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ARALASH TIPDAGI TENGLAMA UCHUN GELLERSTEDT VA BUZILISH CHIZIG‘IDA UMUMIY ULANISH SHARTLI MASALALAR

70540101 «Matematika (Algebra va funksional analiz)» ta’lim yo‘nalishi bo‘yicha
Magistr akademik darajasini olish uchun yozilgan

DISSERTATSIYA

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Temiz Davlat Universiteti fizika-matematika fakulteti «(Algebra va funksional analiz» kafedrasi 2-kurs magistranti Musurmonov Ma'ruf Akram o'g'lining "Aralash tipdagi tenglama uchun Gellerstedt va buzilish chizig'ida umumiy ulanish shartli masalalar" mavzusida 70540101 Matematika ta'lim yo'nalishi bo'yicha magistr darajasini olish uchun yozilgan dissertatsiyasi

ANNOTATSIYASI

Tayanch so'zlar: Gipergeometrik funksiya, ixtiyoriy tartibli integro-differensial operatorlar, Singulyar koeffitsientli elliptik tipdagi tenglamalar, T masala, Trikomi integral tenglamasi, Viner- Xopf integral tenglamasi

Tadqiqot obektlari: Aralash tipdagi tenglama uchun Gellerstedt va buzilish chizig'ida umumiy ulanish shartli masalalar.

Tadqiqot metodlari: Gellerstedt tenglamasi uchun qo'yilgan Bitsadze-Samarskiy masalasi yechimining yagonaligi A.V.Bitsadzening ekstremum prinsipi asosida, yechimning mavjudligi esa integral tenglamalar usulida isbotlangan. Bu erda Gaussning gipergeometrik funksiyalar nazariyasidan, integro- differensial operatorlar nazariyasidan, Singulyar integral nazariyasidan, hamda Fredgolm teoremalaridan keng foydalanilgan.

Ishning amaliy ahamiyati: Magistrlik dissertatsiya ishida to'plangan materiallardan aralash tipdagi tenglamalar uchun nolokal masalaning yechimlari haqida ma'lumotlar olish mumkin. Bundan tashqari chiziqli differensial tenglamalar va Singulyar integral masalalarni echishda foydalanish mumkin.

Dissertatsiya ishining nazariy ahamiyati: olingan natijalar nazariy ahamiyatga ega bo'lib, ulardan aralash turdagi Singulyar koeffitsientli tenglamalar uchun qo'yilgan chegaraviy masalalar nazariyasida keng foydalanish mumkin.

Tadbiq etish darajasi: Ushbu magistrlik dissertatsiyasi bo'yicha asosan singulyar koeffitsientli

$$(sign y)|y|^m u_{xx} + u_{yy} + (\beta_0 / y)u_y = 0 \qquad (1.1)$$

Gellerstedt tenglamasi o’rganiladi. (1.1) tenglama $z = x + iy$, kompleks tekisligining $lmz > 0$ yuqori yarim tekisligida uchlari $A(-1,0)$ va $B(1,0)$ nuqtalarda va yuqori yarim tekislikda joylashgan $\Gamma : y = f(x)$ chizig’i bilan, $lmz < 0$ pastki yarim tekislikda esa (1.1) tenglamaning AC va BC xarakteristikalari bilan chegaralangan bir bog’lamli D sohada o’rganiladi.

Dissertatsiya (1.1) tenglama uchun F. Trikomi, V.I. Jegalov[13], A. M. Naxushev[14], masalalar shartlarini barchasini o’zida birlashtirib yaxlit bir masala sifatida ta’riflangan masalaning korrekt ekanligi isbotlash maqsad qilib qo’yilgan.

(1.1) tenglama uchun Bitsadze-Samarskiy masalasining sharti noma’lum funktsiyaning kasr tartibli hosilalarining qiymatlarini parallel xarakteristikalarda berilgan masalaning korrektligi o’rganiladi.

Olingan natijalar va ularning yangiligi:

- Aralash tipdagi tenglama uchun Gellerstedt Buzilish chizig‘ida ulanish shartlari umumiy bo‘lgan hol uchun masalasi o‘rganilgan.

Qo‘llanilish sohasi: Oliy ta’lim muassasalari.

ANNOTATION

Key words: Geometric function, integro-differential operators of arbitrary order, elliptic type equations with singular coefficients, problem, Tricomi integral equation, Wiener-Hopf integral equation.

Objects of research: Gellerstedt equation with singular coefficient, Bitsadze-Samarsky problem for Gellerstedt equation.

Purpose: to solve the Bitsadze-Samarsky conditional problem of the form (0.6) for the Gellerstedt equation (0.5) with a singular coefficient based on the conditions of the general connection Karatoprakliev. To prove that the solution of the described problem exists and is unique.

Research methods: the uniqueness of the solution of the Bitsadze-Samarsky problem for the Gellerstedt equation is proved based on the extremum principle of A. V. Bitsadze, and the existence of the solution is proved by the method of integral equations. Gauss' theory of hypergeometric functions, theory of integro differential operators, singular integral theory, and Fredholm theorems are widely used here.

Practical significance: The Bitsadze-Samarsky problem for the Gellerstedt equation with a singular coefficient is studied. - The connection conditions at the fault line are studied for the general case

M U N D A R I J A

KIRISH

- Matematika hamma aniq fanlarga asos. Bu fanni yaxshi bilgan bola aqlli, keng tafakkurli bo‘lib o‘sadi, istalgan sohada muvaffaqiyatli ishlab ketadi.

Sh. Mirziyoyev

Hozirgi kunda ilm–fanga Prezidentimiz tomonidan alohida e’tibor berilmoqda. Ayniqsa, 2023 yilning Prezidentimiz tomonidan “Insonga e’tibor va sifatli ta’lim yili” deb e’lon qilinishi hamda bu yilda matematika, kimyo, biologiya va geologiya fanlarini rivojlantirishga alohida e’tibor berilishi biz yosh matematiklar uchun katta imkoniyatlar yaratdi.

Jamiyat ijtimoiy sohasining eng muhim tarkibiy qismlaridan biri ta’lim tarbiya sohasi bo’lib, uning rivojisi yosiy –huquqiy, iqtisodiy va ma’naviy sohalarga bevosita ta’sir etadi hamda ijtimoiy sohalar me’yoriy mohiyatini, kamolot darajasini belgilab beradi.

O’zbekistonda ta’lim tizimining isloh qilishning dasturiy hujjatlarida ta’kidlanganidek, mamlakatimiz ta’lim tizimi xodimlari oldida raqobat bardosh kadrlar tayyorlash, ta’lim tarbiya jarayonini jahon andozalar darajasiga yetkazishni ta’minlash asosiy vazifa qilib qo’yilgan.

Shu ma’noda olib qaraganda, yoshlarning yangi avlodi istiqbol masalalarini kun tartibiga dadil qo’yadigan va uni yecha oladigan, fikr yuritishning yuksak madaniyatini egallagan, siyosiy hamda ijtimoiy iqtisodiy hayotda o’ziga mustaqil yo’l topa oladigan qobiliyatga ega bo’lishi kerak.

Ushbu magistrlik dissertatsiyasi mavzusi ana shu talab va vazifalardan kelib chiqib tanlandi.

Buziluvchan giperbolik, elliptik turdagi va aralash turdagi tenglamalar uchun chegaraviy masalalar nazariyasi, zamonaviy xususiy hosilali differensial tenglamalar nazariyasining asosiy yo‘nalishlardan biri hisoblanadi va u muhim amaliy masalalarni echishda qo‘llaniladi. Buziluvchan giperbolik, elliptik va aralash turdagi tenglamalar nazariyasini rivojlanishi dastlab G. Darbu [36], F.Trikomi [38], e. Holmgren [42] va S. Gellerstedtlarning [41] mos ravishda 1894, 1923, 1927 va 1938 yillarda e’lon qilingan fundamental ishlaridan boshlangan.

Aralash turdagi

$$T(u) = yu_{xx} + u_{yy} = 0 \qquad (0.1)$$

tenglama uchun birinchi fundamental tadqiqotlarni italyan matematigi Franchesko Trikomi bajargan. U hozirgi vaqtda uning nomi bilan ataluvchi quyidagi Trikomi masalasini ta’riflagan va echgan: $z = x + iy$ kompleks tekisligining $y > 0$ yarim tekisligida, uchlar $A(0,0)$ va $B(1,0)$ nuqtalarda bo‘lgan Γ silliq Jordan chizig‘i bilan, $y < 0$ yarim tekislikda esa (0.1) tenglamaning, AC va BC xarakteristikalari bilan chegaralangan bir bog‘lamli Ω sohada (0.1) tenglamaning ushbu

$$u(x, y) = \varphi(x, y), (x, y) \in \Gamma, \qquad (0.2)$$

$$u(x, y) = \psi(x), \ (x, y) \in AC, \qquad (0.3)$$

$$\lim_{y \to -0} u_y = \lim_{y = +0} u_y, \qquad (0.4)$$

Shartlarni qanoatlantiruvchi regulyar yechimi, $u(x, y)$ topilsin.

Bu ishlardan keyin buziluvchan va aralash turdagi tenglamlar uchun chegaraviy masalalar nazariyasi ko‘p yo‘nalishlarda o‘rganildi va rivojlantirildi. Xususan, Trikomi masalasi umumiyroq aralash turdagi tenglamalar uchun [2,3,4,10,11,12,13] ishlarda, Trikomi masalasining har xil modefikatsiyasi [1,9,14-20,30-35,40,41] ishlarda spektral masalalar [26,27,35] ishlarda o‘rganildi. Eng muhim natijalar va adabiyotlar ro‘yxati A. V. Bitsadze [2], M. M. Smirnov [36], M. S. Salaxitdinov [29], T. D. Djuraev [10], A. M. Naxushev [25], E. I. Moiseev [23], A. P. Soldatov [37], A.I.Kojanov [13] monografiyalarida keltirilgan.

1970-yillarning boshiga aralash turdagi tenglamalar uchun chegaraviy masalalar nazariyasi yangi yo‘nalishda rivojlanishida A. V. Bitsadze va A. A.Samarskiylarning (Dan SSSR, 1969, t. 185, № 4, 739-740) ishi mazkur sohada tub burilish yasadi.

Shu bilan birga kichik hadli Singulyar koeffitsientli aralash tipdagi tenglamalar va buzilish chizig‘iga ega bo‘lgan tenglamalar uchun chekli sohalarda chegaraviy masalalarni echishga ustozimiz M.Mirsaburovning 200 dan ko‘p ilmiy maqolalari chop etilgan bo‘lib, shu jumladan ustozimiz shogirdlari bilan bir qator ilmiy ishlar bag‘ishlanganlar [21-24].

Keyingi yillarda bizning respublikamizda va xorijiy davlatlarda shu asnoda universitetimizning matematik analiz kafedrasida ilmiy rahbarim PhD N. Boltayev va ilmiy maslahatchim PhD N. Xurramov bilan kichik hadli Singulyar koeffitsientli aralash tipdagi tenglamalar uchun chegaraviy masalalar nazariyasi faollik bilan o‘rganilmoqda.

Magistrlik dissertatsiyasi mavzusining asoslanishi va uning dolzarbligi: Ushbu magistrlik dissertatsiyasi ham Singulyar koeffitsientli

$$y^m u_{xx} + u_{yy} + \frac{\beta_0}{y} u_y = 0 \qquad (0.5)$$

tenglama uchun Bitsadze-Samarskiy tipidagi masalani o‘rganishga bag‘ishlangan. Magistrlik dissertatsiyasida Bitsadze-Samarskiy sharti (0.5) tenglama yechimini Γ chiziq va AB buzilish chizig‘ida

$$u(x, \sigma_0(x)) = \mu(x) u(x, 0) + \rho(x) \qquad (0.6)$$

ko‘rinishda bog‘laydi, bu erda $y = \sigma_0(x)$ (0.5) tenglamaning normal chizig‘i (Γ chiziq). Ulanish shartlari esa umumiy ko‘rinishda beriladi, ya’ni G. Karatoprakliev (Dan SSSR 1964, t. 158, №, s 271-274) shartlari beriladi.

Ta’riflangan masala yechimining yagonaligi ekstremum prinsipi asosida, mavjudligi esa integral tenglamalar nazariyasi yordamida isbotlanadi.

Tadqiqot obyekti va predmeti: Singulyar koeffitsientli Gellerstedt tenglamasi (0.5) uchun (0.6) ko‘rinishda bo‘lgan Bitsadze-Samarskiy shartli masalani umumiy ulanish Karatoprakliev shartlari asosida echish. Ta’riflangan masala yechimi mavjud va yagona ekanligini isbotlash.

Tadqiqot maqsadi va vazifalari: Gellerstedt tenglamasi uchun qo‘yilgan

Bitsadze-Samarskiy masalasi yechimining yagonaligi A.V. Bitsadzening ekstremum prinsipi asosida, yechimning mavjudligi esa integral tenglamalar usulida isbotlangan. Bu erda Gaussning gipergeometrik funksiyalar nazariyasidan, integro- differensial operatorlar nazariyasidan, Singulyar integral nazariyasidan, hamda Fredgolm teoremalaridan keng foydalanilgan.

Ilmiy yangiligi quyidagilardan iborat

1.Singulyar koeffitsientli Gellerstedt tenglamasi uchun Bitsadze-Samarskiy masalasi buzilish chizig‘ida ulanish shartlari umumiy bo‘lgan hol uchun o‘rganilgan.

Tadqiqotning asosiy masalalari va farazlari: shundan iboratki olingan natijalar nazariy ahamiyatga ega bo‘lib, ulardan aralash turdagi Singulyar koeffitsientli tenglamalar uchun qo‘yilgan chegaraviy masalalar nazariyasida keng foydalanish mumkin.

