan va $2 \leq s \leq 15$ va $N = 2^{25}$ lar uchun optimal koeffitsientlar jadvali olingan.

```
DOUBLE PRECISION S1, S2, D4,,D5 D1 D3, V2., S1 „ P1 „ P, P2, V3, P3,
P4
DIMENSION IA
(20,25), Q(20,25)
 INTEGER Z, Z1 Z2,
S4, V, N, S, E, F, D2, Q
 OPEN (4, FILE = '
IMCOAF1.DAT)
```

```
OPEN (6, PILE ='
[MKOAF1 REZ *)
99 FORMAT ('N! P! (2)! A
(3)! A (4)! A (5)! 1 A (6)! A
(7)! A (8)! A (9)! A (10)')
WRITE (*, 99)
WRITE (6.99)
READ (4.199)KD
FORMAT (10X,I2)
DO 100 KK=1, KD
READ (4.15), L,E,L1
15 FORMAT (4X,I2, 4X,I2,4X,I2,4X,II2)
DO 45 S=3.L1
51 FORMAT( S = ' I2,' P=',F8.3)
DO 44 N=2. L
DO 4 I=1,S
IA(I,1)=1
CONTINUE
DO 9 K=2. N
DO 1 I=1 S-1
Q (I, K) = 0 (I, K-1) * (1 + (-1) * 2) + (2 ** (K-1)) * Z
S1 = 0
S2=0
C1=2*(N-K)
I5=2**K
B2=2.**(-K)
DO 7 J=1. I5.2
F=J
V=1
P1=B2
```

```
CALL SUB (F, V, P1,D4)
P=01+D4
IF (I. FQ.1) GO TO 80
DO 5 J1=1,I-1
F=J
E=E**(I+K)
V = Q (J1.K)
P1=B2
CALL SUB (F. V. P1,D4)
P=P*(C1+D4)
CONTINUE


80 Z1=1
IA(I,K)=IA(I,K-1)*(1+(E-1)*Z1)+2**(K-1)*Z1
F=J
V=IA(I,K)
P1=B2
XALL SUB(F,V,P1,D4)
Z2=0
P4=P*(C1+D4)
IA(I,K)=IA(I,K-1)*(1+(T-1)*Z2)+2**(K-1)*Z2
F=J
V=IA(I,K)
P1=B2
CALL SUB (F,F1.P1,D4)
P2=P*(C1+D4)
P3=1
B3=2.**(I-K)
DO 16 J2=I+1,S-1
```

```
F=J
V=Q(J2,K-1)
P1=B3
CALL SUB (F,V,P1,D4)
P3=P3*(C1+2+D4)
CONTINUE
S1=S2=P3*P4
S2=S2=P3*P2
7 CONTINUE
S1=B2*S1
S2=B2*S2
IF (S1.LT.S2) GO TO 81
Z=Z2
GO TO  1
Z=Z1
GO TO 1
CONTINUE
9CONTINUE
WRITE                                                                    (*,20)
N,P,Q,(1,N),Q(2,N),Q(3,N),Q(4,N),Q(5,N),2Q(6,N),Q(7,N),Q(8,N),Q(9,N),
Q(10,N)
WRITE                                                                    (6,20)
N,P,Q,(1,N),Q(2,N),Q(3,N),Q(4,N),Q(5,N),2Q(6,N),Q(7,N),Q(8,N),Q(9,N),
Q(10,N)
20 FORMAT (I2,1X,D25.12,10(I10))
44 CONTINUE
45 CONTINUE
100 CONTINUE
   END
SUBROUTINE SUB (F,V,P1,D4)
```

```
INTEGER F,D2,V
DOUBLE PRECISION D4,P1,D1,D3
D1=F*V*P1
D2=IDNINT(D1)
D5=DABS(D1-D2)
D4=1,/D5
1 FORMAT (5X,'F=',I3,'  V=',I3,' P1=',D25.12)
2 FORMAT (5X,'D1=',D25.12,'D2=',I3,'D5='.D25.12,'D4=',D25.12)
RETURN
END
```

II BOB. VOLTERNING INTEGRAL TENGLAMALARINI YECHISH USULLARI

2.1-§. Volterning tenglamalarini taqribiy echish.

Integral tenglamalar nazariyasidan ma'lumki, agarda $K(x,s)$ yadro $R\{a \leq s \leq x \leq b\}$ sohada uzluksiz funksiya, $f(x)$ –esa $[a,b]$ uzluksiz funksiya bo'lsa, u holda

$$y(x) - \lambda \int_a^{\infty} K(x,s)\, y(s)\, ds = f(x) \qquad (2.1)$$

Volterrning 2-tur integral tenglamasi λ ning ixtiyoriy qiymatida yagona $y(x)$ echimga ega bo'ladi.

Bu echimni

$$y(x) = \sum_{k=0}^{\infty} \lambda^k \varphi_k(x) \qquad (2.2)$$

ko'rinishda izlash mumkin.

(2.2) ni (2.1) ga qo'yib, λ ning bir xil darajalari oldidagi kaeffisentlarni tenglashtirib, quyidagini olamiz

$$\varphi_0(x) = f(x); \quad \varphi_{k+1}(x) = \int_a^{\infty} K(x,s)\, \varphi_k(s) ds\,. \qquad (2.3)$$

Agar

$$N = \max_{a \leq x \leq b} |f(x)|\,, \qquad M = \max_R |K(x,s)|.$$

bo'lsa, u holda

$$|\varphi_k(x)| \leq \frac{M^k (b-a)^k N}{k!}\,. \qquad (2.4)$$

bo'ladi.

Bundan, agar biz (2.1) integral tenglamaning taqribiy ildizi sifatida (2.2) qatorning n -xususiy olsak:

$$y_n(x) = \sum_{i=0}^{n} \lambda^k \varphi_k(x)$$

U holda, uning qoldig'ini

$$|y(x) - Y_n(x)| = \left|\sum_{k-n+1}^{\infty} \lambda^k \varphi_k(x)\right| \leq \sum_{k-n+1}^{\infty} \frac{|\lambda|^k M^k (b-a)^k N}{k!}. \qquad (2.5)$$

ko'rinishda baholanishi mumkin.

2.2-§. Volterning integral tenglamalarini taqribiy echimi xatoligini baholash.

Judayam qo'pol shu bilan birgalikda juda oddiy xatolik baxosi quidagicha: L bilan $|\lambda|M(b-a)$ ko'paytmani belgilaymiz va (2.5) bahodan $\frac{L^{n+1}N}{(n+1)^!}$ ni chiqaramiz,

$$|y(x) - Y_n(x)| \leq \frac{L^{n+1}N}{(n+1)^!}\left\{1 + \frac{L}{n+2} + \frac{L^2}{(n+2)(n+3)} + \cdots\right\}$$

hosil bo'ladi. Katta qavs ichidagi qatorni

$$1 + \frac{L}{n+2} + \left(\frac{L}{n+2}\right)^2 + \left(\frac{L}{n+2}\right)^3 + \cdots$$

U holda quidagi bahoni olamiz

$$|y(x) - Y_n(x)| \leq \frac{L^{n+1}N}{(n+1)^!} \cdot \frac{cd}{1 - \frac{1}{n+2}}$$

Shunda faraz qilamiz n shunachalik kata bo'lsinki, unda $L < n + 2$ bo'lsin. Agar (2.3) da kvadraturani hisoblab bo'lmasa, u holda ularni hisoblablash uchun teng uzoqlashgan absissali kvadratur formulalarni qo'llashlash mumkin.

Masalan, umumlashgan trapetsiya formulasini qo'llaymiz. Agar $[a, b]$ kesmani s ta teng qisimga bo'sak va

$h = \frac{b-a}{s}$; $x_k = a + kh$; $K(x_i, x_j) = K_{ij}$; $\varphi_n(x_k) = = \varphi_{nk}$, φ_{nk} ning taqribiy qiymatini $\bar{\varphi}_{nk}$ deb belgilasak, u holda

$$\varphi_{n+1k} = \int_0^{x_k} K(x_k, s)\,\varphi_n(s)ds$$

$$\approx \frac{h}{2}\left[K_{k_0\varphi_{n_0}} + 2\left(K_{k_1\varphi_{n_1}} + K_{k_2\varphi_{n_2}} + \cdots + +K_{k_{k-1}\varphi_{n_{k-1}}}\right) + K_{k_k\varphi_{n_k}}\right]$$

yoki

$$\bar{\varphi}_{n+1,k} = \frac{h}{2}\left[K_{k_0}\bar{\varphi}_{n_0} + 2\left(K_{k_1}\bar{\varphi}_{n_1} + K_{k_2}\bar{\varphi}_{n_2} + \dots + K_{k_{k-1}}\bar{\varphi}_{n_{k-1}}\right) + K_{k_k}\bar{\varphi}_{n_k}\right]$$

$$(k = 0, 1, 2, \dots, S).$$

ga ega bo'lamiz.