Tadqiqot mavzusi bo‘yicha adabiyotlar sharhi (tahlili): [30] adabiyotda Pulkin S.P. tomonidan

$$(sign y)|y|^m u_{xx} + u_{yy} + (\beta_0 / y)u_y = 0.$$

$u(x,y)$ funksiya $\Omega^- \setminus (OC_0 \cup OC_1)$ sohada yuqoridagi tenglamaning R_1 sinfga tegishli bo‘lgan umumlashgan echimi ifodalangan. [5] adabiyotda Bitsadzening ekstrimum prinspiga doir teorema va uning isboti keltirilib berilgan. [17] adabiyotda S.G.Mixlinning

$$A(x)\rho(x) - \lambda\int_{-1}^{c}\left(\frac{1}{t-x} - \frac{1}{1-xt}\right)\rho(t)dt = g(x), -1 \le x \le c.$$

tenglamaga takomillashtirilgan Karlemanning regulyarlashtirilgan usuli qo’llanilgan. [20] adabiyotda yechimning mavjudligini keltirilayotganda Bols prinspidan foydalanilgan. [22] adabiyotda M.Mirsaburov elliptik tipdagi tenglamalar uchun yangi turdagi chegaraviy masalalarni o‘rgangan.

Dissertatsiya ishining aprobatsiyasi: dissertatsiya ishida olingan ilmiy natijalar Termiz davlat universiteti “Matematik analiz” kafedrasi qoshida faoliyat ko‘rsatayotgan “Matematikaning zamonaviy masalalari” ilmiy metodik seminarda ma’ruza qilingan.

Ilmiy ish natijalari yuzasidan e’lon qilingan materiallar: Magistrant o‘zining ilmiy izlanishlari natijasida jami 4 ta maqola va tezislar e’lon qilgan.

Tadqiqotda qo‘llanilgan metodikaning tavsifi: Magistrlik dissertatsiya ishida to‘plangan materiallardan aralash tenglamalar uchun nolokal masalaning yechimlari haqida ma’lumotlar olish mumkin. Bundan tashqari chiziqli differensial tenglamalar va Singulyar integral masalalarni echishda foydalanish mumkin.

Dissertatsiya ishining nazariy va amaliy ahamiyati: Dissertatsiyadan olingan natijalar amaliy ahamiyatga ega, ulardan buziladigan elliptik va giperbolik tipdagi tenglamalar uchun qo‘yilgan lokal va nolokal shartli chegaraviy masalalarni yechishda keng foydalanish mumkin.

Ish tuzilmasining tavsifi: Magistrlik dissertatsiyasi kirish qismi, 3 bob, 10 paragraf hamda o‘z ichiga 44 adabiyotni olgan foydalanilgan adabiyotlar ro‘yxatidan iborat. Belgilashlar ikki raqamli bo‘lib, ular orasi nuqta bilan ajratilgan. Birinchi raqam paragraf nomerini bildiradi, ikkinchi son esa tartib nomerini bildiradi. Masalan, 2.1-teorema yozuvi-ikkinchi paragrafning 1- teoremasi ekanligini bildiradi, yoki (2.8) belgilash 2- paragrafdagi 8- formula ekanligini anglatadi.

Birinchi bob uch paragrafdan iborat. Mazkur bobda quyidagi asosiy tushuncha va materiallar o‘rganilgan

1.1-§. Gipergeometrik funksiya.

1.2-§. Ixtiyoriy tartibli integro-differensial operatorlar.

1.3 -§. Singulyar koeffitsientli elliptik tipdagi tenglamalar uchun Dirixle masalasining qo‘yilishi va yechimining tahlili.

Ikkinchi bob uchta paragrafdan iborat bo‘lib, u quyidagilardan iborat

2.1-§. T masalasining qo‘yilishi.

2.2-§. T masalasi yechimining yagonaligi.

2.3-§. T masalasi yechimining mavjudligi.

Uchinchi bob to’rtta paragrafdan iborat bo‘lib, u quyidagilardan iborat

3.1-§. Singulyar integral tenglamalar sistemasini keltirib chiqarish.

3.2-§. Trikomi integral tenglamasini regulyarizatsiyalash.

3.3-§. $\nu_1(x)$ va $\nu_0(x)$ noma`lum funksiyalar o`rtasidagi birinchi va ikkinchi funksional munosabatlar.

Dissertatsiya oxirida xulosa va foydalanilgan adabiyotlar ro‘yxati keltirilgan.

Olingan natijalarning qisqacha bayoni. Ushbu dissertatsiyada aralash tipdagi tenglama uchun Gellerstedt va umumiy ulanish shartli masala ta’riflangan bo‘lib, masala yechimining yagonaligi va mavjudligi isbotlangan.

I-BOB. ARALASH TIPDAGI TENGLAMA UCHUN GELLERSTEDT VA BUZILISH CHIZIG‘INI O’RGANISHNING NAZARIY ASOSLARI.

1.1§ Gipergeometrik funksiya.

1.1.1 $\Gamma(z)$ gamma-funksiyasi:

$$\Gamma(z)=\int_0^\infty e^{-t}t^{z-1}dt,\quad (\operatorname{Re} z>0) \tag{1.1}$$

Eylerning ikkinchi tur integrali yordamida aniqlanadi. (1.1) integralni ikki integral yig‘indisi orqali ifodalaymiz [2,38]

$$\Gamma(z)=\int_0^1 e^{-t}t^{z-1}dt+\int_1^\infty e^{-t}t^{z-1}dt=P(z)+Q(z) \tag{1.2}$$

$P(z)$ funksiyani $\operatorname{Re} z>0$ yarim tekislikda regulyar funksiya ekanligini ko‘rsatish qiyin emas, $Q(z)-$butun funksiya (butun kompleks tekislikda golomorf). SHunday qilib, (1.1) formula $\operatorname{Re} z>0$ yarim tekislikda regulyar funksiyani aniqlaydi. $\Gamma(z)$ funksiyani butun kompleks tekislikka analitik davom ettirish mumkin. Haqiqatdan ham e^{-t} funksiyani darajali qatorga yoyib bu yoyilmani t^{z-1} ga ko‘paytirib va $[0,1]$ kesmada hadma had integrallab ushbu funksional qatorga kelamiz.

$$P(z)=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\frac{1}{z+n},\quad (\operatorname{Re} z>0) \tag{1.3}$$

(1.3) qatorning hadlari $z\neq 0,-1,-2,\ldots$ nuqtalardan tashqari barcha nuqtalarda regulyar funksiyalardir va (1.3) qator $|z+k|\geq\delta>\ 0(k=0,1,2,\ldots;\delta>0$-ixtiyoriy kichik son) sohada tekis yaqinlashuvchi. Veyershtrass teoremasiga ko‘ra (1.3) qator yig‘indisi meromorf funksiyadir, $z=-n$ nuqtalar bu funksiyaning oddiy qutblari bo‘lib, bu qutblardagi funksiyaning chegirmalari $res(P(-n))=\frac{(-1)^n}{n!}$ ga tengdir. (1.3) funksiya $\operatorname{Re} z>0$ yarim tekislikda $P(z)$ integral bilan ustma-ust tushadi, demak (1.3) qator $P(z)$ integralning analitik davomidan iboratdir. SHunday qilib (1.2) dagi ikkinchi yig‘indi $Q(z)-$butun funksiya bo‘lgani uchun $\Gamma(z)$ funksiya $z=-n$ nuqtalarda oddiy qutblarga, hamda bu nuqtalarda mos ravishda $\frac{(-1)^n}{n!}$ ga teng bo‘lgan chegirmalarga ega bo‘lgan meromorf funksiyadir. $\Gamma(z)$ uchun ushbu funksional munosabatlar o‘rinlidir

$$\Gamma(1+z)=z\Gamma(z),\ \Gamma(z)\Gamma(1-z)=\frac{\pi}{\sin\pi z},\ \Gamma(2z)=\frac{2^{2z-1}}{\sqrt{\pi}}\Gamma(z)\Gamma\left(z+\frac{1}{2}\right)$$

$$\Gamma(z)\Gamma(-z)=-\frac{\pi}{z\sin\pi z}\ (1.4)$$

$$\Gamma(1)=1, \Gamma(n+1)=n!, \Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$$

$\Gamma(z)$ funksiya butun kompleks tekislikda nollarga ega emas, demak $\frac{1}{\Gamma(z)}-$butun funksiyadir.

1.1.2 Beta-funksiyasi. $B(p,q)$ beta-funksiya:

$$B(p,q)=\int_0^1 t^{p-1}(1-t)^{q-1}dt\ ,\ \mathrm{Re}\,p>0\ ,\ \mathrm{Re}\,q>0\ (1.5)$$

Eylerning birinchi tur integrali yordamida aniqlanadi. $B(p,q)$ funksiya $\Gamma(z)$ orqali

$$B(p,q)=\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \tag{1.6}$$

formula yordamida ifodalanadi.

1.1.3 Gaussning gipergeometrik funksiyasi.

Gauss tenglamasi. Buziladigan giperbolik va elliptik tipdagi tenglamalar nazariyasida ushbu

$$z(1-z)\omega''(z)+\left[c-(a+b+1)z\right]\omega'(z)-ab\omega(z)=0\,,$$

(1.7)

Gauss tenglamasining yechimlari fundamental ahamiyatga ega, bu erda a,b,c – parametrlar bo‘lib, ular ixtiyoriy kompleks yoki haqiqiy sonlar bo‘lishi mumkin. (1.7) tenglama uchta: 0, 1 , ∞ regulyar maxsus nuqtalarga ega.O‘zgaruvchilarni maxsus almashtirish yordamida buziluvchan giperbolik va elliptik tipdagi tenglamalar (1.7) tenglamaga olib kelinishi mumkin va bu tenglamaning yechimlaridan mos ravishda Riman funksiyasini, Grin funksiyasini tuzishda fundamental ahamiyatga ega.Dastlab (1.7) tenglamaning yechimini $z=0$ nuqta atrofida topamiz. Yechimni

$$\omega_1(z)=\sum_{n=0}^{\infty}c_n z^n\,,$$

(1.8)

darajali qator ko‘rinishida izlaymiz. Bu erda c_n -hozircha noma’lum sonlar. (1.8) dan ushbu hosilalarni hisoblaymiz

$$\omega_1'(z)=\sum_{n=0}^{\infty}c_n n z^{n-1}=\sum_{n=1}^{\infty}c_n n z^{n-1}$$

$$\omega_1''(z)=\sum_{n=0}^{\infty}c_n n(n-1)z^{n-2}=\sum_{n=2}^{\infty}c_n n(n-1)z^{n-2}.$$

Endi bu hosilalarni (1.7) tenglamaga qo‘yib quyidagi munosabatni hosil qilamiz.

$$\sum_{n=0}^{\infty}\left[c_{n+1}(n+1)(n+c)-c_n(n+a)(n+b)\right]z^n=0$$

bu erdan z^n oldidagi umumiy koeffitsientni nolga tenglashtirib, ushbu

$$c_{n+1}=\frac{(a+n)(b+n)}{(n+1)(c+n)}c_n\,,\quad (n=0,1,2\ldots;c\neq -n)$$

rekurrent formulaga kelamiz.(1.7) tenglamaning bir jinsli ekanligidan foydalanibumumiyatlikni buzmasdan $c_0 = 1$ deb qabul qilamiz va

$$\omega_1(z) = F(a,b,c;z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{n!(c)_n} z^n = \\ = 1 + \frac{a \cdot b}{1 \cdot c} z + \frac{a \cdot (a+1) b (b+1)}{1 \cdot 2 \cdot c(c+1)} z^2 + \cdots, \tag{1.9}$$

Gaussning gipergeometrik qatoriga kelamiz, bu erda

$$(a)_0 = 1, (a)_n = a(a+1)\cdots(a+n-1) = \frac{\Gamma(a+n)}{\Gamma(a)}$$

belgilashlar kiritilgan. Dalamber alomatiga ko‘ra (1.9) darajali qatorning yaqinlashish radiusi $R = 1$ ekanligini ko‘rsatish qiyin emas. Demak (1.9) darajali qator $|z| \le q < 1$ doirada absalyut va tekis yaqinlashadi. Raabe alomati yordamida (1.9) gipergeometrik qator uchun ushbu tasdiqlarni isbotlash qiyin emas;

Agar $\operatorname{Re}(c-a-b) > 0$ bo‘lsa (1.9) qator $|z| = 1$ aylanada tekis va absalyut yaqinlashadi;

Agar $-1 \le \operatorname{Re}(c-a-b) \le 0$ bo‘lsa (1.9) qator $|z| = 1$ aylananing $|1-z| < \delta$ $(\delta > 0)$ etarli kichik son) doiradan tashqarida yotgan bo‘lagida tekis va absolyut yaqinlashadi;

Agar $\operatorname{Re}(c-a-b) < -1$ bo‘lsa, (1.9) qator $|z| = 1$ aylanada uzoqlashuvchi bo‘ladi.

Gipergeometrik funksiyalarning sodda xossalarini keltiramiz, bu xossalar (1.9) darajali qatorning ko‘rinishidan bevosita kelib chiqadi.

1^0. Agar $a = -n$ yoki $b = -n$ bo‘lsa, bu erda $n = 0,1,2,\ldots,$ (1.9) darajali qator uziladi, ya’ni $F(-n,b,c;z)$ yoki $F(a,-n,c;z)$ n-darajali ko‘phadga aylanadi;

2^0. $F(a,b,c;z)$ gipergeometrik funksiya a va b parametrlarga nisbatan simmetrikdir, ya’ni $F(a,b,c;z) = F(b,a,c;z)$.

3^0. $b=c$ bo‘lganda

$$F(a,b,b;z)=(1-z)^{-a}$$

(1.10)

tenglikka ega bo‘lamiz. (1.7) tenglamaning ikkinchi yechimini topish uchun $\omega(z)$ o‘rniga

$$\omega(z)=z^q u(z)$$

(1.11)

formula yordamida yangi funksiya kiritamiz, bu erda q-hozircha ixtiyoriy noma’lum son. (1.11) tenglikni (1.7) tenglamaga qo‘yib ushbu tenglamaga ega bo‘lamiz

$$z(1-z)u''+\left[2q+c-(2q+a+b+1)z\right]u'-\left[-\frac{q(q-1+c)}{z}+q(q+a+b)+ab\right]u=0.$$

Bu tenglamada $q=1-c$, deb olsak, u holda oxirgi tenglama

$$z(1-z)u''(z)+[c_1-(a_1+b_1+1)z]u'(z)-a_1b_1u=0$$

tenglamaga aylanadi bu erdagi parametrlar $a_1=a+1-c$, $b_1=b+1-c$, $c_1=2-c$ tengliklar bilan aniqlanadi .

SHunday qilib (1.9) va (1.11) ga asosan (1.7) tenglamaning ikkinchi yechimi

$\omega_2(z)=z^{1-c}F(a+1-c,b+1-c,2-c;z)$ (1.12)

ko‘rinishda bo‘ladi, bu erda $2-c\neq 0,-1,-2,\ldots$(1.7) tenglamaning (1.9) yechimida $c\neq -n$ shart bajarilishi kerak edi, endi biz (1.12) ga asosan (1.7) tenglamaning yechimini $c=-n,$ holida ham hosil qilishimiz mumkin;

$$\omega(z)=z^{n+1}\sum_{m=0}^{\infty}\frac{(a+n+1)_m(b+n+1)_m}{(2+n)_m m!}z^m=$$
$$=z^{n+1}F(a+n+1,b+n+1,n+2;z).$$

(1.13)

(1.7) tenglamaning topilgan $\omega_1(z)$ va $\omega_2(z)$ yechimlari chiziqli erkli, demak uning umumiy yechimi

$$\omega(z) = c_1 F(a,b,c;z) + c_2 z^{1-c} F(a+1-c, b+1-c, 2-c; z) \quad (1.14)$$

formula bilan beriladi, bu erda c_1 va c_2 ixtiyoriy o'zgarmas sonlardir (1.7) tenglamaning yechimini $z=1$ maxsus nuqta atorofida hosil qilish uchun z ni $1-z$ ga almashtirish etarlidir. Bu holda (1.7) tenglama parametrlari $a_1 = a$, $b_1 = b$, $c_1 = a+b-c-1$ lardan iborat bo'lgan gipergeometrik tenglamaga aylanadi. Bu holda (1.7) tenglamaning $z=1$ maxsus nuqta atrofida

$$\omega_3(z) = F(a, b, 1+a+b-c; 1-z),$$

(1.15)

$$\omega_4(z) = (1-z)^{c-a-b} F(c-b, c-a, 1+c-a-b; 1-z),$$

chiziqli erkli yechimlarini hosil qilish qiyin emas, bu erda $c-a-b$ butun sonlar bo'lmasligi kerak.

Nihoyat, (1.7) tenglamaning yechimlarini cheksiz uzoqlashgan maxsus nuqta: $z=\infty$ atrofida topish uchun erkli o'zgaruvchi z va $\omega(z)$ funksiyani ushbu formulalar yordamida almashtiramiz. $\varsigma = \dfrac{1}{z}$, $\omega(z) = \varsigma^a u(\varsigma)$, bu holda (1.7) tenglama $u(\varsigma)$ funksiyaga nisbatan parametrlari $a_1 = a$, $b_1 = b+a-c$, $c_1 = 1+a-b$ bo'lgan gipergeometrik funksiyaga aylanadi. SHunday qilib, (1.7) tenglamaning $z=\infty$ maxsus nuqta atrofidagi chiziqli erkli yechimlari ushbu ko'rinishda bo'ladi

$$\omega_5(z) = z^{-a} F\left(a, 1+a-c, 1+a-b; \frac{1}{z}\right),$$
$$\omega_6(z) = z^{-b} F\left(b, 1+b-c, 1+b-a; \frac{1}{z}\right),$$

(1.16)

bu erda $a-b$ butun sonlar bo‘lmasligi kerak.

SHunday qilib, biz (1.7) Gauss tenglamasining oltita asosiy yechimlarini gipergometrik funksiyalar orqali ifodaladik.

1.1.4 Gipergeometrik funksiyani analitik davom etdirish. (1.7) tenglamaning $|z|<1$doirada aniqlangan regulyar yechimi $\omega_1(z) = F(a,b,c;z)$ ni zo‘zgaruvchining butun kompleks tekisligiga analitik davom ettirish mumkin. $F(a,b,c;z)$ funksiyaning analitik davomini ham $F(a,b,c;z)$ simvol bilan belgilaymiz va u (1.9) qatorning $|z|<1$ doiradan tashqariga davom ettirilgan analitik funksiyasining bosh shoxchasini ifodalaydi. $F(a,b,c;z)$ gipergeometrik funksiyani analitik davom ettirishni xususan Eylerning ushbu

$$\int_0^1 t^{a-1}(1-t)^{c-a-1}(1-zt)^{-b}\,dt = \frac{\Gamma(a)\Gamma(c-a)}{\Gamma(c)} F(a,b,c;z) \ ,$$

(1.17)

$0 < \mathrm{Re}\, a < \mathrm{Re}\, c$, $|\arg(1-z)| < \pi$,

gipergeometrik integrali yordamida amalga oshirish mumkin. (1.17) tenglikningchap tomonidagi integral$(1,\infty)$ kesimli butun kompleks tekislikda regulyar funksiyani beradi. (1.17) tenglikni isbotlash uchun, analitik davom ettirish prinsipiga ko‘ra, uni $|z|<1$ doira ichida tekshirish etarlidir $(1-zt)^{-b}$ funksiyani zt $(|zt|<1)$ ning darajalari bo‘yicha binomial qatorga yoyamiz, va bu yoyilmani $t^{a-1}(1-t)^{c-a-1}$ ga ko‘paytirib t bo‘yicha $(0,1)$ oraliqda hadma-had integrallaymiz va (1.5) formulani qo‘llab (1.17) munosabatgakelamiz.(1.17) formula

$\mathrm{Re}(c-a-b)>0$, $c\neq 0,-1,-2,\ldots$. bo‘lganda $F(a,b,c;1)$ ni hisoblashga imkon beradi:

$$F(a,b,c;1)=\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}.$$

(1.18)

(1.17) integralda $t=\frac{1-s}{1-zs}$,$1-t=\frac{s(1-z)}{1-zs}$, $1-tz=\frac{1-z}{1-zs}$, $dt=-\frac{(1-z)ds}{(1-zs)^2}$

almashtirishni bajarib ushbu

$$\frac{\Gamma(a)\Gamma(c-a)}{\Gamma(c)}F(a,b,c;z)=(1-z)^{c-a-b}\frac{\Gamma(c-a)\Gamma(a)}{\Gamma(c)}F(c-a,c-b,c;z),$$

ya’ni

$$F(a,b,c;z)=(1-z)^{c-a-b}F(c-a,c-b,c;z)$$

(1.19)

$|\arg(1-z)|<\pi$ formulani hosil qilamiz.(1.19) formula avtotransformatsiya formulasi deyiladi.Argumentlari z va $1-z$ bo‘lgan gipergeometrik funksiyalar o‘rtasida funksional munosabatlarni keltirib chiqaramiz. $|z|<1$ va $|1-z|<1$ doiralarning kesishmasida (1.7) tenglama yechimi $\omega_1(z)=F(a,b,c;z)$ shu tenglamaning chiziqli erkli yechimlari $\omega_3(z)$ va $\omega_4(z)$ larning chiziqli kombinatsiyasi orqali ifodalanadi:

$$F(a,b,c;z)=AF(a,b,a+b-c+1;1-z)+$$
$$+B(1-z)^{c-a-b}F(c-a,c-b,c-a-b+1;1-z), \quad (1.20)$$
$$c-a-b\neq 0,\pm1,\pm2,\pm3,\ldots;\ |\arg(1-z)|<\pi.$$

$F(a,b,c;z)$ funksiya a,b,c parametrlarning analitik funksiyasidan iborat, demak (1.20) formuladagi. A va B koeffitsientlar ham a,b,c parametlarning analitik funksiyasi bo‘ladi. A va B koeffitsientlarni aniqlashda biz a,b,c parametlarga shunday «qulay» shartlarni qo‘yamizki, ular analitik davom ettirish prinsipi yordamida shunday minimal shartlarga keltiriladiki, bu shartlarda oxirgi natija ma’noga ega bo‘lmagan ifodalarga ega bo‘lmaydi. (1.20) tenglamaning o‘ng tomoni $z=1$ nuqtada a,b,c $(c\neq -n)$ parametrlarining ixtiyoriy qiymatida ma’noga ega, chap tomonining chekliligi $\mathrm{Re}(c-a-b)$ ifodaning ishorasiga bog‘liq.

1. $\mathrm{Re}(c-a-b)>0$ bo‘lsin, u holda (1.20) da $z=1$ deb hisoblab ushbu

$$A=F(a,b,c;1)=\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}$$ tenglikka kelamiz.

2. $\mathrm{Re}(c-a-b)<0$ bo‘lsin. (1.20) tenglikning chap tomoniga avtotransformatsiya formulasi (1.19) ni qo‘llab, hosil bo‘lgan tenglikni $(1-z)^{a+b-c}$ ifodaga ko‘paytirib va oxirgi natijada $z=1$ deb hisoblab ushbu

$$B=F(c-a,c-b,c;1)=\frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}$$

tenglikka kelamiz.Aniqlangan A va B koeffitsientlarni (1.20) tenglikga qo‘yib ushbu

$$\begin{aligned}F(a,b,c;z)=\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}F(a,b,a+b-c+1;1-z)+\\+\frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}(1-z)^{c-a-b}\times\\\times F(c-a,c-b,c-a-b+1;1-z),\end{aligned}$$

(1.21)

$c-a-b\neq 0,\pm1,\pm2,\ldots.\quad |\arg(1-z|<\pi$

Bols formulasini hosil qilamiz [1]. (1.21) formula $F(a,b,c;z)$ gipergeometrik funksiyani $|z|<1$ sohadan $|1-z|<1$, $|\arg(1-z)|<\pi$ sohaga analitik davomini beradi. (1.17) formuladagi integralda integral o‘zgaruvchisini $t=1-s$ formula bilan almashtirib ushbu

$$\int_0^1 t^{a-1}(1-t)^{c-a-1}(1-zt)^{-b}dt =$$
$$= (1-z)^{-b}\int_0^1 s^{c-a-1}(1-s)^{a-1}(1-\frac{z}{z-1}s)^{-b}ds$$

yoki (1.17) ni hisobga olib

$$F(a,b,c;z) = (1-z)^{-b}F(c-a,b,c;\frac{z}{z-1})$$

(1.22)

formulaga kelamiz. $\operatorname{Re} z < \frac{1}{2}$ bo‘lganda, $\left|\frac{z}{z-1}\right| < 1$ tengsizlik o‘rinli. SHunday qilib, (1.22) formula $F(a,b,c;z)$ funksiyani $|z|<1$ doiradan $\operatorname{Re} z < \frac{1}{2}$ yarim tekislikka analitik davomini beradi. (1.22) formulada z o‘rniga $1-z$ almashtirish bajarib (1.22) formulani ushbu

$$F(a,b,c;1-z) = z^{-b}F(c-a,b,c;\frac{z-1}{z}) \ .$$

(1.23)

ko‘rinishda yozib olamiz.Gipergeometrik funksiyalar o‘rtasida boshqa funksional munosabatlarni, keltirib chiqarilgan formulalarning kombinatsiyalari yordamida hosil qilish mumkin. Masalan (1.22) va (1.21) formulalarni ketma-ket qo‘llab ushbu

$$F(a,b,c;z) = \frac{\Gamma(c)\Gamma(b-a)}{\Gamma(c-a)\Gamma(b)}(1-z)^{-a}F(a,c-b,a-b+1;\frac{1}{1-z}) +$$
$$+\frac{\Gamma(c)\Gamma(a-b)}{\Gamma(c-b)\Gamma(a)}(1-z)^{-b}F(c-a,b,b-a+1;\frac{1}{1-z}) \ ,$$

(1.24)

$a-b \neq 0,\pm1,\pm2,\cdots; \ |\arg(1-z)| < \pi$

funksional munosabatni hosil qilamiz, bu formula $F(a,b,c;z)$ gipergeometrik funksiyani $|z|<1$ doiradan $|z-1|>1,\ |\arg(1-z)|<\pi$ sohaga analitik davom ettirish imkonini beradi. YAna (1.24) va (1.22) formulalarni ketma-ket qo'llab, ushbu

$$F(a,b,c;z)=\frac{\Gamma(c)\Gamma(b-a)}{\Gamma(c-a)\Gamma(b)}(-z)^{-a}F(a,a-c+1,a-b+1;\frac{1}{z})+$$
$$+\frac{\Gamma(c)\Gamma(a-b)}{\Gamma(c-b)\Gamma(a)}(-z)^{-b}F(b,b-c+1,b-a+1;\frac{1}{z}),$$

(1.25)

$a-b\neq 0,\pm1,\pm2,\ldots;\ |\arg(-z)|<\pi,\ |\arg(1-z)|<\pi$.

formulaga ega bo'lamiz, bu formula $F(a,b,c;z)$ gipergeometrik funksiyani $|z|<1$ doiradan $|z|>1, |\arg(z)|<\pi$ sohaga analitik davomini beradi.Nihoyat (1.21) va (1.23) formulalarni ketma-ket qo'llab, ushbu

$$F(a,b,c;z)=\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}z^{-a}\times$$
$$\times F(a,a-c+1,a+b-c+1;\frac{z-1}{z})+$$
$$+\frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}z^{a-c}(1-z)^{c-a-b}\times$$
$$\times F(c-a,1-a,c-a-b+1;\frac{z-1}{z}),$$

(1.26)

$$c-a-b\neq 0,\pm1,\pm2,\ldots,\ |\arg z|<\pi,\quad |\arg(1-z)|<\pi$$

formulaga ega bo'lamiz, bu formula $F(a,b,c;z)$ gipergeometrik funksiyani $|z|<1$ doiradan $\operatorname{Re} z>\frac{1}{2}$ sohaga analitik davom ettirish imkonini beradi.

1.2§. Ixtiyoriy tartibli integro-differensial operatorlar.

$f(x)$-funksiya $L(a,b),\ a<b<\infty$ sinfga tegishli bo'lgani xtiyoriy funksiya bo'lsin. Ushbu

$$D_{a,x}^{\lambda} f(x) = \begin{cases} \dfrac{1}{\Gamma(-\lambda)} \displaystyle\int_a^x \dfrac{f(t)dt}{(x-t)^{1+\lambda}}, & \text{агар} \quad \lambda < 0, \\ \dfrac{d^{n+1}}{dx^{n+1}} D_{a,x}^{\lambda-(n+1)} f(x), & \text{агар} \quad \lambda > 0, \end{cases}$$

(1.27)

$$D_{x,b}^{\lambda} f(x) = \begin{cases} \dfrac{1}{\Gamma(-\lambda)} \displaystyle\int_x^b \dfrac{f(t)}{(t-x)^{1+\lambda}} dt, & \text{агар} \quad \lambda < 0, \\ (-1)^{n+1} \dfrac{d^{n+1}}{dx^{n+1}} D_{x,b}^{\lambda-(n+1)} f(x), & \text{агар} \quad \lambda > 0, \end{cases}$$

belgilashlarni kiritamiz, bu erda $D_{a,x}^{\lambda}$ va $D_{x,b}^{\lambda}$ ifodalar agar $\lambda<0$ bo'lsa $-\lambda$ kasr tartibli integral operatorni, agar $\lambda>0$ bo'lsa Liuvill ma'nosidagi umumlashgan hosilani beradi. $n=[\lambda]-\lambda$ sonining butun qismi.