$\bar{\varphi}_{n+1,k}$ ni hisoblab, (2.1) integral tenglamaning taqribiy echimi

$Y_{n,k} = \sum_{i=0}^{n} \lambda' \bar{\varphi}_{i,k} \qquad (k = 0, 1, 2, \dots, S)$

formula bilan topilgan x_k tugunlarda olamiz.

Simpson umumlashgan formulasi qo'llaganda, $[a, b]$ kesmani $2s$ ta teng qisimga bo'lamiz ,

$$h = \frac{b-a}{2s}; \quad x_k = a + kh.$$

U holda

$$\varphi_{n+1,2k} = \int_a^{x_{2k}} K(x_{2k}, s)\ \varphi_n(s)ds,$$

Integralni echishda Simpson formulasini qo'llab

$$\bar{\varphi}_{n+1,2k} = \frac{h}{3}\{K_{2k,0}\bar{\varphi}_{n_0} + 4(K_{2k,1}\bar{\varphi}_{n_1} + K_{2k,2}\bar{\varphi}_{n_2} + K_{2k,3}\bar{\varphi}_{n_3} + \dots$$
$$+ K_{2k,2k-1}\bar{\varphi}_{n,2k-1})\}$$
$$+ 2\left(K_{2k,2}\bar{\varphi}_{n,2} + K_{2k,4}\bar{\varphi}_{n,4} + \dots + K_{2k,2k-2}\bar{\varphi}_{n,2k-2}\right)$$
$$+ K_{2k,2k}\bar{\varphi}_{n,2k} \qquad (n = 0, 1, 2, \dots; k = 0, 1, 2, \dots s)$$

ga ega bo'lamiz.

Toq k larda $\bar{\varphi}_{n+1,k}$ ning qiymati interpolyasilash yo'li bilan topiladi.

(2.1) Integral tenglamani echish uchun tenglamadagi integralni to'g'ridan to'g'ri biror kvadratur formulaning yig'indisi bilan almashtirish usulini qo'llash mumkin. Masalan, umumlashgan trapitsiya formulasini qo'llashda oraliqni n ta bo'lakka bo'lib,

$x_0 = a;\ x_1 = a + h;\ \ .\ .\ .\ ;\ x_n = a + nh = b$ larda

$$y_k - \lambda \int_a^{x_k} K(x_k, s)\, y(s) ds$$

$$\approx y_k - \frac{h\lambda}{2}\Big[K_{k,0} y_0 + 2\big(K_{k,1} y_1 + K_{k,2} y_2 + \cdots + + K_{k,k} y_{k-1}\big)$$

$$+ K_{k,k} y_k\Big] \approx f(x_k),$$

yoki

$$y_k - \frac{h\lambda}{2}\left[K_{k,0} Y_0 + 2\big(K_{k,1} Y_1 + K_{k,2} Y_2 + \cdots + K_{k,k} Y_{k-1}\big) + K_{k,k} y_k\right] - f(x_k) = 0,$$

ga ega bo‘lamiz.

Shunda

$$y_k = \frac{1}{1 - \frac{\lambda h}{2} K_{k,k}} \left\{ f_k + \frac{\lambda h}{2} K_{k,0} Y_0 + h\lambda \sum_{i=1}^{k-1} K_{k,i} Y_i \right\}.$$

Manosi [22] toq raqamlari uchun interpolatsiya orqali axtarishga majbur bo‘lamiz.

Tenglamaning taxminiy echimi uchun (2.4) integralni to‘g‘ri almashtrish usuli bilan ham qo‘llash mumkin, tenglamaga kirgan har qanday kvadrat formulasining yakuniy yig‘indisi bo‘yicha.

Masalan trapetsiyaning umumlashtrilgan formulasini qo‘llasak chiziqni teng qisimlarga bo‘lish orqali quidagi vujudga keladi

$$y_k - \lambda \int_a^{x_k} K(x_k, s)\, y(s) ds \approx$$

$$\approx y_k - \frac{h\lambda}{2}\left[K_{k,0} y_0 + 2\big(K_{k,1} y_1 + K_{k,2} y_2 + \cdots + K_{k,k-1} y_{k-1}\big) + K_{k,k} y_k\right] \approx f(x_k),$$

yoki

$$y_k - \frac{h\lambda}{2}\left[K_{k,0} Y_0 + 2\big(K_{k,1} Y_1 + K_{k,2} Y_2 + \cdots + K_{k,k-1} Y_{k-1}\big) + K_{k,k} y_k\right] - f(x_k) = 0,$$

shunda

$$y_k = \frac{1}{1 - \frac{\lambda h}{2} K_{k,k}} \left\{ f_k + \frac{\lambda h}{2} K_{k,0} Y_0 + h\lambda \sum_{i=1}^{k-1} K_{k,i} Y_i \right\}.$$

Shunday qilib, y_k hamma qiymatlarini qadamma-qadam topamiz .

Volterraning 1-tur

$$\lambda \int_a^x K(x,s)y(s)ds = f(x)$$

integral tenglamasiga kelsak, $K(x,s)$ va $f(x)$ uzluksiz differensiallanuvchi, $K(x,x) > \alpha > 0$, uni Volterraning 2-tur integral tenglamasiga keltirish mumkin. (2.1) tenglamaning ikkala qismini differensiallab

$$\lambda K(x,x)y(x) + \lambda \int_a^\infty K'_x(x,s)y(s)ds = f'(x)$$

ga ega bo'lamiz va $y(x)$ Volterraning

$$y(x) + \int_a^\infty \frac{K'_x(x,s)}{K(x,x)} y(s)ds = \frac{1}{\lambda}\frac{f'(x)}{K(x,x)}.$$

2-tur integral tenglamasining echimi bo'ladi.

Masalan

$$y(x) - \int_0^\infty e^{-x-8} y(s)ds = \frac{e^{-x} + e^{-3x}}{2}.$$

Integral tenglamaning echimini toping.

Yechish: 1-usul

yechimni quidagicha topamiz

$$y(x) \approx y_4(x) = \varphi_0(x) + \varphi_1(x) + \varphi_2(x) + \varphi_3(x) + \varphi_4(x).$$

Bu erda

$$\varphi_0(x) = f(x);\ \ \varphi_k(x) = \int_a^\infty K(x,s)\varphi_{k-1}(s)ds.$$

Integrallash natijasida quidagilarga ega bo'lamiz :

$$\varphi_0(x) = \frac{1}{2}(e^{-x} + e^{-3x}),$$

$$\varphi_1(x) = \int_0^x e^{-x-s}\, \varphi_0(s)ds = \frac{1}{8}(3e^{-x} - 2e^{-3x} - e^{-5x}),$$

$$\varphi_2(x) = \int_0^x e^{-x-s}\varphi_1(s)ds = \frac{1}{48}(5e^{-x} - 9e^{-3x} + 3e^{-5x} + e^{-7x}),$$

$$\varphi_3(x) = \int_0^x e^{-x-s}\varphi_2(s)ds = \frac{1}{384}(7e^{-x} - 20e^{-3x} + 18e^{-5x} - 4e^{-7x} - e^{-9x})$$

$$\varphi_4(x) = \int_0^x e^{-x-s}\varphi_3(s)ds$$

$$= \frac{1}{3840}(9e^{-x} - 35e^{-3x} + 50e^{-5x} - 30e^{-7x} + 5e^{-9x} + e^{-11x})$$

bu erdan

$$Y_4(x) = \frac{1}{3840}[3839e^{-x} + 5e^{-3x} - 10e^{-5x} + 10e^{-7x} - 5e^{-9x} + e^{-11x}]$$

Bu tenglamaning aniq echimi

$$y(x) = e^{-x}$$

Taqqoslash uchun aniq echimi va taxminiy echimlarning manosini keltiramiz [31] quidagicha

$$y(0) = 1{,}00000 \qquad y(1) = 0{,}36788,$$

$$Y_4(0) = 1{,}00000 \qquad Y_4(1) = 0{,}36783$$

III bob. Sonlar nazariyasi usullarida tuzilgan to'rlar yordamida Volterra integral tenglamalarni taqribiy echish.

3.1-§. Funksiyalarni davriylashtirish

E_s^α sinfiga kiruvchi funksiyalar uchun olingan har qanday kvadratur formula H_s^α sinfidagi davriy funksiyalar uchun ham o'rinli bo'ladi , chunki 7 - lemma ning birinchi xulosasiga ko'ra, bunday funksiyalar E_s^α sinfiga tegishlidir

Keling , H_s^α sinfidan ixtiyoriy funksiya uchun kvadratur formulalarini qurish masalasini E_s^α sinfidagi kvadratur formulalari masalasiga keltirish mumkinligini ko'rsatalaylik $\alpha \geq 2$ bo'lganda $f(x_1, \dots, x_s)$ funksiyaning eng oddiy davriyligi $\varphi(x_1, \dots, x_s) \in H_s^\alpha$ shartni qanoatlantiruvchi funksiyani topish masalasi deb qaraymiz

$$\left.\begin{aligned} &\varphi(x_1, \dots, x_{\nu-1}, 1, x_{\nu+1}, \dots, x_s) = \varphi(x_1, \dots, x_{\nu-1}, 0, x_{\nu+1}, \dots, x_s) \\ &\int_0^1 \dots \int_0^1 f(x_1, \dots, x_s)\, dx_1 \dots dx_s = \int_0^1 \dots \int_0^1 \varphi(x_1, \dots, x_s)\, dx_1 \dots dx_s \end{aligned}\right\}$$

$$(\nu = 1, 2, \dots, s) \tag{3.1}$$

Agar bu shartlar bajarilsa va ularning birinchisi nafaqat φ funksiyaning o'zi balki uning x_ν dan $\alpha - 2$ tartibgacha bo'lgan hosilalari uchun ham amal qilsa, mos qo'yilgan φ funksiyani f funksiyani davriylashtirish to'liq deb ataymiz

Qisqaroq qilib yozish uchun odatiy belgilar bilan bir qatorda biz quyidagi qisqartmalardan ham foydalanamiz :

$$(m, Q) = m_1 x_1 + \dots + m_s x_s \quad ,$$

$$\int_{G_s} f(Q)\, dQ = \int_0^1 \dots \int_0^1 f\ (x_1, \dots, x_s)\ dx_1 \dots dx_s$$

$$f^{n_1, \dots, n_s}(Q) = \frac{\partial^{n_1 + \dots + n_s}}{\partial x_1^{n_1} \dots \partial x_s^{n_s}} f(x_1, \dots, x_s) \tag{3.2}$$

$$\frac{\partial^n}{\partial x_\nu^n} f[Q_\nu(x)] = \frac{\partial^n}{\partial x_\nu^n} f(x_1, \dots, x_{\nu-1}, x, x_{\nu+1}, \dots, x_s)$$

Ushbu belgilardan foydalanib , davrlashtirish shartlarini quyidagi ko'rinishda yozib olamiz :

$$\begin{cases} 1^{\circ}. \dfrac{\partial^n}{\partial x_\nu^n} \varphi [\, Q_\nu (1)] = \dfrac{\partial^n}{\partial x_\nu^n} \varphi [\, Q_\nu (0)] \\ \nu = 1, 2, \dots , s; \; n = 0, 1, \dots , \alpha_1 - 2), \\ 2^{\circ}. \displaystyle\int_{G_s} f\,(Q)\; dQ = \int_{G_s} \varphi\,(Q)\; dQ \,. \end{cases} \qquad (3.3)$$

Bu erda $\alpha_1 = 2$ yoki $\alpha_1 = \alpha$ lardan birini tanlab , mos ravishda eng oddiy yoki to'la davriylashtirish shartlarini olamiz .

Funksiyalarni davriylashtirish imkonini beradigan quyidagi usul yordamida tekshirish oson bo'ladi [18].

$\Psi\,(x) \in E_1^{\alpha+1}$ - quyidagi shartlarni bajaruvchi ixtiyoriy monoton funksiya bo'lsin

$$\Psi(0) = 0\,, \Psi(1) = 1\,, \Psi^n(1) = \Psi^n(0) = 0 \; (n = 1,2, \dots \alpha_1 - 1) \qquad (3.4)$$

$x_\nu = \Psi(Z_\nu) \quad (\nu = 1,2, \dots \mathrm{s})$ ni hisobga olib va o'zgaruvchilarni almashtirishni amalga oshirgan holda quyidagini hosil qilamiz

$$\int_0^1 \dots \int_0^1 f\;(x_1, \dots , x_s)\; dx_1, \dots , dx_s = \int_0^1 f[\Psi(Z_1), \dots , \Psi(Z_s)] \times$$
$$\Psi'\,(Z_1\,) \dots \Psi'\,(Z_s) dZ_1 \dots dZ_s \qquad (3.5)$$

tanlaymiz

$$\varphi\,(\,x_1\,, \dots , x_s\,) = \mathrm{f}[\Psi(x_1), \dots , \Psi(x_s)] \quad \Psi'\,(x_1) \dots \Psi'\,(x_s) \qquad (3.6)$$

$f \in H_s^\alpha$ va $\Psi \in H_s^{\alpha+1} \in H_1^{\alpha+1}$ bo'lganligi uchun φ funksiya $\varphi^{\alpha,\dots,\alpha}\;(Q)$ hosilaga ega va shu bilan birga H_s^α sinfiga tegishli bo'ladi. Bundan tashqari, (3.5) formuladan φ funksiya uchun 2° shart bajariladi. Nihoyat (3.4) tufayli $\nu = 1,2, \dots s$ da quyidagi tengliklar bajariladi

$$\frac{\partial^n}{\partial x_\nu^n} \varphi[\, Q_\nu(1)] = \frac{\partial^n}{\partial x_\nu^n} [\, Q_\nu(0)] \qquad (n = 0, 1, \dots \alpha_1 - 2\,,)$$

Bu erda 1° shart butunligicha bajarilishini ta'minlaydi . Shunday qilib , funksiyani davriyrashtirish (3.6) tenglik bilan aniqlangan f funksiyasi yordamida amalga oshirilishi mumkin .

Funksiyalarni davriylashtirish bizga H_s^{α} sinfidagi kvadratura formulalarini qurish masalasini E_s^{α} sinfidagi shunga o'xshash masalaga qisqartirish imkonini beradi. Bunday qisqartirish imkoniyati quyidagi lemmaga asoslanadi :

Lemma 12. $\alpha \geq 2, 2 \leq \alpha_1 \leq \alpha$ va $\varphi(x_1, \dots, x_s) \alpha H_s^{\alpha}(C)$ bo'lsin.

Agar

$$\frac{\partial^n}{\partial x_v^n}\varphi[Q_v(1)] = \frac{\partial^n}{\partial x_v^n}\varphi[Q_v(0)], \ (v = 1,2, \dots s; n = 0,1, \dots, \alpha_1 - 2) \quad (3.7)$$

Tenglik bajarilsa, u holda $\varphi(\{x_1\}, \dots, \{x_s\})$ funksiya $H_s^{\alpha_1}(C)$ sinfiga mansub hisoblanadi.

Isbot . (3.7) tenglikni differensiallash va n ni n_v ga almashtirib

$$\varphi^{n_1, \dots, n_s}[Q_v(1)] = \varphi^{n_1, \dots, n_s}[Q_v(0)] \quad (3.8)$$

Hosil qilamiz , bu erda

$$0 \leq n_j \leq \begin{cases} j = v \text{ bo'lganda}, \alpha_1 - 2 \\ j \neq v \text{ bo'lganda}, \ \alpha_1 . \end{cases}$$

$\tau_1, \dots, \tau_s$ kattaliklari quyidagi tenglik orqali aniqlangan bo'lsin

$$\tau_v = \begin{cases} 0 \ , \text{agarda } m = 0, \\ 1, \text{ agarda } m \neq 0 . \end{cases} \quad (v = 1, 2, \dots, s \ ).$$

Furye funksiyasi $\varphi(\{x_1\}, \dots, \{x_s\})$ koeffitsienlarini $C(m_1, \dots, m_s)$ orqali belgilab olamiz :

$$C(m_1, \dots, m_s) = \int_0^1 \dots \int_0^1 \varphi\ (x_1, \dots, x_s)\, e^{-2\pi i(m_1 x_1 + \dots + m_s x_s)} dx_1 \dots dx_s$$

va $(\alpha_1 - 1)\tau_v$ - har bir x_v o'zgaruvchi bo'yicha bo'laklab karrali integral olamiz. (3.8) tenglikdagi integrallangan hadlarning nolga aylanishini bilgan holda va qisqartma belgilardan foydalanib quyidagi ifodani hosil qilamiz

$$C(m_1, \dots, m_s) = \frac{(2\pi)^{-(\alpha_1 - 1)(\tau_1 + \dots + \tau_s)}}{(\overline{m_1} \dots \overline{m_s})^{(\alpha_1 - 1)}} \left| \int_{G_s} \varphi^{(\alpha_1 - 1)\tau_1, \dots, (\alpha_1 - 1)\tau_s}(Q) \times \right.$$

$$\left. e^{-2\pi i(m, Q)} dQ \right| \quad (3.9)$$

Yana bo'laklab integrallaymiz va keyingi $H_s^{\alpha}(C)$ sinfi ta'rifidagi bahodan foydalanamiz

$$|\varphi^{n_1, \dots, n_s}(Q)| \leq C \quad (0 \leq n_v \leq \alpha, \ v = 1,2, \dots, s)$$

Chunki endi har bir bo‘laklab integrallashda 3 ta yangi integral paydo bo‘ladi , u holda (3.9) ifodadan

$$C(m_1, \dots, m_s) = \frac{3^{\tau_1+\dots+\tau_s} C}{(2\pi)^{\alpha_1(\tau_1+\dots+\tau_s)}(\overline{m_1} \dots \overline{m_s})^{\alpha_1}} \leq \frac{C}{(\overline{m_1} \dots \overline{m_s})^{\alpha_1}}$$

hosil qilamiz .