Ta'rifga asosan

$$D_{a,x}^{0} f(x) = D_{x,b}^{0} f(x) = f(x).$$

(1.28)

deb hisoblaymiz.

1.2.1 Kasr tartibli integral operatorlarning xossalari. Kasr tartibli integral operatorlar uchun ushbu teoremalar o'rinlidir, qulaylik uchun $\lambda=-\alpha<0$, $\alpha>0$ deb hisoblaymiz.

1.1. Teorema. *Agar* $f(x)\in L_p(a,b)$, $p>1$, $a<b<\infty$, $0<\alpha<\dfrac{1}{p}$, $q=p/(1-p\alpha)$, *bo'lsa, u holda* $D_{a,x}^{-\alpha} f(x)\in L_q(a,b)$, *shu bilan birga*

$$\left(\int_a^b \left|D_{a,x}^{-\alpha} f(x)\right|^q dx\right)^{\frac{1}{q}} \le k\left(\int_a^b \left| f(x)\right|^p dx\right)^{\frac{1}{p}},$$

(1.29)

tengsizlik o'rinli bu erda $k = k(p,\alpha)$ *faqat* p *va* α *ga bog'liq* .

Natija. *Agar* $p>1$ *va* $\alpha = \frac{1}{p}$ *bo'lsa, u holda* $D_{a,x}^{-\alpha} f(x)$ operator chegaralangan.

1.2. Teorema. $f(x)$ *funksiya* (a,b) *oraliqda* $\alpha \in (0,1]$ *ko'rsatkich va* M *Gelder o'zgarmasi bilan* (a,b) *intervalda Gyolder shartini qanoatlantiradi deyiladi, agar ixtiyoriy* $x_1, x_2 \in (a,b)$ *uchun*

$$\left| f(x_1) - f(x_2)\right| \le M\left|x_2 - x_1\right|^{\alpha}$$

(1.30)

tengsizlik bajarilsa. Qisqacha qilib bunday funksiyalar H *yoki* $H(\alpha)$ *shartini qanoatlantiradi deyiladi.*

Agar $\alpha > 1$ bo'lsa, (1.30) dan ko'rinib turibdiki, $f'(x) \equiv 0$ ya'ni $f(x) = const$.

$H(\alpha)$ shartni qanoatlantiruvchi funksiyalarning xossalarini keltiramiz.

1^0 . Agar $f(x)$ funksiya (a,b) intervalda $\left| f'(x)\right| < M$ chekli hosilaga ega bo'lsa, u holda $f(x)$ funksiya Gyolder shartini $\alpha = 1$ ko'rsatkich bilan qanoatlantiradi (Lipshits sharti).

2^0 . Agar $f(x)$ funksiya (a,b) chekli intervalda α ko'rsatkich bilan Gyolder shartini qanoatlantirsa, u holda bu funksiya $\beta < \alpha$ ko'rsatkich bilan ham Gyolder shartini qanoatlantiradi.

SHunday qilib, kichik α uchun kengroq funksiyalar sinfi mos keladi. Eng tor sinf bu, Lipshits shartini qanoatlantiruvchi funksiyalar sinfidir.

3^0. Agar $f_1(x)$ va $f_2(x)$ (a,b) oraliqda mos ravishda $H(\alpha_1)$ va $H(\alpha_2)$ shartlarni qanoatlantirsa, u holda $f_1(x)+f_2(x)$, $f_1(x)\cdot f_2(x)$, $\dfrac{f_1(x)}{f_2(x)}$ $(f_2(x)\neq 0)$ funksiyalar $\alpha=\min(\alpha_1,\alpha_2)$ shart bilan Gyolder shartini qanoatlantiradi.

1.3. Teorema. $p>1$, $\dfrac{1}{p}<\alpha<\dfrac{1}{p}+1$ *yoki* $p=1$, $1\leq\alpha<2$ *bo'lib* $f(x)\in L_p(a,b)$, *bo'lsa, u holda* $D_{a,x}^{-\alpha}f(x)$ (a,b) *oraliqda* $\alpha-\dfrac{1}{p}$ *ko'rsatkich bilan Gyolder shartini qanoatlantiradi.*

1.4. Teorema. *k va* α *sonlari uchun* $k\geq 0,\ \alpha>0,\ k+\alpha<1$ *shartlar bajarilsin. Agar* $f(x)$ *funksiya* (a,b) *oraliqda k ko'rsatkich bilan Gyolder shartlarini qanoatlantirsa va kichik* $x-a$ *lar uchun* $f(x)=O\left((x-a)^k\right)$ *bo'lsa u holda* $D_{a.x}^{-\alpha}f(x)$ (a,b) *oraliqda* $k+\alpha$ *ko'rsatkich bilan Gyolder shartini qanoatlantiradi, shu bilan birga kichik* $x-a$ *lar uchun*

$$D_{a,x}^{-\alpha}f(x)=O\left((x-a)^{k+\alpha}\right)$$

bo'ladi.

1.5. Teorema. *Agar* $g(x)\in C^{(0,\lambda)}[a,b]$ *bo'lsa, u holda* $g(x)$*ni* $g(x)=g(a)+D_{a.x}^{-\alpha}f(x)$, *ko'rinishda ifodalash mumkin, bu erda* $f(x)\in C^{(0,\lambda-\alpha)}(a,b)$, $0<\alpha<\lambda\leq 1$.

1.2.2. Kasr tartibli differensial operatorlar xossasi.

1^0. $\lambda=n+1$bo'lsin, u holda (1.27) va (1.28) munosabatlarga ko'ra

$D_{a,x}^{n+1}f(x)=\dfrac{d^{n+1}}{dx^{n+1}}f(x)$ va $n=0$ uchun $D_{a,x}^{1}f(x)=\dfrac{d}{dx}f(x)$ tengliklar o'rinli.

2^0. Kasr tartibli differensial operatorlar uchun ekstremum prinsipi. Musbat, kamaymaydigan $\omega(t)$ funksiya hamda $f(t)$ funksiyalar $[a,b]$ kesmada uzluksiz bo'lsin. Agar $[a,b]$ kesmada, $f(t)$ funksiya o'zining musbat maksimumiga (manfiy minimumiga) $t=x$ nuqtada erishsa, $a<x<b$ va bu nuqtaning ixtiyoriy kichik atrofida $\omega(t)f(t)$ ko'paytma $\gamma>\alpha$ ko'rsatkich bilan Gyolder shartini qanoatlantirsa, u holda

$$D_{a,x}^{\alpha}\omega(x)f(x)>0\ \left(D_{a,x}^{\alpha}\omega(x)f(x)<0\right).$$

(1.31)

Agar $\omega(t)$-musbat o'smaydigan funksiya bo'lsa, yuqoridagiga o'xshash natijani $D_{x,b}^{\alpha}$ operator uchun ham olish mumkin. Ixtiyoriy tartibli integro-differensial operatorlar kompozitsiyalari uchun o'rinli bo'lgan ba'zi bir munosabatlarni keltiramiz.

1^0. *Agar* $f(x)\in L(a,b)$ *bo'lsa, ixtiyoriy* $\alpha>0$ *va deyarli barcha* $x\in(a,b)$ *uchun*

$$D_{a,x}^{\alpha}D_{a,x}^{-\alpha}f(x)=f(x)$$

(1.32)

tenglik o'rinli.

2^0. $D_{a,x}^{\alpha}f(x)\in L(a,b)$ *bo'lsin, u holda deyarli barcha* $x\in(a,b)$ uchun

$$D_{a,x}^{-\alpha}D_{a,x}^{\alpha}f(x)=f(x)-\sum_{k=1}^{n}\left[D_{a,x}^{\alpha-k}f(x)\right]_{x=a}\frac{(x-a)^{\alpha-k}}{\Gamma(\alpha-k+1)},\ n-1<\alpha\le n.$$

(1.33)

tenglik o'rinlidir.

(1.32) va (1.33) tengliklarning umumlashmalarini keltiramiz.

3^0. $f(x)\in L(a,b)$ *bo'lsin. U holda:*

1) *agar* $\beta\ge\alpha>0$ *bo'lsa, u holda*

$$D_{a,x}^{\alpha}D_{a,x}^{-\beta}f(x)=D_{a,x}^{-(\beta-\alpha)}f(x)\ ,\ x\in(a,b),$$

(1.34)

2) *agar* $\alpha > \beta \geq 0$ *bo'lib,* $f(x)$ *funksiyaning* (a,b) *da* $D_{a,x}^{\alpha-\beta} f(x)$ hosilasi mavjud bo'lsa, u holda

$$D_{a,x}^{\alpha} D_{a,x}^{-\beta} f(x) = D_{a,x}^{\alpha-\beta} f(x), \; x \in (a,b) \qquad (1.35)$$

tenglik o'rinlidir.

4^0. $f(x) \in L(a,b)$ bo'lib, uning kasr tartibli hosilasi $D_{a,x}^{\beta} f(x) \in L(a,b)$ bo'lsin, bu erda $n-1 < \beta \leq n \; (n \geq 1)$, u holda ixtiyoriy $\alpha > 0$ son uchun

$$D_{a,x}^{-\alpha} D_{a,x}^{\beta} f(x) = D_{a,x}^{\beta-\alpha} f(x) - \sum_{k=1}^{n} \left[D_{a,x}^{\beta-k} f(x) \right]_{x=a} \frac{(x-a)^{\alpha-k}}{\Gamma(\alpha-k+1)}$$

(1.36)

tenglik o'rinlidir.

5^0. $f(x) \in C^{q-1}(a,b)$ va $f^{(q)}(x) \in L(a,b)$ bo'lsin $(q \geq 1)$ u holda ixtiyoriy $\alpha \; (0 < \alpha \leq q)$ uchun $D_{a,x}^{\alpha} f(x)$ hosila mavjud, shu bilan birga $n-1 < \alpha \leq n$ bo'lsa, u holda deyarli barcha $x \in (a,b)$ uchun ushbu

$$D_{a,x}^{\alpha} f(x) = \sum_{k=0}^{n-1} \frac{f^{(k)}(a)(x-a)^{k-a}}{\Gamma(k-\alpha+1)} + D_{a,x}^{-(n-\alpha)} f^{(n)}(x)$$

(1.37)

tenglik o'rinlidir.

Biz keyinchalik foydalanadigan ba'zi-bir ayniyatlarni isbotsiz keltiramiz.

1.1 Lemma. *Agar* $0 < \alpha, \beta < 1$ *va* $x^{-\alpha} f(x), \; x^{-\beta} f(x) \in L(a,b)$ *bo'lsa, u holda deyarli barcha* $x \in (a,b)$ *uchun*

$$D_{a,x}^{-\beta} (x-a)^{-\beta} D_{a,x}^{-\alpha} (x-a)^{-\alpha} f(x) = D_{a,x}^{-\alpha} (x-a)^{-\alpha} D_{a,x}^{-\beta} (x-a)^{-\beta} f(x)$$

(1.38)

ayniyat o'rinlidir.

1.2 Lemma. *Agar* $0<2\alpha<1$ *va* $(x-a)^{-\alpha} f(x)$, $(b-x)^{-\alpha} f(x) \in L(a,b)$ *bo'lsa, u holda deyarli barcha* $x \in (a,b)$ *uchun*

$$D_{a,x}^{\alpha}(x-a)^{2a-1} D_{a,x}^{\alpha-1}(x-a)^{-\alpha} f(x)= \\ =(x-a)^{\alpha-1} D_{a,x}^{2\alpha-1} f(x), \tag{1.39}$$

$$D_{x,b}^{\alpha}(b-x)^{2\alpha-1} D_{x,b}^{\alpha-1}(b-x)^{-\alpha} f(x)=(b-x)^{\alpha-1} D_{x,b}^{2\alpha-1} f(x).$$

(1.40)

munosabatlar o'rinli bo'ladi.

1.3 Lemma. *Agar* $0<2\beta<1$ *va* $(x-a)^{\beta-1} f(x), (b-x)^{\beta-1} f(x) \in L(a,b)$

bo'lsa, u holda deyarli barcha $x \in (a,b)$ *uchun ushbu*

$$D_{a,x}^{1-\beta}(x-a)^{1-2\beta} D_{a,x}^{-\beta}(x-a)^{\beta-1} f(x)=(x-a)^{-\beta} D_{a,x}^{1-2\beta} f(x),$$

(1.41)

$$D_{x,b}^{1-\beta}(b-x)^{1-2\beta} D_{x,b}^{-\beta}(b-x)^{\beta-1} f(x)=(b-x)^{-\beta} D_{x,b}^{1-2\beta} f(x),$$

(1.42)

ayniyatlar o'rinli.