Bu erdan , shunday qilib φ $(\{x_1\}, \dots, \{x_s\})$ funksiyasi o‘zgaruvchilarning har biri uchun yagona cheksiz davrga ega bo‘lganligi sababli, E_s^{α} sinfiga ko‘ra lemma shartlariga bo‘ysunadi.

12-lemma shartlarini qanoatlantiruvchi funksiya uchun quyidagi misolni ko‘rib chiqamiz.

$B_r(x) - r$ o‘lchovli Bernulli ko‘phadi bo‘lsin. Shunday qilib Lemma 6 ning ikkinchi shartiga binoan

$$B_r'(x) = rB_{r-1}(x) \ (r \geq 1)$$

u holda, oydinlashadiki, $n \leq r$ da

$$B_r^{(n)}(x) = r(r-1) \dots (r-n+1)B_{r-n}(x)$$

Ammo aynan shu lemmaning birinchi shartiga ko‘ra

$$B_{r-n}(1) = B_{r-n}(0), \quad r - n \geq 2$$

va bundan kelib chiqadiki

$$B_r^{(n)}(1) = B_r^{(n)}(0), \quad n = 0,1, \dots, r-2$$

Bu tengliklardan foydalangan holda , ishonch hosil qilish mumkinki , funksiya uchun $r \geq 2$ bo‘lganda

$$\varphi(x_1, \dots, x_s) = B_1(x_1) \dots B_r(x_s)$$

$\alpha_1 = \alpha = 1$ da bajariluvchi (2.11) ning shartlari bilan mos keluvchi

$$\frac{\partial^n}{\partial x_\nu^n} \varphi[Q_\nu(1)] = \frac{\partial^n}{\partial x_\nu^n} \varphi[Q_\nu(0)]$$

$$\nu = 0,1,2, \dots s; \quad n = 1, 2, \dots, \mathrm{r} - 2 ,$$

shartni bajaradi. Shunday qilib $\varphi\ (x_1, \dots, x_s) \in H_s^r$ ekanligi ma'lum bo'lsa, u holda $\varphi\ (x_1, \dots, x_s)$ funksiyasi 12-lemma ning barcha shartlarini qanoatlantiradi va bundan tashqari

$$B_1(\{x_1\}) \dots\ B_r(\{x_s\}) \in H_s^r \qquad (s \geq 1,\ r \geq 2)\ . \tag{3.10}$$

Endi 12 - lemmaning ta'rifidagi bitta natijaga ko'ra funksiyalarni davriylashtirish H_s^α sinfidagi kvadratur formulalarida qurilgan masalalarida $2 \leq \alpha_1 \leq \alpha$ bo'lganda $E_s^{\alpha_1}$ sinfi masalalaridagi kabi namoyon bo'lishini ta'riflaymiz.

Natija .

$$f\ (x_1, \dots, x_s) \in H_s^\alpha, \qquad \alpha \geq 2\ va\ \ \varphi\ (x_1, \dots, x_s)$$

Funksiyalari oddiy usulda hosil qilingan yoki $f\ (x_1, \dots, x_s)$ funksiyasini umumiy davriylashrishdan hosil qilingan bo'lsin . U holda

$$\int_0^1 \dots \int_0^1 f\ (x_1, \dots, x_s)\ dx_1 \dots dx_s =$$

$$\int_0^1 \dots \int_0^1 \varphi\,(\{x_1\}, \dots, \{x_s\})\ \ dx_1, \dots dx_s \tag{3.11}$$

bu erda $\varphi\,(\{x_1\}, \dots, \{x_s\})$ funksiyasi mos ravishda yoki E_s^2 sinfiga yoki E_s^α sinfiga tegishli bo'ladi .

Rostan ham , φ funksiyasi f funksiyasini davriylashtirish yo'li bilan hosil qilinganligi sababli , bu funksiya uchun (3.3) sharti bajariladi . Bu shartlarning ikkinchisidan foydalanib, (3.9) tenglikni hosil qilamiz ; (3.4) shartning birinchisi 12 - lemmaning sharti bilan mos keladi shuningdek, $\varphi\,(\{x_1\}, \dots, \{x_s\}) \in E_s^{\alpha_1}$, bu erda mos ravishda $\alpha_1 = 2$ yoki $\alpha_1 = \alpha$.

Oddiy davriylashtirishning bir nechta usullarini ko'rib chiqamiz.

Oddiy davriylashtirishning birinchi usuli quyidagi aniq tenglikka asolangan

$$\int_0^1 f(x)\,dx = \int_0^1 \frac{f(x) + f(1-x)}{2}\,dx \tag{3.12}$$

$S = 1$ uchun tanlaymizki

$$\varphi\,(x_1) = \frac{f(x) + f(1-x)}{2}\,. \tag{3.13}$$

Agar $f(x_1) \in H_1^\alpha$ bo'lsa, u holda $\varphi(x_1)$ funksiyasi ham aniq H_1^α sinfiga tegishli bo'ladi. Shuningdek

$$\varphi(x_1) = \frac{f(0) + f(1)}{2} = \varphi(1),$$

$\alpha_1 = 2$ uchun yozib olingan (48) shartining birinchisi bajariladi. (3.12) ga ko'ra bu shartlarning ikkinchisi bajariladi. Natijada, $\varphi(x_1)$ funksiyasi yordamida $f(x_1)$ funksiyasini oddiy davriylashtirish haqidagi masala echiladi .

$S > 1$ da (3.13) tenglikni har o'zgaruvchi $x_1, \dots, x_s$ bo'yicha birin–ketin tadbiq qilib chiqamiz:

$$\left.\begin{aligned} \varphi_1(x_1, \dots, x_s) &= \frac{1}{2}[f(x_1, \dots, x_s) + f(1 - x_1, x_2, \dots, x_s)], \\ \varphi_2(x_1, \dots, x_s) &= \frac{1}{2}[\varphi_1(x_1, \dots, x_s) + \varphi_1(1 - x_1, x_2, \dots, x_s)] \\ &\dots\dots\dots\dots\dots\dots \\ \varphi_s(x_1, \dots, x_s) &= \frac{1}{2}[\varphi_s(x_1, \dots, x_s) + \varphi_s(x_1, x_2, \dots, 1 - x_s)] \end{aligned}\right\} \quad (2.18)$$

$\varphi(x_1, \dots, x_s) = \varphi_s(x_1, \dots, x_s)$ ni tanlaymiz. $f(x_1, \dots, x_s) \in H_s^\alpha$ shartidan $s = 1$ bo'lgani kabi, bevosita $\varphi(x_1, \dots, x_s) \in H_s^\alpha$ bo'ladi.

$$\int_0^1 \cdots \int_0^1 f\ (x_1, \dots, x_s)\ dx_1 \dots dx_s$$

$$= \int_0^1 \cdots \int_0^1 \frac{1}{2} \lfloor f(x_1, \dots, x_s) + f\ (1 - x_1, x_2, \dots, x_s) \rfloor dx_1 \dots dx_s$$

$$= \int_0^1 \cdots \int_0^1 \varphi_1(x_1, \dots, x_s)\ dx_1 \dots dx_s = \cdots$$

$$= \int_0^1 \cdots \int_0^1 \varphi_s(x_1, \dots, x_s)\ dx_1 \dots dx_s,$$

(3.3) shartning ikkinchisi bilan mos keladi . $\alpha_1 = 2$ da bajariladigan shartlardan birinchisi bevosita (3.14) tenglik uchun ham o'rinli ekanligi kelib chiqadi . Shu yo'l bilan $\varphi\ (x_1, \dots, x_s) = \varphi_s\ (x_1, \dots, x_s)$ funksiyalari , $f\ (x_1, \dots, x_s)$ funksiyani oddiy davriylashtirish haqidagi masalani echadi

Oddiy davriylashtirishning ikkinchi usuli o'zgaruvchilarni jo'n ravishda almashtirish orqali hosil qilish mumkin, (3.6) tenglikda aytilganidek

$$\varphi\ (x_1, \dots, x_s) = f\ [\Psi(x_1) \dots \Psi(x_s)]\ \Psi'(x_1) \dots \Psi'(x_s)$$

va masalan $\Psi(x) = sin^2 \frac{\pi}{2} x$ Shuningdek

$$\Psi'(x) = \frac{\pi}{2} \sin 2\pi x$$

u holda

$\Psi(0) = 0, \quad \Psi(1) = 1, \quad \Psi'(x_1) = \Psi'(1) = 0$

Shu yo'l bilan (2.8) shart $\alpha_1 = 2$ yordamida bajariladi va bevosita

$$\varphi\ (x_1, \dots, x_s) = \frac{\pi^2}{2} \sin \pi x_1 \dots \sin \pi x_s f\ (sin^2 \frac{\pi}{2} x_1, \dots, sin^2 \frac{\pi}{2} x_s)$$

funksiyasi yordamida oddiy davriylashtiriluvchi $f\ (x_1, \dots, x_s)$ funksiya amalga oshadi .

Oddiy davriylashtirishning uchinchi usuli quyodagi aniq tenglikdan foydalanishga asoslangan

$$\int_0^1 f(x)\,dx = \int_0^1 \left[f(x) + \left(x - \tfrac{1}{2}\right) (f(0) - f(1))dx\right] \tag{3.15}$$

$S = 1$ bo'lganda tanlaymiz

$$\varphi(x) = f(x) + \left(x - \tfrac{1}{2}\right)\left(f(0) - f(1)\right) \qquad (3.16)$$

Agar $f\ (x_1) \in H_1^\alpha$, u holda $\varphi(x_1) \in H_s^\alpha$ bo'lishi ma'lum . (3.3) ning ikkinchi sharti (2.19) tengligi bilan mos keladi . Shuningdek

$$\varphi\,(0) = \frac{f(0) + f(1)}{2} = \varphi\,(1)$$

bu shartlardan birinchisi ham bajariladi, hamda, bevosita, $\varphi(x_1)$ funksiyasi $f(x_1)$ funksiyani oddiy davriylashtirish haqidagi masalani echadi.

$s > 1$ da (2.20) tenglikni birin – ketin tadbiq qilish yo'li bilan

$$\left.\begin{aligned}
&\varphi_1\,(x_1,\dots,x_s) = f\,(x_1,\dots,x_s) + \left(x_1 - \tfrac{1}{2}\right)[f\,(0,x_1,\dots,x_s) - \\
&f\,(1,x_1,\dots,x_s)] \\
&\dots\dots\dots\dots\dots\dots\dots\dots \\
&\varphi_s\,(x_1,\dots,x_s) \\
&\qquad = \varphi_{s-1}\,(x_1,\dots,x_s) \\
&\qquad + \left(x_s - \frac{1}{2}\right)[\varphi_{s-1}\,(x_1,\dots,x_s,0) - \varphi_{s-1}\,(x_1,\dots,x_s, \quad 1)]
\end{aligned}\right\} \quad (3.17)$$

$\varphi\,(x_1,\dots,x_s) = \varphi_s\,(x_1,\dots,x_s)$ tanlaymiz. $f \in H_s^\alpha$ shartdan φ funksiyaning ta'rifiga ko'ra bevosita $\varphi \in H_s^\alpha$ ekanligi kelib chiqadi . (3.17) tenglikdan foydalanib , aniqlaymizki , jo'n induksiya funksiya ϕ (3.3) ifoda qoniqtiradi. . Shunday qilb , $\varphi\,(x_1,\dots,x_s) = \varphi_s\,(x_1,\dots,x_s)$ funksiya yordamida $f\,(x_1,\dots,x_s)$ oddiy funksiya davomiyligi mavjud bo'ladi. Endi to'la davomiylikning bir necha usullarini ko'ramiz.

Agar $\alpha \geq 2$ va $f\,(x_1,\dots,x_s) - H_s^\alpha$ sinfining mustaqil funksiyasi bo'ladi.

Aniqroq aytganganda , umumiy davriylashtirilgan $f\ (x_1, \dots, x_s)$ funksiya (3.4) ifodada ko'rsatilgan , $\varphi\ (x_1, \dots, x_s)$ funksiya yordamda keltirilgn b'lishi mumkin

$$\varphi\ (x_1, \dots, x_s) = f\ [\Psi(x_1) \dots \Psi(x_s)]\, \Psi'(x_1) =$$
$$f\ [\Psi(x_1) \dots \Psi(x_s)]\, \Psi'(x_1) \dots \Psi'(x_s) \qquad (3.18)$$

$\Psi(x) - H_s^{\alpha+1}$ sinfda mustaqil monoton funksiya va bu shartlarni bajaradi

$$\Psi(0) = 0 \qquad \Psi(1) = 1 \quad \Psi^n(0) = \Psi^n(1) = 0$$
$$(n = 1,2, \dots \alpha - 1) \qquad (3.19)$$

$\Psi(x)$ funksiya o'rniga boshqa funksiya qo'yish mumkinligini ko'rsatamiz

$$\Psi_\alpha(x) = (2\alpha - 1)C_{2\alpha-1}^{\alpha-1} \int_x^0 [\, t(1-t)^{\alpha-1}]\, dt.$$

3.2-§. Ko'p o'lchovli Volter integral tenglamalarni sonlar nazariyasi usullari yordamida taqribiy echish.

Bu paragrifda sonlar nazariyasi to'ri metodi bilan iteratsiya usulini birgalikda qo'llab,

$$\varphi(x_1, x_2, \cdots, x_s) = \int_0^x \cdots \int_0^x \sum_{r=1}^q K_r(x_1, x_2, \cdots, x_s, y_1, y_2, \cdots, y_s)\varphi_r(y_1, y_2, \cdots, y_s)dy_1 dy_2 \cdots dy_s + f(x_1, x_2, \cdots, x_s) \qquad (3.20)$$

2-tur Volter integral tenglamasini echishni ko'rib chiqamiz.

(3.20) tenglamani echish uchun qo'shimcha tasdiq va ta'riflar kiritamiz.

Ta'rif-1. $f(x_1 \dots, x_n)$ funksiya E_n^α sinfga tegishli deyiladi, agarda

$$S_f(m_1, \dots, m_n) = 0((\overline{m_1}\ , \dots, \overline{m_n})^{-\alpha})$$

baho bajarilsa, bunda α birdan katta xaqiqiy son va «0» belgidag i o'zgarmas $m_1, \dots, m_n$ ga bog'liq emas.

Bu konsttantaning qiymatini ko'rsatish kerak bo'lsa, Ye_n^a o'rniga $Ye_n^a(S)$ ni yozish kerak va oldingi bahoni

$$|S_f(m_1,\dots,m_n)| \le \frac{C}{(\overline{m}_1 \dots \overline{m}_n)^a}$$

tengsizlik bilan almashtirish kerak.

Faraz qilamiz, $0 \le t \le v - r$ intervaldan olingan ixtiyoriy t butun son uchun $\Omega, 1xx_1x_2 \ . \ . \ . x_v0$ sohada aniqlangan $\Phi(x_1, x_2, \dots, x_v)$ funksiya

$$\frac{\partial^t \Phi(x_1, \dots, x_v)}{\partial x_{g_1} \partial x_{g_2} \dots \partial x_{g_t}} \qquad (3.22)$$

hosilaga ega bo'lsin, bunda $g_1, g_2, \dots, g_t$ lar v dan oshmaydigan shunday sonlarki, ular uchun $\mu \ne \varrho$ da va ixtiyoriy $\mu (1 \le \mu \le t)$ va $v\ (1 \le v \le r)$, $g_\mu \ne g_v$ larda $g_\mu \ne g_\varrho$ bo'ladi.

$h(\Phi, s, r)$ orqali (3.22)hosila modulining $g_1, g_2, \dots, g_t$ lar bo'yicha yuqori chegarasini belgilaymiz.

$$(\overline{m}_{s-r+1} \dots \overline{m}_{s_0})^{-1} \sum_{1 \le h_v \le s-r} \int_0^1 e^{-2\pi i m_{h_1} x_1} dx_1 \int_0^{x_1} e^{-2\pi i m_{h_2} x_2} dx_2 \int_0^{x_2} \dots$$

$$\int_0^{x_{j_1-2}} e^{-2\pi i m_{h_{j_1-1}} x_{j_1-1}} dx_{j_1-1} \int_0^{x_{j_1-1}} e^{i b_1 x_{j_1}} dx_{j_1} \int_0^{x_{j_1}} e^{-2\pi i m_{h_{j_1}} x_{j_1+1}} dx_{j_1+1} \int_0^{x_r} \dots$$

$$\int_0^{x_{j_1+1}} \dots \int_0^{x_{j_2-1}} \int_0^{x_r} e^{i b_r x_{j_r}} dx_{j_r} \int_0^{x_{s_0-1}} e^{-2\pi i m_{h_{s_0}} x_{s_0}} \chi\Phi(x_1, \dots, x_{s_0}) dx_{s_0},$$

yig'indini (s, r) tartibli (*) yig'indi deb aytamiz

(bu erda $s_0 s$, $\overline{m} = \begin{cases} 1, & m = 0, \\ m, & m \ne 0, \end{cases}$ $b_1, b_2, \dots, b_r$ lar $(m_{s-r+1}, \dots, m_{s_0})$

sonlarning biror chiziqli kombinatsiyasi) va $\sum_{*}(s, r)$ kabi belgilaymiz

Lemmaning isbotini [3] dan ko'ring.