1.4 Lemma. *Agar* $\varphi(x) \in C^{(0,\alpha)}(a,b)$ *bo'lsa, u holda ushbu*

$$D_{x,b}^{-\alpha}\varphi(x)=\cos\alpha\pi D_{a,x}^{-\alpha}\varphi(x)+\frac{\sin\alpha\pi}{\pi}\int_a^b\left(\frac{b-x}{b-t}\right)^{\alpha}\frac{D_{a,t}^{-\alpha}\varphi(t)}{t-x}dt \quad ,$$

(1.43)

$$D_{a,x}^{-\alpha}\varphi(x)=\cos\alpha\pi D_{x,b}^{-\alpha}\varphi(x)-\frac{\sin\alpha\pi}{\pi}\int_a^b\left(\frac{x-a}{t-a}\right)^{\alpha}\frac{D_{t,b}^{-\alpha}\varphi(t)}{t-x}dt \quad ,$$

(1.44)

$$D_{x,b}^{-\alpha}\varphi(x)=\cos\alpha\pi D_{a,x}^{-\alpha}\varphi(x)+\frac{\sin\alpha\pi}{\pi\Gamma(\alpha)}\int_a^x\frac{(x-t)^{\alpha-1}}{(t-a)^{\alpha}}dt\int_a^b\frac{(z-a)^{\alpha}\varphi(z)dz}{z-t}$$

(1.45)

$$D_{a,x}^{-\alpha}\varphi(x)=\cos\alpha\pi D_{x,b}^{-\alpha}\varphi(x)-\frac{\sin\alpha\pi}{\pi\Gamma(\alpha)}\int_x^b\frac{(t-x)^{\alpha-1}}{(b-t)^{\alpha}}dt\int_a^b\frac{(b-z)^{\alpha}\varphi(z)}{z-t}dz$$

(1.46)

ayniyatlar o'rinlidir $(0<\alpha<1)$:

Natija. $\Phi(x)$ *funksiya* $D_{a.,x}^{-\alpha}\varphi(x)$ *Abel integrali yordamida ifodalansin, u holda ushbu ayniyatlar o'rinlidir:*

$$D_{a,x}^{-\alpha}D_{t,b}^{\alpha}\Phi(t)=\cos\alpha\pi\Phi(x)-\frac{\sin\alpha\pi}{\pi}\int_a^b\left(\frac{x-a}{t-a}\right)^{\alpha}\frac{\Phi(t)dt}{t-x},$$

(1.47)

$$D_{x,b}^{-\alpha}D_{a,t}^{\alpha}\Phi(t)=\cos\alpha\pi\Phi(x)-\frac{\sin\alpha\pi}{\pi}\int_a^b\left(\frac{b-x}{b-t}\right)^{\alpha}\frac{\Phi(t)}{t-x}dt \quad .$$

(1.48)

1.5. Lemma. $\varphi(x)\in C^{(0,\lambda)}(a,b)$ *bo'lsin, u holda ushbu ayniyatlar o'rinlidir*

$$D_{a,x}^{\alpha}D_{t.,b}^{-\alpha}\varphi(t)=\cos\alpha\pi\varphi(x)+\frac{\sin\alpha\pi}{\pi}\int_a^b\left(\frac{t-a}{x-a}\right)^{-\alpha}\frac{\varphi(t)dt}{t-x},$$

(1.49)

$$D_{x,b}^{\alpha}D_{a,t}^{-\alpha}\varphi(t)=\cos\alpha\pi\varphi(x)-\frac{\sin\alpha\pi}{\pi}\int_a^b\left(\frac{b-t}{b-x}\right)^{\alpha}\frac{\varphi(t)dt}{t-x}.$$

(1.50)

1.3§. Aralash tipdagi tenglama uchun gellerstedt va buzilish chizig'ini o'rganish bo'yicha olib borilgan ilmiy tadqiqot ishlari va ularning tahlili.

$z=x+iy$ kompleks tekisligining yuqori $\operatorname{Im}z>0$ yarim tekisligida

$$y^m u_{xx} + u_{yy} + (\beta_0 / y)u_y = 0, \qquad (1.51)$$

tenglamani o‘rganamiz, bu erda m, β_0 o‘zgarmas sonlar bo‘lib, $m > 0$, $-(m/2) < \beta_0 < 1$ shartlarni qanoatlantiradi.

Ω – chekli bir bog‘lamli soha bo‘lib, uchlari $A(-a,0)$ va $B(a,0)$, nuqtalarda bo‘lgan va $y > 0$ yarim tekislikda yotuvchi silliq Jordan chizig‘i $\Gamma : x = x(s), y = y(s)$ bu erda s parametr $\overset{\cup}{MB}$ yoy uzunligi, hamda $y = 0$ o‘qining AB kesmasi bilan chegaralangan bo‘lsin. (1.51) tenglama uchun Ω sohada Dirixle masalasini o‘rganamiz.

Dirixle masalasi. Ω sohada (1.51) tenglamaning ushbu

$$u\big|_{\Gamma} = \varphi(s) \quad 0 \le s \le l\ ;\ u(x,0) = \tau(x),\ x \in I\ , \qquad (1.52)$$

shartlarni qanoatlantiruvchi regulyar yechimi $u(x,y) \in C(\overline{\Omega}) \cap C^2(\Omega)$ topilsin, bu erda s Γ chiziqning BM yoyi uzunligi,

l –butun Γ chiziq yoyi uzunligi: $\varphi(s)$ va $\tau(x)$–berilgan uzluksiz funksiyalar, shu bilan birga $\tau(-a) = \varphi(l), \tau(a) = \varphi(0)$, $I = (-a, a), y = 0$ o‘qining intervali.

Ekstremum prinsipi Ω sohada (2.1) tenglamaning $u(x,y)$ regulyar

yechimi hech bir $(x,y) \in \Omega$ nuqtada o‘zining musbat maksimumiga va manfiy minimumiga erishmaydi.

Isboti. Ushbu
$$\upsilon(x,y) = u(x,y) / A(y) \qquad (1.53)$$

funksiyani qaraymiz, bu erda

$$A(y) = e^{d^{1-\beta_0}} - \varepsilon e^{y^{1-\beta_0}},$$

d – bu Ω soha diametri, $0 < \varepsilon < 1$. Bevosita hisoblashlar yordamida

$$E(u) = A(y) E_1(\upsilon)$$

tenglikni to‘g‘riligiga ishonch hosil qilish qiyin emas, bu erda

$$E_1(\upsilon) = y^m \upsilon_{xx} + \upsilon_{yy} + \frac{1}{y}\left(\beta_0 + 2yA_y\right)\upsilon_y + \frac{1}{A}\left(\frac{\beta_0}{y}A_y + A_{yy}\right)\upsilon, \quad (1.54)$$

$$A_y = -\varepsilon(1-\beta_0)e^{y^{1-\beta_0}}y^{-\beta_0},$$

$$A_{yy} = -\varepsilon(1-\beta_0)^2 e^{y^{1-\beta_0}}y^{-2\beta_0} + \varepsilon\beta_0(1-\beta_0)e^{y^{1-\beta_0}}y^{-\beta_0-1},$$

$$\frac{1}{A}\left(\frac{\beta_0}{y}A_y + A_{yy}\right) = -\frac{\varepsilon}{A}(1-\beta_0)^2 e^{y^{1-\beta_0}}y^{-2\beta_0} < 0 \qquad (1.55)$$

(1.55) tengsizlikka asosan (1.54) tenglama yechimi $\vartheta(x,y)$ Ω soha ichidagi hech bir (x_0,y_0) nuqtada o‘zining musbat maksimumiga erishmaydi. Haqiqatdan ham, teskarisini faraz qilaylik (x_0,y_0) nuqtada $\vartheta(x,y)$ funksiya o‘zining musbat maksimumiga erishsin u holda bu nuqtada

$$\frac{\partial\vartheta(x_0,y_0)}{\partial x} = 0, \frac{\partial\vartheta(x_0,y_0)}{\partial y} = 0,$$

$\frac{\partial^2\vartheta(x_0,y_0)}{\partial x^2} \le 0,\ \frac{\partial^2\vartheta(x_0,y_0)}{\partial y^2} \le 0$ bo‘lgani uchun (1.53) dan $E_1(\vartheta) < 0$. Bu esa $E_1(\vartheta) = 0$ tenglikka ziddir. Aynan shu mulohazalarni takrorlab $\vartheta(x,y)$ funksiya Ω sohaning hech bir ichki nuqtasida o‘zining manfiy minimumga erishmasligini ko‘rsatish mumkin.

SHunday qilib, (1.53) ga asosan (1.51) tenglamaning regulyar yechimi $u(x,y)$ o‘zining musbat maksimumi va manfiy minimumini Ω sohaning ichki nuqtalarida qabul qilmaydi.

2.1-teorema. Ω *sohada (1.51) tenglama uchun qo‘yilgan Dirixle masalasining yechimi mavjud bo‘lsa, u yagonadir*.

Isboti. Faraz qilaylik, qo‘yilgan masala ikkita u_1 va u_2 yechimlarga ega bo‘lsin, u holda berilgan tenglama va chegaraviy shartlar chiziqli bo‘lgani uchun $w = u_1 - u_2$ funksiya(1.51) tenglamani va bir jinsli

$$w|_{\Gamma} = (u_1 - u_2)|_{\Gamma} = 0 \ ; \ w(x,0) = u_1(x,0) - u_2(x,0) = 0 \qquad (1.56)$$

shartlarni qanoatlantiradi. Ekstremum prinsipiga ko‘ra $\overline{\Omega}$ sohada uzluksiz $w(x,y)$ funksiya o‘zining ekstremumlarini faqat $\partial\overline{\Omega} = \Gamma \cup AB$ da qabul qiladi, ya’ni

$$0 = \min_{(x,y)\in\partial\overline{\Omega}} w(x,y) \le \underset{(x,y)\in\overline{\Omega}}{w}(x,y) \le \max_{(x,y)\in\partial\overline{\Omega}} w(x,y) = 0 .$$

Bundan esa $w(x,y) \equiv 0$, $(x,y) \in \overline{\Omega}$.

2.1- teorema isbot bo‘ldi.

Dirixle masalasining Grin funksiyasi va yechimi. Ω sohada (1.51) tenglama uchun Dirixle masalasining Grin funksiyasi deb quyidagi shartlarni qanoatlantiruvchi $G_2(x,y;x_0,y_0)$ funksiyaga aytiladi:

1) Ω sohaning (x_0, y_0) nuqtasidan tashqari barcha nuqtalarida (1.51) tenglamaning regulyar yechimi;

2) $$G_2(x,y;x_0,y_0)\big|_{\Gamma \Upsilon \overline{A}B} = 0 \quad (1.57)$$

chegaraviy shartni qanoatlantiradi;

3)uni

$$G_2(x,y;x_0,y_0) = q_2(x,y;x_0,y_0) + \vartheta_2(x,y;x_0,y_0) \qquad (1.58)$$

shaklda ifodalash mumkin, bu erda

$$q_2(x,y;x_0,y_0) = k_2\left(\frac{4}{m+2}\right)^{4\beta-2} \times$$
$$\times r_1^{-2\beta}(1-\sigma)^{1-2\beta} F(1-\beta, 1-\beta, 2-2\beta; 1-\sigma)$$

(1.59)

(1.51) tenglamaning fundamental yechimi, $\vartheta_1(x,y;x_0,y_0)\,\Omega$ sohaning barcha nuqtalarida (1.51) tenglamaning regulyar yechimi.

Grin funksiyasini tuzish uning regulyar qismi $\vartheta_2(x,y;x_0,y_0)$ ni topishga olib kelinadi, $\vartheta_2(x,y;x_0,y_0)$ funksiya (1.57) va (1.58) tengliklarga asosan

$\vartheta_2(x,y;x_0,y_0)\big|_{\Gamma} = -q_2(x,y;x_0,y_0)\big|_{\Gamma}$, (1.60)

$\vartheta_2(x,0;x_0,y_0) = 0$ (1.61)

chegaraviy shartlarni qanoatlantirishi kerak $\vartheta_2(x,y;x_0,y_0)$ funksiyani ikki qatlam potensiali shaklida izlaymiz:

$$\vartheta_2(x,y;x_0,y_0) = \int_0^l \mu_2(t;x_0,y_0)\eta^{\beta_0}(t)A_t[\xi(t),\eta(t);x,y]dt\,.$$

(1.62)

Ikki qatlam potensiali $W^{(2)}(x,y)$ uchun o‘rinli bo‘lgan (1.39) munosabatlarning birinchisidan hamda (1.60) chegaraviy shartlardan foydalanib $\mu_2(t;x_0,y_0)$ zichlik uchun quyidagi integral tenglamani hosil qilamiz

$$\begin{gathered}\mu_2(s;x_0,y_0) - 2\int_0^l K_2(s,t)\,\mu_2(t;x_0,y_0)dt = \\ = 2q_2(x(s),y(s);x_0,y_0)\end{gathered}\,.$$

(1.63)

(1.63) tenglamaning o‘ng tomoni s argumentning uzluksiz funksiyasidir $((x_0,y_0)\in\Omega$, $(x(s),y(s)\in\Gamma)$ 2.7 lemmaga asosan (1.63) integral tenglamaga Fredgolm nazariyasini qo‘llash mumkin.

$\lambda = 2$ $K_2(s,t)$ yadroning xarakteristik soni bo‘lmasligi isbotlangan edi [33, 89 b.], demak (1.63) tenglama yechimi mavjud va uning uzluksiz yechimini

$$\mu_2(s;x_0,y_0)=2q_2(x(s),y(s);x_0,y_0)+$$
$$+4\int_0^l R_2(s,t;2)\,q_2(\xi(t),\eta(t);x_0,y_0)dt$$

(1.64)

shaklda ifodalash mumkin, bu erda $R_2(s,t;2)$ orqali $K_2(s,t)$ yadroning rezolventasi belgilangan $(x(s),y(s))\in\Gamma$. (1.64) dan $\mu_2(s;x_0,y_0)$ ni (1.62) ga qo‘yib ushbu

$$\vartheta_2(x,y;x_0,y_0)=2\int_0^l q_2(\xi,\eta;x_0,y_0)\eta^{\beta_0}(t)A_t[\xi(t),\eta(t);x,y]dt+$$
$$+4\int_0^l\int_0^l \eta^{\beta_0}(t)A_t[q_2(\xi(t),\eta(t);x,y)]R_2(t,s;2)q_2(x(s),y(s);x_0,y_0)dtds$$

tenglikka kelamiz.

Dirixle masalasi uchun Grin funksiyasining regulyar qismi $\vartheta_2(x,y;x_0,y_0)$ ni oddiy qatlam potensiali shaklida ifodalash mumkin:

$$\vartheta_2(x,y;x_0,y_0)=\int_0^l \rho_2(t;x_0,y_0)\,q_2(\xi(t),\eta(t);x,y)dt\,,$$

(1.66)

bu erda

$$\rho_2(s;x_0,y_0)=2y^{\beta_0}(s)A_s[q_2(x(s),y(s));x_0,y_0]+$$
$$+4\int_0^l R_2(t,s;2)\,A_t[q_2(\xi(t),\eta(t));x_0,y_0]dt$$

(1.67)

YA’ni, $\rho_2(s;x_0,y_0)$ ushbu

$$\rho_2(s;x_0,y_0)-2\int_0^l K_2(t,s)\,\rho_2(t;x_0,y_0)dt=$$
$$=2y^{\beta_0}(s)A_s[q_2(x(s),y(s));x_0,y_0] \qquad (1.68)$$

integral tenglamaning yechimi.