[1], [2] ishlarda E_s^α sinfda Fredgol 2-tur integral tenglamasining taqribiy echimi optimal koeffitsientlarda berilgan. E_s^α sinfda kvadratur formula integrallash oralig'i birlik kub bo'yicha bo'lgan hol uchun qurilgan.

Volter integral tenglamasida o'zgaruvchi integrallash oralig'ida bo'ladi va Volter integral tenglamasini Fredgolm integral tenglamasiga keltirishda yadroning silliqlik darajasi pasayadi.

Shuning uchun yadro $K_{ij} \in H_{2s}^\alpha$ va ozod had $f \in H_s^\alpha$ bo'lgan holni ko'ramiz.

Qaralayotgan (3.20) tenglamani

$$\varphi(P) = \int_0^x \cdots \int_0^x \sum_{r=1}^{q} K_r(P,Q)\varphi_r(Q)d\bar{Q} + f(P), \qquad (3.23)$$

ko'rinishda yozib olaylik.

Faraz qilaylik, (3.20) tenglamaning ozod hadi va yadrosi mos ravishda $H_s^\alpha(C_1)$ va $H_{2s}^\alpha(C_2)$ sinflarga tegishli bo'lsin, u holda

$$f(P) \in H_s^\alpha(C_1), \quad K_r(P,Q) \in H_{2s}^\alpha(C_2)$$
$$\tau = 1, 2, \cdots, q. \qquad (3.24)$$

Quyidagi teorema o'rinli bo'ladi.

Teorema. Agar $f(P)$, $K_\tau(P,Q)$, $(\tau = 1, \ldots, q)$ funksiyalar (3.22) shartni qanoatlantirsa, u holda quyidagi munosobat o'rinli bo'ladi

$$\varphi(P) - f(P) - \frac{1}{N}\sum_{k=1}^{N}\sum_{n=1}^{M}\frac{x^n}{n!}\Psi(P, PM_{k,n}) = O(N^{-\alpha+\varepsilon}),$$

bunda

$$M_{k\,v} = \left(\left\{\frac{a_{s(v-1)+1}k}{N}\right\}, \cdots, \left\{\frac{a_{sv}k}{N}\right\}\right), \quad k = 1, 2, \cdots, N; \; v = 1, 2, \cdots, n,$$

va $\{A\}$ eng yaqin butun songacha bo'lgan masofani anglatadi.

Isbot. Bu tenglama echimini

$$\varphi(P) = \sum_{n=0}^{\infty} \lambda^n \varphi_n(P)$$

Neyman qatori ko'rinishda qidiramiz. Bu erda λ -kichik parametr, $\varphi_{ij}(P)$ funksiya esa ta'rifga mos.

(3.23) ni (3.20) ga qo'yib, integrallash amali bilan summani hisoblash amallarining o'rnini almashtirib

$$\sum_{n=0}^{\infty}\lambda^n\varphi_n(P)=\sum_{n=1}^{\infty}\sum_{j=1}^{q}\lambda^n\int_0^x\cdots\int_0^x\left(K_j(P,Q)\varphi_{jn-1}(Q)dQ+f(P)\right),$$

ni hosil qilamiz.

λ ning bir xil darajalari oldidagi koeffitsientlarni tenglashtirib, ba'zi almashtirishlarni bajargandan so'ng quyidagiga ega bo'lamiz

$$\varphi_0(P)=f_j(P)\ldots\varphi_n(P)=$$

$$=\int_0^x\cdots\int_0^x\int_0^{y_1}\cdots\int_0^{y_{s(n-1)}}\left(K_1(P,Q_1)\prod_{m=2}^{n}K_m(Q_{m-1},Q_m)f_m(Q_m)\right)d\bar{Q}$$

unda $d\bar{Q}=dy_1\ \cdots\ dy_n$.

Quyidagicha belgilashlar kiritamiz

$G_s=\{0\le x_1\le 1,0\le x_2\le 1,\cdots,0\le x_s\le 1\}$ -birlik s o'lchovli kub,

$G_s(x)=\{0\le x_1\le x,\ 0\le x_2\le x,\cdots,\ 0\le x_s\le x\}$ qirrasi x ga teng bo'lgan s o'lchovli kub.

$$G_s(x)=\{0\le x_1\le x,0\le x_2\le x,\cdots,0\le x_n\le x\}$$

kubda $\Psi_n(x_1,\ .\ .\ .\ ,x_n)$ funksini quyidagicha aniqlaymiz:

$$\Psi_n(x_1,\cdots,x_n)=$$

$$=\sum_{j_1,j_2,\ldots,j_n=1}^{q}K_{jj_1}(P,Q_1)\prod_{m=2}^{n}K_{j_{m-1}j_m}(Q_{m-1},Q_m)f_{j_m}(Q_m).$$

U holda

$$\varphi_{jn}(P)=$$

$$=\int_0^x\cdots\int_0^x\int_0^{y_1}\cdots\int_0^{y_{s(n-1)}}\left(\sum_{j_1,j_2,\ldots,j_n=1}^{q}K_{jj_1}(P,Q_1)\prod_{m=2}^{n}K_{j_{m-1}j_m}(Q_{m-1},Q_m)f_{j_m}(Q_m)\right)d\bar{Q}$$

$$=\int_{G_{ns}(x)}\Psi_{jn}(x_1,\cdots,x_{sn})d\bar{Q}$$

$K(P,Q)$ va $f(P)$ lar chegaralangan bo'lgani uchun $C_s > 0$ shundayki, unda

$$\left|\varphi_j(P) - \sum_{n=0}^{\infty} \varphi_{jn}(P)\right| \le \frac{C_2^{M+1}}{(M+1)!}, \; j = 1, \; . \; . \; . \; , g$$

bo'ladi.

$\Psi_n(P,Q)$ funksiyani

$$\cdots \int_0^{y_{h_{n-1}}} \sum_{j_1, j_2, \ldots, j_n = 1}^{q} \begin{matrix} K_{jj_1}(P, y_{h_1}) K_{j_1 j_2}(y_{h_1}, y_{h_2}) \cdots \\ K_{j_{n-1} j_n}(y_{h_{n-1}}, y_{h_n}) f_{j_m}(y_{h_n} \end{matrix}) dy_{h_n} \int_{G_s(x)} \Psi_n(x_1, \cdots, x_{sn}) d\bar{Q}$$

$$=$$

$$= n! \int_{G_s(x)} dy_{h_1} \int_0^{y_{h_1}} dy_{h_2} \int_0^{y_{h_2}} \; . \; . \; . \int_0^{y_{n-1}} \sum_{j_1, j_2, \ldots, j_n = 1}^{q} K_{jj_1}(P, y_1)$$

$$\times \sum_{\substack{1 \le h_k \le n \\ h_k \ne h_m}} \times \prod_{m=2}^{n} K_{j_{m-1} j_m}(y_{m-1}, y_m) f_{j_m}(y_n) dy_n$$

$$= n! \; \varphi_{jn}(P) \int_{G_s(x)} dy_{h_1} \int_0^{y_{h_1}} dy_{h_2} \int_0^{y_{h_2}} \; . \; . \; .$$

Bo'ladigan qilib tanlaymiz. Bundan

$$\varphi_n(P) = \frac{1}{n!} \int_0^x \cdots \int_0^x \Psi_{jn}(x_1, \cdots, x_{sn}) d\bar{Q} = \frac{1}{n!} \int_{G_{ns}(x)} \Psi_{jn}(P, Q) d\bar{Q},$$

U holda

$$\varphi_n(P) = \int_0^1 \cdots \int_0^1 \frac{x^n}{n!} \Psi_{jn}(Py_1, \cdots, Py_n) \, d\bar{Q} = \int_{G_{ns}} \frac{x^n}{n!} \Psi_{jn}(P, Q) d\bar{Q} \,,$$

$$\mathrm{F}(Q) = \frac{x^n}{n!} \Psi_n(Py_1, \cdots, Py_n) \tag{3.25}$$

belgilash kiritamiz, u holda

$$\varphi_n(P) = \int_{G_s} \mathrm{F}(Q)d\bar{Q}\,,$$

Ma'lumki agar $f_1(P)$ va $f_2(P)$ funksiyalar mos ravishda $H_s^{\alpha}(C_1)$ va $H_s^{\alpha}(C_2)$ sinflarga tegishli bo'lsa, u holda ixtiyoriy B_1 va B_2 lar uchun

$$B_1 f_1 + B_2 f_2 \in H_s^{\alpha} \qquad (|B_1|C_1 + |B_2|C_2)f_1 f_2 \in H_s^{\alpha}(AC_1C_2)$$

bo'ladi, bunda

$$A \le \left(2^{\alpha+1}\left(3+\frac{2}{\alpha-1}\right)\right)^n.$$

U holda, $\alpha = 1$ ekanligidan

$$K_{jj_1}(P, Q_{h_1})K_{j_1j_2}(Q_{h_1}, Q_{h_2})\ .\ .\ .\ K_{j_{n-1}j_n}(Q_{h_{n-1}}, Q_{h_n})f_{j_m}(Q_{h_n})$$

funksiya $y_1, \ .\ .\ .\ , y_s$ o'zgaruvchilarning funksiyasi sifatida $H_s^1(AC_1^nC_2)$ sinfga tegishli bo'ladi, bundan esa

$$\Psi_n(Q) \in H_s^1(Aq^nC_1^nC_2) \tag{3.26}$$

ekani kelib chiqadi.

$\mathrm{F}_j(Q)$ funksiyaning Fure koeffitsientining bahosini keltiramiz

$$C(m_1, \cdots m_n) = \int_0^1 \cdots \int_0^1 e^{-2\pi i(m_1y_1 + \ .\ .\ .+ y_n m_n)}\mathrm{F}_j(y_1, \quad \cdots, y_n)\,dy_1 \cdots dy_n$$

$$= \frac{p^n}{n!}\sum_{\substack{1\le h_k\le n\\ h_k\ne h_m}} \int_{G_s} e^{-2\pi i m_{h_1}y_1}dy_{h_1}\int_0^{y_1} e^{-2\pi i m_{h_2}y_2}dy_{h_2}\int_0^{y_2}\cdots\int_0^{y_{n-1}} e^{-2\pi i m_n y_n}\times$$

$$\times \sum_{j_1,j_2,\ .\ .\ .\ ,j_n=1}^{q} K_{jj_1}(P, Py_1)\prod_{m=2}^{n} K_{j_{m-1}j_m}(Py_{m-1}, Py_m)f_{j_m}(Py_n)dy_n$$

Bundan ko'rinadiki, $C(m_1, \cdots m_n)$ koeffitsient (s, 0) tartibli (*) turdagi yig'indini hosil qiladi.

Lemma2. Yuqorida qilingan farazlar o'rinli bo'lganda quyidagi baho o'rinli bo'ladi:

$$\left|\sum\nolimits_{*}(s,r)\right| \leq \frac{(s-r)!}{\bar{m}_1 \cdot \bar{m}_2 \ldots \bar{m}_{s_0}} L(\Phi, \mathrm{s}, \mathrm{r}).$$

Lemmaning isbotini [8] dan qarang.

Lemma 2 dan,

$$|C(m_1, \cdots, m_n)| \leq \frac{1}{n!} \cdot \frac{n!}{\bar{m}_1 \ldots \bar{m}_n} \cdot Aq^n C_1^n C_2 = \frac{Aq^n C_1^n C_2}{\bar{m}_1 \ldots \bar{m}_n}.$$

ni olamiz.

Ixtiyoriy $A(P)$ uzluksiz funksiya uchun

$$\int_0^1 \cdots \int_0^1 A(P) dx_1 \cdots dx_s = \int_0^1 \cdots \int_0^1 A(2P) dx_1 \cdots dx_s$$

tenglik o'rinli.

Ko'rinib turibdiki $A(2P)$ funksiya uzluksiz va xar bir o'zgaruvchisi bo'yicha bir davrga ega.

(3.36) dan,

$$\int_{G_{ns}} \frac{P^n}{n!} \Psi_{jn}(Py_1, \cdots, Py_n)\, d\bar{Q} = \int_{G_{ns}} \frac{P^n}{n!} \Psi_{jn}(2Py_1, \cdots, 2Py_n)\, d\bar{Q}.$$

ekani kelab chiqadi.

Quyidagicha belgilash kiritamiz

$$\mathrm{F}_{jn}{}^{*}(y_1, \quad \cdots, y_n) = \frac{x^n}{n!} \Psi_{jn}(2Py_1, \quad \cdots, 2Py_n)$$

$\mathrm{F}_{jn}{}^{*}(y_1, \ldots, y_n)$ funksiyani Steklov bo'yicha almashtiramiz, ya'ni

$$\mathrm{W}_{jn}(y_1, \cdots, y_n) = \frac{1}{(2\Delta)^n} \int_{-\Delta}^{\Delta} \cdots \int_{-\Delta}^{\Delta} \mathrm{F}_{jn}{}^{*}(y_1 + t_1, \cdots, y_n + t_n)\, dy_1 \cdots dy_n$$

ni kiritamiz

Preobrazuem funksiyu $\mathrm{F}_{jn}{}^{*}(y_1, \ldots, y_n)$ po Steklovu, to est vvedem funksiyu

$$\mathrm{W}_{jn}(y_1, \cdots, y_n) = \frac{1}{(2\Delta)^n} \int_{-\Delta}^{\Delta} \cdots \int_{-\Delta}^{\Delta} \mathrm{F}_{jn}{}^{*}(y_1 + t_1, \cdots, y_n + t_n)\, dy_1 \cdots dy_n$$

N natural sonda $\left(y_1^{(k)},\cdots,y_n^{(k)}\right)=\left(\left\{\frac{a_{s(v-1)+1}k}{N}\right\},\ \cdots\ ,\left\{\frac{a_{sv}k}{N}\right\}\right)$ bo'lgan holda

$$\int_0^1\cdots\int_0^1 \mathrm{F}_{jn}{}^*(y_1,\cdots,y_n)\,dy_1\cdots dy_n-\frac{1}{N}\sum_{k=1}^{N}\mathrm{F}_{jn}{}^*\left(y_1{}^{(k)},\cdots,y_n{}^{(k)}\right)=$$

$$=\left(\int_0^1\cdots\int_0^1 W_{jn}(Py_1,\cdots,Py_n)\,dy_1\cdots dy_n\right.$$

$$\left.-\frac{1}{N}\sum_{k=1}^{N}\mathrm{W}_{jn}{}^*\left(y_1{}^{(k)},\cdots,y_n{}^{(k)}\right)\right)$$

ning modulini baholaymiz,

Lemma3. Agar $\Phi\in E_s^{\alpha}(S)$, $\varepsilon_1\in(0,\alpha-1)$ ning qanday bo'lishidan qat'iy nazar $N=p$ da optimal koeffitsientlar bilan qurilgan

$$\int_0^1 . . . \int_0^1 \Phi(x_1,x_2,. . . ,x_s)dx_1dx_2 . . . dx_s$$

$$=\frac{1}{N}\sum_{k=1}^{N}\Phi\left(\left\{\frac{a_{s(v-1)+1}k}{N}\right\},\ . . . ,\left\{\frac{a_{sn}k}{N}\right\}\right)-R,$$

kvatratur formulaning xatoligi uchun

$$|R|\le C\frac{\left(6\alpha+\frac{12\alpha^2}{\varepsilon_1}\right)^{2\alpha s}}{p^{\alpha-\varepsilon_1}}$$

baho o'rinli.

3-lemmani hisobga olsak, $\varepsilon>0$ da (3.24), (3.25), (3.26) lardan

$$\varphi_j(P)-f_j(P)-\frac{1}{N}\sum_{k=1}^{N}\sum_{n=1}^{M}\frac{P^n}{n!}\sum_{j_1,j_2,\dots,j_n=1}^{q}K_{lj_1}\left(P,Py_1{}^{(k)}\right)$$

$$\times\prod_{l=2}^{n}K_{j_{l-1}j_l}\left(2Py_{l-1}{}^{(k)},2Py_l{}^{(k)}\right)f_{j_l}\left(2Py_l{}^{(k)}\right)-DN^{-\alpha+\varepsilon}$$

ni hosil qilamizgda

$D = \frac{A(qC_1)^n C_2 (6\alpha\varepsilon + 12\alpha^2)^{2\alpha s}}{\varepsilon^{2\alpha s}}$.

Teorema isbotlandi.

XULOSA

Ushbu dissertatsiya ishining 1-bobida optimal koeffitsentlarni xisoblash algoritmi o'rgani chiqilgan, bunda sonlar nazariyasining ta'rif va qoidalaridan foydalanilgan, dastur tuzilgan va EHM da natija olingan. Optimal koeffitsentlar jadvali keltirilgan.

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3-bobda Sonlar nazariyasi usullari yordamida tuzilgan to'rlarda Volter integral tenglamalarni taqribiy echish masalasi ko'rib chiqilgan. Unda funksiyalarni davriylashtirish masalasi o'rganilgan va quyidagi

$$\varphi\,(x_1, x_2, \cdots, x_s) =$$

$$= \int_0^x \cdots \int_0^x \sum_{r=1}^{q} K_r(x_1, x_2, \cdots, x_s, y_1, y_2, \cdots, y_s)\varphi_r(y_1, y_2, \cdots, y_s)dy_1 dy_2 \cdots dy_s$$

$$+ f(x_1, x_2, \cdots, x_s),$$

ko'p o'lchovli Volter integral tenglamasi taqribiy echilgan va xatoligi ko'rsatilgan.