(1.66) ga oddiy qatlam potensiali konormal hosilasi uchun o‘rinli bo‘lgan (1.44) formulalardan birinchisini qo‘llab quyidagi tenglikni hosil qilamiz

$$2y^{\beta_0}(s)A_s\left[\rho_2(x(s),y(s));x_0,y_0\right]\Big|_i =$$
$$=\rho_2(s;x_0,y_0)+2\int_0^l K_2(t,s)\,\rho_2(t;x_0,y_0)dt \qquad (1.69)$$

Endi (1.68) va (1.69) tenglamalardan

$$y^{\beta_0}(s)A_s\left[G_2(x(s),y(s));x_0,y_0\right]=\rho_2(s;x_0,y_0) \qquad (1.70)$$

tenglikka kelamiz va bu tenglikka asosan (1.66) formulani ushbu

$$\vartheta_2(x,y;x_0,y_0)=\int_0^l q_2(\xi(t),\eta(t);x,y)A_t\left[G_2\xi(t),\eta(t);x_0,y_0\right]dt \qquad (1.71)$$

ko‘rinishda yozish mumkin.

2.11-lemma. *Agar* (x_0,y_0) *nuqta* Ω *soha ichida joylashgan bo‘lsa, u holda* $G_2(x,y;x_0,y_0)$ *Grin funksiyasi* (x,y) *va* (x_0,y_0) *nuqtalarga nisbatan simmetrikdir.*

2.11 lemmaning isboti 2.10 lemmaning isbotiga o‘xshash bajariladi.

Ω_0 normal soha uchun Grin funksiyasi quyidagi ko‘rinishda bo‘ladi

$$G_{02}(x,y;x_0,y_0)=q_2(x,y;x_0,y_0)-\left(\frac{a}{R}\right)^{2\beta}q_2(x,y;\bar{x}_0,\bar{y}_0),$$

(1.72)

bu erda

$$R^2=x_0{}^2+\frac{4}{(m+2)^2}y_0{}^{m+2},\ \bar{x}_0=\frac{a^2}{R^2},\quad \bar{y}_0^{(m+2)/2}=\frac{a^2}{R^2}y_0^{(m+2)/2}$$

Grin funksiyasi $G_{02}(x,y;x_0,y_0)$ ning regulyar qismi

$$\vartheta_{02}(x,y;x_0,y_0)=-\left(\frac{a}{R}\right)^{2\beta}q_2(x,y;\bar{x}_0,\bar{y}_0)$$

ni ushbu

$$\vartheta_{02}(x,y;x_0,y_0)=-\int_0^l\rho_2(s;x,y)\vartheta_{02}(x(s),y(s);x_0,y_0)ds \qquad (1.73)$$

shaklda ifodalash mumkin, bu erda $\rho_2(s;x,y)$ (1.68) integral tenglamaning yechimi. Endi (1.58) dan (1.73) ni ayirib, Ushbu

$$H_2(x,y;x_0,y_0)=G_2(x,y;x_0,y_0)-G_{02}(x,y;x_0,y_0)=$$
$$=\vartheta_2(x,y;x_0,y_0)-\vartheta_{02}(x,y;x_0,y_0)$$

tenglikni hosil qilamiz. Bu tengliklardan Grin funksiyasining simmetrikligini hisobga olib, ushbu tenglikka kelamiz.

$$H_2(x,y;x_0,y_0)=\vartheta_2(x_0,y_0;x,y)-\vartheta_{02}(x,y;x_0,y_0) \qquad (1.74)$$

(1.74) tenglikni (1.66), (1.73) va (1.) ifodalarga asosan ushbu

$$\begin{aligned}H_2(x,y;x_0,y_0)&=\int_0^l\rho_2(t;x,y)q_2(\xi(t),\eta(t);x_0,y_0)+\\&+\int_0^l\rho_2(t;x,y)\vartheta_{02}(\xi(t),\eta(t);x_0,y_0)dt=\\&=\int_0^l\rho_2(t;x,y)\vartheta_{02}(\xi(t),\eta(t);x_0,y_0)dt\end{aligned} \qquad (1.75)$$

shaklida yozib olamiz.

2.6-teorema. Q*uyidagi formula*

$$\begin{aligned}u(x,y)=&\int_{-a}^{a}\tau(t)\left[\eta^{\beta_0}\frac{\partial G_2(t,\eta;x,y)}{\partial\eta}\right]\Big\|_{\eta=0}dt-\\&-\int_0^l\varphi(\xi(s))\eta^{\beta_0}(s)A_s\left[G_2(\xi(s),\eta(s);x,y)\right]ds\end{aligned} \qquad (1.76)$$

Ω sohada (2.1) tenglama uchun Dirixle masalasining yechimini beradi, bu erda $\tau(x)\in C[-a,a]$, $\varphi(s)\in C[0,l]$, *shu bilan birga* $\varphi(0)=\tau(a)$, $\varphi(l)=\tau(-a)$

Izoh. (1.75) va (1.72) formulalarga asosan (1.76) yechimni quyidagi ko‘rinishda yozish mumkin.

$$u(x,y)=\int_{-a}^{a}\tau(t)\left[\eta^{\beta_0}\frac{\partial G_2(t,\eta;x,y)}{\partial\eta}+\eta^{\beta_0}\frac{\partial H_2(t,\eta;x,y)}{\partial\eta}\right]\Big|_{\eta=0}dt-$$

$$-\int_0^l\phi(s)\{\eta^{\beta_0}(s)A_s[G_{02}(\xi(s),\eta(s);x,y)]+ \qquad (1.77)$$

$$+\eta^{\beta_0}(s)A_s[H_2(\xi(s),\eta(s);x,y)]\}ds,$$

bu erda

$$\eta^{\beta_0}\frac{\partial G_{02}(t,0;x,y)}{\partial\eta}\Big|_{\eta=0}=k_2(1-\beta_0)y^{1-\beta_0}\times$$

$$\times\left\{\left[(x-t)^2+\frac{4}{(m+2)^2}y^{m+2}\right]^{\beta-1}-\left[\left(a-\frac{xt}{a}\right)^2+\frac{4t^2}{(m+2)^2}y^{m+2}\right]^{\beta-1}\right\}$$

Dirixle masalasining yechimini (1.77) shaklda ifodalash aralash tipdagi tenglamalar uchun qo‘yilgan chegaraviy masalalarni echishda juda qulaydir.

Agar o‘rganilayotgan soha Ω_0 -normal soha bo‘lsa, yechim juda sodda ko‘rinishda bo‘ladi:

$$u(x,y)=k_2(1-\beta_0)y^{1-\beta_0}\int_{-1}^{1}\tau(t)\left\{\left[(x-t)^2+\frac{4}{(m+2)^2}y^{m+2}\right]^{\beta-1}-\right.$$

$$\left.-\left[\left(a-\frac{xt}{a}\right)^2+\frac{4t^2}{a^2(m+2)^2}y^{m+2}\right]^{\beta-1}\right\}dt-k_2(1-\beta)(m+2)\left(a^2-R^2\right)y^{1-\beta_0}\times$$

$$\times\int_0^l\varphi(\xi(s))\left(r_1^2\right)^{\beta-2}F(1-\beta,2-\beta,2-2\beta;1-\sigma)d\xi(s).$$

(1.78)

I BOB YUZASIDAN XULOSALAR.

Dissertatsiya ishining birinchi bobida asosiy tushunchalar, ta'riflar keltirib o'tilgan. Shuningdek, aralash tipdagi tenglama uchun Gellerstedt va buzilish chizig'ini umumiy ulanish shartli masalani o'rganishning nazariy asoslari haqida so`z yuritilgan.

Shuningdek birinchi bobda $\Gamma(z)$ gamma-funksiyasi:

$$\Gamma(z)=\int_0^{\infty} e^{-t}t^{z-1}dt, \quad (\operatorname{Re} z>0),$$

Beta-funksiyasi. $B(p,q)$ beta-funksiya:

$$B(p,q)=\int_0^1 t^{p-1}(1-t)^{q-1}dt, \ \operatorname{Re} p>0, \ \operatorname{Re} q>0$$

Gaussning gipergeometrik funksiyasi, Gauss tenglamasi, buziladigan giperbolik va elliptik tipdagi tenglamalar nazariyasida ushbu

$$z(1-z)\omega''(z)+\left[c-(a+b+1)z\right]\omega'(z)-ab\omega(z)=0$$

Gauss tenglamasining yechimlari, ixtiyoriy tartibli integro-differensial operatorlar, kasr tartibli integral operatorlarning xossalari va aralash tipdagi tenglama uchun gellerstedt va buzilish chizig'ida umumiy ulanish shartli masalalarini o'rganish bo'yicha olib borilgan ilmiy tadqiqot ishlari va ularning tahlili o'rganilgan.

II-Bob. SOHA ICHIDA BUZILADIGAN SINGULYAR KOEFFITSIENTLI ELLIPTIK TURDAGI TENGLAMA UCHUN UMUMIY ULASH SHARTLI MASALANING QO‘YILISHI , YECHISH METODIKASI VA BAYONI.

D - soha $z = x + iy$ kompleks tekisligining $y > 0$ yarim tekislikda yotuvchi, uchlari $A = A(-1,0), \mathrm{B} = \mathrm{B}(1,0)$ nuqtalarda bo‘lgan silliq $\Gamma: y = \sigma_0(x)$ chizig‘i Bilan, $y < 0$ yarim tekislikda esa

$$\mathrm{E}(u) = sign y |y|^m u_{xx} + u_{yy} + (\beta_0 / y) u_y = 0 \qquad (2.1)$$

tenglamaning $\mathrm{A}C$ va $\mathrm{B}C$ xarakteristikalari bilan chegaralangan soha bo‘lsin.

$y = \sigma_0(x)$ funksiyadan $\sigma_0(x) \in C(\bar{I}) \cap C^2(I),\ \sigma_0''(x) < 0,\ \forall x \in I = (-1,1)$ shartni qanoatlantirishini talab qilamiz, shu bilan birga $(d,0)$, $\sigma_0(d) = 0$ bo‘lsin.

D sohaning elliptik va giperbolik qismlarini mos ravishda D_1 va D_2 orqali belgilaymiz.

Ushbu magistrlik dissertatsiyasida Bitsadze-Samarskiy sharti (2.1) tenglama yechimini qiymatlarini Γ chiziqda va buzilish chizig‘i kesmasi AB da bog‘laydi. SHu bilan birga buzilish chizig‘idagi ulanish shartlari uzilishga ega bo‘lishi mumkin, ya’ni ulanish shartlari umumiy ko‘rinishda beriladi.

2.1-§. T masalasining yechish metodikasi.

T **- masalasi**. D aralash sohada (2.1) tenglamaning ushbu shartlarni qanoatlantiruvchi yechimi topilsin:

1) $u(x, y)$ har bir $\overline{D_1}$ va $\overline{D_2}$ sohalarda uzluksiz;

2) $u(x,y)\in C^2(D_1)$ va bu sohada (1) tenglamaning regulyar yechimi;

3) $u(x,y)\in R_1$ ya’ni $u(x,y)$ funksiya (1) tenglamaning regulyar yechimi;

4) $u(x,y)$ funksiya ushbu chegaraviy shartlarni qanoatlantiradi:

$$u(x,\sigma_0(x)) = \mu(x)u(x,0) + \varphi(x), x\in \bar{I} \qquad (2.2)$$

$$u\big|_{AC} = \psi(x), \quad x\in[-1,0], \qquad (2.3)$$

bu erda $\mu(x),\psi(x)$ o‘z aniqlanish sohasida uzluksiz funksiyalar;

5) parabolik buzilish chizig‘i kesmasi AB da ushbu ulanish shartlari bajariladi

$$u(x,-0) = a(x)u(x,+0) + \bar{\gamma}(x) \quad x\in \bar{I}, \qquad (2.4)$$

$$\lim_{y\to -0}(-y)^{\beta_0}\frac{\partial u}{\partial y} = b(x)\lim_{y\to +0} y^{\beta_0}\frac{\partial u}{\partial y} + \delta(x), \quad x\in I \qquad (2.5)$$

bu erda $a(x),\bar{\gamma}(x),b(x),\delta(x)$ - berilgan uzluksiz funksiyalar bo‘lib,

$a(x)\neq 0, b(x)\neq 0, \forall x\in \bar{I}, \gamma(-1)=0$.

Ta’riflangan masalani o‘rganish uchun D_1 sohada Dirixle masalasi yechimini beruvchi Xolmgren formulasidan va D_2 sohada shakli o‘zgargan Koshi masalasi yechimini beruvchi Darbu formulalaridan foydalanamiz.

Dirixle masalasi: D_1 sohada (1) tenglamaning $C(\overline{D}_1)$ I $C^2(D_1)$ sinfga tegishli va ushbu

$$u(x,0) = \tau(x), \quad x\in \bar{I} \qquad (2.6)$$

$$u\big|_{\Gamma} = \varphi_0(s), \qquad (2.7)$$

shartlarni qanoatlantiruvchi regulyar yechimi topilsin, bu erda s Γ chiziqning $B(-1,0)M(x,y)$ yoy uzunligi.