Ushbu dissertatsiyada bajarilgan ishlar magistrlik ishiga qo'yilgan talabga noliq javob beradi.

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Termiz davlat universiteti magistratura bo'limi matematika yo'nalishi bitiruvchisi Ismoilov Muhriddin Mamatqobil o'g'lining "Sonlar nazariyasi usullaridan foydalanib 2-tur Volterra integral tenglamasini taqribiy yechish" mavzusida 70540101 – matematika ta'lim yo'nalishi bo'yicha magistr akademik darajasini olish uchun yozgan dissertatsiyaga raxbar

Xulosasi

Dissertatsiya ishi matematikaning dolzarb sohalaridan biri integral tenglamalarni taqribiy yechishga bag'ishlangan. Bunda iteratsiya usuli bilan sonlar nazariyasi usullari birlashtirib Volterra 2-tur integral tenglamasi taqribiy yechilgan va xatolik baholangan.

Magistrlik dissertatsiyaning asosiy maqsadi iteratsiya usuli bilan sonlar nazariyasi usullarini birlashtirib Volterra 2-tur integral tenglamasini taqribiy yechish va xatolikni baholashdan iborat.

Dissertatsiya ishi kirish, uchta bob, xulosa va foydalanilgan adabiyotlar ro'yxatidan iborat.

Kirish qismida masalaning dolzarbligi, maqsad va vazifalari, tadqiqot ob'ekti va predmeti, ilmiy yangiligi va farazlari hamda bajarilgan ishlar tasnifi keltirilgan, shuningdek, tadqiq etilayotgan masalalarning holati haqida so'z yuritilib, asosiy tushunchalar qisqacha keltirilgan.

Dissertatsiya ishining birinchi bobida sonlar nazariyasining muxim ta'rif va teoremalari keltirilgan va ular asosida optimal koeffisientlar tushunchasi kiritilib, ularni hisoblash uchun dastur tuzilgan. Ikkinchi bobda esa Volterraning integral tenglamalarini yechishning sonlar nazariyasiga bog'liq bo'lmagan usullari keltirilgan va xatoligi ko'rsatilgan.

Uchinchi bobda esa funksiyalarni davriylashtirish masalasi ko'rilgan. Bu bobning 2-§ da ko'p o'lchovli Volterra integral tenglamalarni sonlar nazariyasi usullari yordamida taqribiy yechgan.

Ushbu magistrlik dissertatsiyasi ilmiy va uslubiy tarzda yozilgan bo'lib, u magistrlik dissertatsiyasiga qo'yilgan barcha talablarga mos keladi. Dissertatsiyani himoyaga tavsiya etaman.

Ilmiy rahbar: **f.m.f.n. I.M.Abirayev**

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Ishdagi asosiy natijalar quyidagilar:

- Optimal koeffitsentlarni hisoblangan.
- Parallelopipedal to'r qurilgan.
- Volterraning integral tenglamalarini taqribiy yechilgan va xatoligini baholagan
- Funksiyalarni davriylashtirish.
- Ko'p o'lchovli Volter integral tenglamalarni sonlar nazariyasi usullari yordamida taqribiy yechish.

Ilmiy ish nazariy xarakterda bo'lib, undan matematika, fizika va texnikaning ko'plab masalalari hamda iqtisodiy masalalarni yechishda foydalanish mumkin. Shuningdek shu sohada ilmiy izlanishlar olib boruvchi mutaxassislar va talabalarga maxsus kurs hamda seminarlar o'tishda foydalanish mumkin.

Umuman bajarilgan ishni matematika yo'nalishi bo'yicha magistrlik dissertatsiyaga qo'yilgan talablarga javob beradi va uning muallifi Ismoilov Muhriddin Mamatqobil o'g'li matematika bo'yicha magistr akademik darajasini olishga munosib deb hisoblayman.

Magistrlik dissertatsiyani Davlat Attestatsiya Komissiyasi yig'ilishida himoya qilishga tavsiya qilaman.

Taqrizchi

Termiz Davlat universitetining "Algebra va geometriya"

Kafedrasi professori, fizika-matematika fanlari doktori prof. I.Allakov

2.06.2023

Termiz davlat universiteti magistratura bo'limi matematika yo'nalishi bitiruvchisi Ismoilov Muhriddin Mamatqobil o'g'lining "Sonlar nazariyasi usullaridan foydalanib 2-tur Volterra integral tenglamasini taqribiy yechish" mavzusida 70540101 – matematika ta'lim yo'nalishi bo'yicha magistr akademik darajasini olish uchun yozgan dissertatsiyaga

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$$= \int_0^x \cdots \int_0^x \sum_{r=1}^q K_r(x_1, x_2, \cdots, x_s, y_1, y_2, \cdots, y_s)\varphi_r(y_1, y_2, \cdots, y_s)dy_1 dy_2 \cdots dy_s$$

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Taqrizchi

DTPI katta o'qituvchisi f.m.f.n. O.Qosimov

Hujjat tekshirish natijalari

ANTIPLAG

Tekshiruvchi: Abdurazzoqov Javohir Rustamovich (ID: 304)
Tashkilot:

Hisobot "Antiplag.Uz" servisi tomonidan taqdim etilgan

HUJJAT TO'G'RISIDAGI MA'LUMOTLAR

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Yuklangan vaqti: 30-05-2023 19:43:00
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HISOBOT TO'G'RISIDAGI MA'LUMOTLAR

Qidirish oralig'i: ko'rsatilmagan
Qidirish modullari: ИПС Адилет qidiruv moduli, Internet plyus qidiruv moduli, Переводные заимствования по Интернету (EnRu) qidiruv moduli, BMK dissertatsiyalari qidiruv moduli, Universitetlar halqasi qidiruv moduli, Модуль поиска переводных заимствований qidiruv moduli, Milliy reestr qidiruv moduli, Переводные заимствования по Интернету (UzRu) qidiruv moduli, Tabobat qidiruv moduli, Garant AHT qidiruv moduli, RDK to'plami qidiruv moduli, Shablon iboralar qidiruv moduli, Patentlar qidiruv moduli, Tarjima tekshiruvi en-ru qidiruv moduli, Elektron-kutubxona tizimlari qidiruv moduli, Internet plyus qidiruv moduli, Tarjima tekshiruvi uz-ru qidiruv moduli, Wiley nashriyoti qidiruv moduli, Bibliografiya qidiruv moduli, Rossiya va MDH OAVlari qidiruv moduli, Iqtibos keltirish qidiruv moduli, Iboralarni qayta ishlatish qidiruv moduli, Перефразирование по СПС Гарант: аналитика qidiruv moduli, Переводные заимствования издательства Wiley (RuEn) qidiruv moduli, СПС Гарант: нормативно-правовая документация qidiruv moduli, eLIBRARY.RU qidiruv moduli

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O'zlashtirib olish — topilgan barcha matnli kesishmalar ulushi, tizim hujjatning umumiy hajmiga nisbatan iqtibos keltirishga kiritganlaridan tashqari.
O'z-o'zidan iqtibos keltirishlar — tekshirilayotgan hujjatdagi muallifi yoki hammuallifi tekshirilayotgan hujjatning muallifi bo'lgan manba matni fragmenti bilan mos tushuvchi yoki deyarli mos tushuvchi matn fragmentlarining hujjatning umumiy hajmiga nisbatan ulushi.
Iqtibos keltirish — muallifniki bo'lmagan, biroq tizim ulardan foydalanishni to'g'ri deb hisoblagan matnli kesishmalarning hujjatning umumiy hajmiga nisbatan ulushi. Bunga GOST bo'yicha qilingan iqtiboslar; umumfoydalanuvchi iboralar; me'yoriy-huquqiy hujjatlar to'plamidan olingan manbalarda topilgan matn fragmentlari kiradi.
Matnli kesishma — tekshirilayotgan hujjatdagi manba matni fragmenti bilan ustma-ust yoki deyarli ustma-ust tushuvchi matn fragmenti.
Manba — tizimda indekslangan va tekshirish o'tkazuvchi qidirish modulida mavjud bo'lgan hujjat.
Originallik — tekshirilayotgan hujjat matnidagi tekshiruv borgan birorta ham manbada topilmagan fragmentlarning hujjatning umumiy hajmiga nisbatan ulushi.
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E'tiboringizni tizim tekshirilayotgan hujjatning tizimda indekslangan matnli manbalar bilan matnli kesishmalarni topishiga qaratamiz. Bunda tizim yordamchi vosita hisoblanadi, o'zlashtirib olishlar yoki iqtibos keltirishlarning to'g'riligi va o'rinliligini hamda tekshirilayotgan hujjat matni fragmentlarining muallifi kimligini aniqlash tekshiruvchining vakolatida qoladi.

№	Hisobotdagi ulushi	Manba	Qidirish moduli
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Tekshirish sanasi 30.05.2023
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