Shakli o‘zgargan Koshi masalasi. D_2 sohada (1) tenglamaning $C(\overline{D}_2)$ I $C^2(D_2)$ sinfga tegishli va ushbu

$$u(x,0) = \tau(x), \quad x\in \bar{I} \qquad (2.8)$$

$$\lim_{y\to -0}(-y)^{\beta_0}\frac{\partial u}{\partial y}=\nu(x),\quad x\in I \tag{2.9}$$

shartlarni qanoatlantiruvchi yechimi topilsin. Bu erda
$\tau(x)\in C(\bar{I})\text{ I } C^2(I),\nu(x)\in C(I).$

Dirixle va shakli o‘zgargan Koshi masalalarining yechimini beruvchi formulalar mos ravishda ushbu ko‘rinishlarda bo‘ladi

$$u(x,y)=k_2(1-\beta_0)y^{1-\beta_0}\times$$
$$\times\int_{-1}^{1}\tau(t)\left\{\left[(x-t)^2+\frac{4}{(m+2)^2}y^{m+2}\right]^{\beta-1}-\left[(1-xt)^2+\frac{4t^2}{(m+2)^2}y^{m+2}\right]^{\beta-2}\right\}dt-$$
$$-k_2(1-\beta)(m+2)\left(1-R^2\right)y^{1-\beta_0}\int_0^{\lambda}\varphi_0(\xi(s))\left(r_1^2\right)^{\beta-2}F(1-\beta,2-\beta,2-2\beta;1-\sigma)d\xi(s),$$

bu erda $F(a,b,c;x)$ - Gaussning gipergeometrik funksiyasi.

$$\left.\begin{matrix}r^2\\ r_1^2\end{matrix}\right\}=(x-t)^2+\frac{4}{(m+2)^2}\left(y^{\frac{m+2}{2}}\ \mu\,\eta^{\frac{m+2}{2}}\right),$$
$$R^2=x^2+\frac{4}{(m+2)^2}y^{m+2},\ \ \sigma=\frac{r^2}{r_1^2},\ (t,\eta)\in\sigma_0,$$

$$u(x,y)=\gamma_1\int_{-1}^{1}\tau\left[x+\frac{2t}{m+2}(-y)^{\frac{m+2}{2}}\right](1-t)^{\alpha-1}(1+t)^{\beta-1}dt+$$
$$+\gamma_2(-y)^{1-\beta_0}\int_{-1}^{1}\nu\left[x+\frac{2t}{m+2}(-y)^{\frac{m+2}{2}}\right](1-t)^{-\beta}(1+t)^{-\alpha}dt, \tag{2.10}$$

bu erda

$$\gamma_1=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}2^{1\alpha-\beta},\gamma_2=-\frac{\Gamma(2-\alpha-\beta)}{(1-\beta_0)\Gamma(1-\alpha)\Gamma(1-\beta)}$$

(2.10) formuladan (2.3) chegaraviy shartga asosan ushbu munosabatni hosil qilamiz

$$\nu_1(x)=\gamma D_{-1,x}^{1-2\beta}\tau_1(x)+\psi_1(x),\quad x\in I \tag{2.11}$$

bu erda

$$\tau_1(x) = u(x,-0),\ x \in \bar{I};\ \nu_1(x) = \lim_{y \to -0} (-y)^{\beta_0} \frac{\partial u}{\partial y},\quad x \in I, \qquad (2.12)$$

noma'lum funksiyalar.

$$\psi_1(x) = -\gamma(\Gamma(\beta)/\Gamma(2\beta))(1+x)^{\beta} D_{-1,x}^{1-\beta}\psi((x-1)/2), \qquad (2.13)$$

$$\gamma = \frac{2\Gamma(2\beta)\Gamma(1-\beta)}{\Gamma(\beta)\Gamma(1-2\beta)}\left(\frac{m+2}{4}\right)^{2\beta},\quad \beta = \frac{m+2\beta_0}{2(m+2)}$$

(2.4) – ulanish shartiga asosan (2.11) tenglikni ushbu ko'rinishda yozib olamiz

$$b(x)\nu(x) = \gamma D_{-1,x}^{1-2\beta} a(x)\tau(x) + \psi_2(x) \qquad (2.14)$$

bu erda

$$\tau(x) = u(x,+0),\ x \in \bar{I};\ \nu(x) = \lim_{y \to +0} y^{\beta_0} \frac{\partial u}{\partial y},\quad x \in I, \qquad (2.15)$$

$$\psi_1(x) = \gamma D_{-1,x}^{1-2\beta} \bar{\gamma}(x) + \psi_1(x) - \delta(x) \qquad (2.16)$$

(2.14) formula $\tau(x)$ va $\nu(x)$ o'rtasida birinchi funksional munosabatdir. (D_2 sohadan I intervalga olib kelingan).

Lemma 1. Agar: 1) $u(x,y) \in C(\overline{D}_1)$ I $C^2(D)$ *bo'lib* $u(x,y)$ *funksiya o'zining eng katta musbat qiymatini (eng kichik manfiy qiymatini)* $(b,0),\in(-1,1)$ *nuqtada qabul qilsa; 2)* $u(x,y)$ *funksiyaning* Γ *chiziqdagi qiymati* $(b,0)$ *nuqtadagi qiymatidan kichik (katta) bo'lsa, u holda*

$$\lim_{y \to +0} y^{\beta_0} \frac{\partial u}{\partial y} < 0 \quad (>0). \qquad (2.17)$$

2.2-§. T **masalasi yechimining yagonaligi.**

Teorema2.1. *T masalasi* $\varphi(x) \equiv 0, \psi(x) \equiv 0$ *va*

$$0 < \mu(x) \le 1, a'(x) \ge 0, a(x) > 0, b(x) > 0 \qquad (2.18)$$

bo'lganda faqat va faqat trivial yechimga ega.

Isbot. $u(x,y)$ funksiya 2.1-teorema shartlarni qanoatlantiruvchi T masalasining yechimi bo‘lsin. Xopf [ ] prinsipiga ko‘ra $u(x,y)$ funksiya o‘zining musbat maksimumi va manfiy minimumiga D_1 soha ichida erishmaydi.

1. Faraz qilaylikki, $u(x,y)$ funksiya o‘zining musbat maksimumiga $\overline{D}_1$ sohaning AB kesmasidagi $P(x_0,0)$ nuqtasida erishsin. Ya’ni

$\max\limits_{(x,y)\in\overline{D}} u(x,y)=u(x_0,0)=\tau(x)>0$ bo‘lsin. Kasr tartibli: $D^{1-2\beta}_{-1,x}a(x)\tau(x)$ hosilaning, $\tau(x)$ funksiyaning maksimum nuqtasidagi qiymati $D^{1-2\beta}_{-1,x}a(x)\tau(x)>0$ tengsizlikni qanoatlantirishidan foydalanib

$$b(x_0)\nu(x_0)=\gamma\, D^{1-2\beta}_{-1,x_0}a(x)\tau(x)>0 \qquad (2.19)$$

tengsizlikka kelamiz, lekin $b(x_0)>0$ bo‘lgani uchun (2.19) dan $\nu(x_0)>0$ ekanligi kelib chiqadi bu tengsizlik ulanish sharti (2.5) ga asosan (2.17)- tengsizlikka qarama-qarshi, bundan $p(x_0 0)$ nuqtada $u(x,y)$ funksiya musbat ekstremumga erishmasligi kelib chiqadi.

SHunday qilib, $u(x,y)$ funksiya o‘zining musbat maksimumiga Γ chiziqda erishar ekan, lekin bir jinsli (2.2) shartga ko‘ra, yuqoridagi mulohazalar, ya’ni $u(x,y)$ funksiyaning AB kesmada erishmasligidan Γ chiziqda ham erishmasligi kelib chiqadi.

SHunday qilib, $u(x,y)$ funksiya o‘zining musbat maksimumiga $\overline{D}_1$ sohaning $A(-1,0)$ va $B(1,0)$ nuqtalarida erishar ekan.

YUqoridagi kabi $u(x,y)$ funksiya o‘zining manfiy minimumiga ham $\overline{D}_1$ sohaning $A(-1,0)$ va $B(1,0)$ nuqtada erishishi kelib chiqadi, lekin $u(A)=u(B)=0$ bundan $u(x,y)\equiv 0$.

2.1 Teorema isbotlandi.

2.3-§. T masalasi yechimining mavjudligi bayoni.

T masalasi yechimining mavjudligini soddalik uchun Γ chiziq (2.1) tenglamaning normal chizig‘i

$$\sigma_0 : x^2 + \frac{4}{(m+2)^2} y^{m+2} = 1$$

bilan ustma-ust tushgan holda o‘rganamiz.

2.2. *Teorema.* T *masalasining yechimi mavjud* .

Isbot. (2.1) tenglama uchun Dirixle masalasi yechimini beruvchi (2.10) formuladan foydalanib, uni y bo‘yicha differensiallaymiz va ushbu

tengliknihosilqilamiz

$$\frac{\partial u}{\partial y} = k_2(1-\beta_0)\int_{-1}^{1} \tau(t) \frac{\partial}{\partial y} y^{1-\beta_0} \times$$

$$\times\left\{\left[(x-t)^2 + \frac{4}{(m+2)^2} y^{m+2}\right]^{\beta-1} - \left[(1-xt)^2 + \frac{4t^2}{(m+2)^2} y^{m+2}\right]^{\beta-1}\right\} dt + \qquad (2.21)$$

$$+ k_2(1-\beta)(m+2)\int_{-1}^{1} \varphi_0(t) \frac{\partial}{\partial y}\left\{\left(1-R^2\right) y^{1-\beta_0}\left(r_1^2\right)^{\beta-2} F(1-\beta, 2-\beta, 2-2\beta; 1-\sigma)\right\} dt$$

Bevosita hisoblashlar yordamida ushbu tenglikni to‘g‘ri ekanligini tekshirib ko‘rish qiyin emas.

$$\frac{\partial}{\partial y}\left\{ y^{1-\beta_0}\left[(x-t)^2 + \frac{4}{(m+2)^2} y^{m+2}\right]^{\beta-1} - y^{1-\beta_0}\left[(1-xt)^2 + \frac{4t^2}{(m+2)^2} y^{m+2}\right]^{\beta-1}\right\} =$$

$$= \frac{m+2}{2} y^{1-\beta_0} \frac{\partial}{\partial t}\left\{ (x-t)\left[(x-t)^2 + \frac{4}{(m+2)^2} y^{m+2}\right]^{\beta-1} - \right. \qquad (2.22)$$

$$\left. - \frac{(1-xt)}{x}\left[(1-xt)^2 + \frac{4t^2}{(m+2)^2} y^{m+2}\right]^{\beta-1}\right\}.$$

Endi (2.21) tenglikning birinchi integralida (22) tenglikni hisobga olgan holda bo‘laklab integrallash operatsiyasini bajarib ushbu tenglikka kelamiz

$$\frac{\partial u}{\partial y}=\frac{k_2(1-\beta_0)(m+2)}{2}y^{-\beta_0}\left\{\tau(t)\left[(x-t)\left((x-t)^2+\frac{4}{(m+2)^2}y^{m+2}\right)^{\beta-1}-\frac{(1-xt)}{x}\times\right.\right.$$

$$\left.\times\left((1-xt)^2+\frac{4t^2}{(m+2)^2}y^{m+2}\right)^{\beta-1}\right]\Bigg|_{-1}^{1}-\int_{-1}^{1}\tau'(t)\times$$

$$\left.\times\left[(x-t)\left((x-t)^2+\frac{4}{(m+2)^2}y^{m+2}\right)^{\beta-1}-\frac{(1-xt)}{x}\left((1-xt)^2+\frac{4t^2}{(m+2)^2}y^{m+2}\right)\right]^{\beta-1}dt\right\}+\tag{2.23}$$

$$+k_2(1-\beta)(m+2)\int_{-1}^{1}\varphi_0(t)\frac{\partial}{\partial y}\left\{\left(1-R^2\right)y^{1-\beta_0}\left(r_1^2\right)^{\beta-2}F(1-\beta,2-\beta,2-2\beta;1-\sigma)\right\}dt$$

(2.23) tenglikni y^{β_0} ga ko‘paytirib va $y\to 0$ da limitga o‘tib ushbu tengsizlikni hosil qilamiz [ ].

$$\nu(x)=-k_2(1-\beta_0)\frac{m+2}{2}\left\{\frac{\tau(1)}{(1-x)^{1-2\beta}}+\frac{\tau(-1)}{(1+x)^{1-2\beta}}+\right.$$

$$+\int_{-1}^{1}\frac{(x-t)\tau'(t)dt}{|x-t|^{2-2\beta}}+(1-2\beta)\int_{-1}^{1}\frac{\tau(t)dt}{(1-xt)^{2-2\beta}}-\tag{2.24}$$

$$\left.-2(1-\beta)(1-x^2)\int_{-1}^{1}\varphi(t)(x^2-2xt+1)^{\beta-2}dt\right\}.$$

(2.24) - tenglikdagi oxirgi integralni (2.4)- chegaraviy shartni hisobga olgan holda quyidagicha almashtiramiz

$$\Phi(x)=\int_{-1}^{1}\varphi_0(t)(x^2-2xt+1)^{\beta-2}dt=\int_{-1}^{1}(\mu(t)\tau(t)+\varphi(t))(x^2-2xt+1)^{\beta-2}dt=$$

$$=\int_{-1}^{1}\mu(t)(x^2-2xt+1)^{\beta-2}\tau(t)dt+f_0(x),$$

bu erda

$$f_0(x)=\int_{-1}^{1}\varphi(t)(x^2-2xt+t)^{\beta-2}dt\tag{2.25}$$

(2.25) tenglikka asosan (2.24) tenglikni ushbu ko‘rinishda yozib olamiz

$$\nu(x) = -k_2(1-\beta_0)\frac{m+2}{2}\left\{\frac{\tau(1)}{(1-x)^{1-2\beta}} + \frac{\tau(-1)}{(1+x)^{1-2\beta}} + \right.$$

$$\left. + \int_{-1}^{1}\frac{(x-t)\tau'(t)dt}{|x-t|^{2-2\beta}} + (1-2\beta)\int_{-1}^{1}\frac{\tau(t)dt}{(1-xt)^{2-2\beta}} - 2(1-\beta)(1-x^2)\int_{-1}^{1}K_0(x,t)\tau(t)dt\right\} + f_1(x). \tag{2.26}$$

Bu erda

$$f_1(x) = k_2(1-\beta)(1-\beta_0)(m+2)(1-x^2)\,f_0(x) \tag{2.26a}$$

$$K_0(x,t) = k_2(1-\beta)(1-\beta_0)(m+2)(1-x^2)\mu(t)(x^2-2xt+1)^{\beta-2} \tag{2.27}$$

-regulyar yadro.

Endi (2.14) va (2.26) munosabatlardan $\nu(x)$ ni yo‘qotib ushbu tenglikka kelamiz.

$$\gamma D_{-1,x}^{1-2\beta}a(x)\tau(x) = -k_2(1-\beta_0)\frac{m+2}{2}b(x)\left\{\frac{\tau(1)}{(1-x)^{1-2\beta}} + \frac{\tau(-1)}{(1+x)^{1-2\beta}} + \right.$$

$$\left. + \int_{-1}^{1}\frac{(x-t)\tau'(t)dt}{|x-t|^{2-2\beta}} + (1-2\beta)\int_{-1}^{1}\frac{\tau(t)dt}{(1-xt)^{2-2\beta}} - 2(1-\beta)(1-x^2)\int_{-1}^{1}K_0(x,t)\tau(t)dt\right\} + \tag{2.28}$$

$$+ f_1(x)b(x) - \psi_2(x).$$

(28) tenglikka $D_{-1,x}^{2\beta-1}$ kasr tartibli integrallash operatorini qo‘llaymiz va $f(-1)=0$ bo‘lganda

$$D_{-1,x}^{2\beta-1}D_{-1,x}^{1-2\beta}f(x) = f(x) \tag{2.29}$$

(2.29) tenglikni isbotini keltiramiz

$$D_{-1,x}^{2\beta-1}D_{-1,x}^{1-2\beta}\tau(x)=\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{x}\frac{D_{-1,t}^{1-2\beta}\tau(t)dt}{(x-t)^{2\beta}}=$$

$$=\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{x}\frac{dt}{(x-t)^{2\beta}}\frac{1}{\Gamma(2\beta)}\frac{d}{dt}\int_{-1}^{t}\frac{\tau(s)}{(t-s)^{1-2\beta}}=$$

$$=\frac{1}{\Gamma(1-2\beta)\Gamma(2\beta)}\int_{-1}^{x}\frac{dt}{(x-t)^{2\beta}}\frac{d}{dt}\int_{-1}^{t}\frac{\tau(s)}{(t-s)^{1-2\beta}}=$$

$$=\frac{1}{\mathrm{B}(2\beta,1-2\beta)}\int_{-1}^{x-\varepsilon}\frac{dt}{(x-t)^{2\beta}}\frac{d}{dt}\int_{-1}^{t}\frac{\tau(s)ds}{(t-s)^{1-2\beta}}=$$

$$=\frac{1}{\mathrm{B}(2\beta,1-2\beta)}\left[(x-t)^{-2\beta}\int_{-1}^{t}\frac{\tau(s)ds}{(t-s)^{1-2\beta}}\Bigg|_{-1}^{x-\varepsilon}-2\beta\int_{-1}^{x-\varepsilon}(x-t)^{-2\beta-1}dt\int_{-1}^{t}\frac{\tau(s)ds}{(t-s)^{1-2\beta}}\right]=$$

$$=\frac{1}{\mathrm{B}(2\beta,1-2\beta)}\left[\varepsilon^{-2\beta}\int_{-1}^{x-\varepsilon}\frac{\tau(s)ds}{(x-\varepsilon-s)^{1-2\beta}}-2\beta\int_{-1}^{x-\varepsilon}\tau(s)ds\int_{s}^{x-\varepsilon}(x-t)^{-2\beta-1}(t-s)^{-(1-2\beta)}dt\right],$$

Ichki integralni hisoblash uchun quyidagi almashtirishlarni bajaramiz

$$t=s+(x-\varepsilon-s)\sigma,\;\; dt=(x-\varepsilon-s)d\sigma$$

$$x-t=x-s-(x-\varepsilon-s)\sigma=(x-s)\left(1-\frac{x-\varepsilon-s}{x-s}\sigma\right)$$

$$t-s=(x-\varepsilon-s)\sigma,$$

u holda quyidagi natijaga kelamiz

$$\int_s^{x-\varepsilon}(x-t)^{-2\beta-1}(t-s)^{-(1-2\beta)}dt=$$

$$\int_0^1(x-s)^{-2\beta-1}\left(1-\frac{x-\varepsilon-s}{x-s}\sigma\right)^{-2\beta-1}(x-\varepsilon-s)^{-(1-2\beta)}\sigma^{-(1-2\beta)}\times$$

$$\times(x-\varepsilon-s)d\sigma=(x-s)^{-(1-2\beta)}(x-\varepsilon-s)^{-2\beta}\times$$

$$\times\int_0^1\sigma^{2\beta-1}\left(1-\frac{x-\varepsilon-s}{x-s}\sigma\right)^{-2\beta-1}d\sigma=$$

$$=(x-s)^{-1-2\beta}(x-\varepsilon-s)^{2\beta}\frac{\Gamma(2\beta)\Gamma(1)}{\Gamma(1+2\beta)}F\left(2\beta,1+2\beta,1+2\beta;\frac{x-\varepsilon-s}{x-\varepsilon}\right)=$$

$$=\frac{(x-s)^{-1-2\beta}(x-\varepsilon-s)^{2\beta}}{2\beta}\left(1-\frac{x-\varepsilon-s}{x-\varepsilon}\right)^{-2\beta}=$$

$$=\frac{(x-s)^{-1}(x-\varepsilon-s)^{2\beta}}{2\beta}\varepsilon^{-2\beta}=$$

$$=\frac{1}{\mathrm{B}(2\beta,1-2\beta)}\left[\varepsilon^{-2\beta}\int_{-1}^{x-\varepsilon}\frac{\tau(s)ds}{(x-\varepsilon-s)^{1-2\beta}}-\varepsilon^{-2\beta}\int_{-1}^{x-\varepsilon}\frac{(x-\varepsilon-s)^{2\beta}}{x-s}\tau(s)ds\right]=$$

$$=\frac{1}{\mathrm{B}(2\beta,1-2\beta)}\left[\varepsilon^{-2\beta}\int_{-1}^{x-\varepsilon}\tau(s)(x-\varepsilon-s)^{2\beta}\left(\frac{1}{x-\varepsilon-s}-\frac{1}{x-s}\right)ds\right]=$$

$$=\frac{1}{\mathrm{B}(2\beta,1-2\beta)}\left[\varepsilon^{1-2\beta}\int_{-1}^{x-\varepsilon}\tau(x)\frac{(x-\varepsilon-s)^{2\beta-1}}{x-s}\right]=$$

$$=\frac{1}{\mathrm{B}(2\beta,1-2\beta)}\left[\varepsilon^{1-2\beta}\tau(x)\int_{-1}^{x-\varepsilon}\frac{(x-\varepsilon-s)^{2\beta-1}ds}{x-s}+\varepsilon^{1-2\beta}\int_{-1}^{x-\varepsilon}\frac{\tau(s)-\tau(x)}{x-s}(x-\varepsilon-s)^{2\beta-1}ds\right]$$

Quyidagi

$$\int_{-1}^{x-\varepsilon}\frac{(x-\varepsilon-s)^{2\beta-1}}{x-s}ds$$

integralni hisoblash uchun

$$s=-1+(x-\varepsilon+1)\sigma$$

$$x-s=1+x-(x-\varepsilon+1)\sigma=(1+x)\left(1-\frac{1+x-\varepsilon}{1+x}\sigma\right)$$

$$x-\varepsilon-s=(x-\varepsilon+1)(1-\sigma)$$

almashtirishlarni bajaramiz

$$=\int_0^1 \frac{(x-\varepsilon+1)^{2\beta-1}(1-\sigma)^{2\beta-1}(x-\varepsilon+1)d\sigma}{(1+x)\left(1-\frac{1-x-\varepsilon}{1+x}\sigma\right)}=$$

$$=\frac{(x-\varepsilon+1)^{2\beta}}{1+x}\int_0^1 (1-\sigma)^{2\beta-1}\left(1-\frac{1+x-\varepsilon}{1+x}\sigma\right)^{-2}=$$

$$=\frac{(x-\varepsilon+1)^{2\beta}}{1+x}\frac{\Gamma(1)\Gamma(2\beta)}{\Gamma(1+2\beta)}F\left(1,1,1+2\beta;\frac{(1+x-\varepsilon)}{1+x}\right)=$$

$$=\frac{(x-\varepsilon+1)^{2\beta}}{1+x}\left(1-\frac{1+x-\varepsilon}{1+x}\right)^{2\beta-1}F\left(2\beta,2\beta,1+2\beta;\frac{(1+x-\varepsilon)}{1+x}\right)=$$

$$=\frac{(x-\varepsilon+1)^{2\beta}}{1+x}\frac{\varepsilon^{2\beta-1}}{(1+x)^{2\beta-1}}\frac{1}{2\beta}\frac{\Gamma(1+2\beta)\Gamma(1-2\beta)}{\Gamma(1)\Gamma(2)}=$$

$$=\left(\frac{(x-\varepsilon+1)}{1+x}\right)^{2\beta}\varepsilon^{2\beta-1}\Gamma(1-2\beta)\Gamma(2\beta)=$$

$$=\left(\frac{(x-\varepsilon+1)}{1+x}\right)^{2\beta}\varepsilon^{2\beta-1}B(2\beta,1-2\beta)$$

u holda quyidagi munosabatga kelamiz

$$\frac{1}{B(2\beta,1-2\beta)}\left[\varepsilon^{1-2\beta}\tau(x)\left(\frac{x-\varepsilon+1}{1+x}\right)^{2\beta}\varepsilon^{2\beta-1}B(2\beta,1-2\beta)\right]=\tau(x)$$

(2.29) xossadan foydalanib ushbu tenglikka ega bo‘lamiz

$$\gamma\, a(x)\tau(x)=-k_2(1-\beta_0)\frac{m+2}{2}\times$$

$$\times\left\{\tau(1)D_{-1,x}^{2\beta-1}(1-x)^{2\beta-1}b(x)+\tau(-1)D_{-1,x}^{2\beta-1}(1+x)^{2\beta-1}b(x)+D_{-1,x}^{2\beta-1}\left(b(x)\int_{-1}^{1}\frac{(x-t)\tau'(t)dt}{|x-t|^{2-2\beta}}\right)-\right. \qquad (2.30)$$

$$-(2\beta-1)D_{-1,x}^{2\beta-1}\left(b(x)\int_{-1}^{1}\frac{\tau(t)dt}{(1-xt)^{2-2\beta}}\right)-$$

$$\left.-2(1-\beta)D_{-1,x}^{2\beta-1}\left((1-x^2)b(x)\int_{-1}^{1}K_0(x,t)\tau(t)dt\right)\right\}+\Phi_0(x),$$

bu erda

$$\Phi_0(x)=D_{-1,x}^{2\beta-1}(b(x)\,f_1(x)-\psi_2(x)) \qquad (2.31)$$

(2.30)- tenglikdagi

$$I_1(x) = D_{-1,x}^{2\beta-1}\left(b(x)\int_{-1}^{1}\frac{(x-t)\tau'(t)dt}{|x-t|^{2-2\beta}} \right) \tag{2.32}$$

$$I_2(x) = D_{-1,x}^{2\beta-1}\left(b(x)\int_{-1}^{1}\frac{\tau(t)dt}{(1-xt)^{2-2\beta}} \right) \tag{2.33}$$

ifodalarni hisoblaymiz.

Dastlab $I_0(x) = \int_{-1}^{1}\frac{(s-t)\tau'(t)dt}{|s-t|^{2-2\beta}}$ integralni hisoblaymiz

$$I_0(x) = \int_{-1}^{1}\frac{(s-t)\tau'(t)dt}{|s-t|^{2-2\beta}} = \lim_{\varepsilon\to 0}\left(\int_{-1}^{s-\varepsilon}\frac{\tau'(t)dt}{(s-t)^{1-2\beta}} - \int_{s+\varepsilon}^{1}\frac{\tau'(t)dt}{(t-s)^{1-2\beta}} \right),$$

bu erda bo‘laklab integrallash formulalaridan foydalanamiz

$$I_1(x) = \lim_{\varepsilon\to 0}\left(\tau(x-\varepsilon)\varepsilon^{2\beta-1} - \tau(-1)(1+s)^{2\beta-1} + \tau(s+\varepsilon)\varepsilon^{2\beta-1} + (2\beta-1)\int_{-1}^{s-\varepsilon}\frac{\tau(t)dt}{(s-t)^{2-2\beta}} - \right.$$
$$\left. - \tau(1)(1-s)^{2\beta-1} + \tau(s+\varepsilon)\varepsilon^{2\beta-1} + (2\beta-1)\int_{s+\varepsilon}^{1}\frac{\tau(t)dt}{(t-s)^{2-2\beta}} \right).$$

Ushbu tengliklarga asosan

$$(2-2\beta)\int_{-1}^{s-\varepsilon}\frac{\tau(t)dt}{(s-t)^{2-2\beta}} = \frac{d}{ds}\int_{-1}^{s-\varepsilon}\frac{\tau(t)dt}{(s-t)^{1-2\beta}} -$$
$$-\tau(s-\varepsilon)\varepsilon^{2\beta-1}(2\beta-1)\int_{s+\varepsilon}^{1}\frac{\tau(t)dt}{(t-s)^{2-2\beta}} = -\frac{d}{ds}\int_{s+\varepsilon}^{1}\frac{\tau(t)dt}{(t-s)^{1-2\beta}} - \tau(s+\varepsilon)\varepsilon^{2\beta-1},$$

bu tengliklarga asosan $I_0(x)$ da $\varepsilon \to 0$ limitga o‘tib ushbu tenglikka kelamiz

$$I_0(x)\int_{-1}^{1}\frac{(s-t)\tau'(t)dt}{|s-t|^{2-2\beta}} = \tau(-1)(1+s)^{2\beta-1} + \frac{d}{ds}\int_{-1}^{s}\frac{\tau(t)dt}{(s-t)^{1-2\beta}} -$$
$$-\tau(1)(1-s)^{2\beta-1} - \frac{d}{ds}\int_{s}^{1}\frac{\tau(t)dt}{(t-s)^{1-2\beta}}. \tag{2.34}$$

Endi (2.32) va (2.33) formulalarni isbotlaymiz.

1. $I_1(x)$ hisoblaymiz

$$I_1(x)=D_{-1,x}^{2\beta-1}\left(b(x)\int_{-1}^{1}\frac{(x-t)\tau'(t)}{|x-t|^{2-2\beta}}\right)=\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{x}\frac{b(s)dt}{(x-s)^{2\beta}}\int_{-1}^{1}\frac{(s-t)\tau'(t)}{|s-t|^{2-2\beta}}dt=$$

$$=\lim_{\varepsilon\to0}\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{x}\frac{b(s)ds}{(x-s)^{2\beta}}\left(\int_{-1}^{s-\varepsilon}\frac{\tau'(t)}{(s-t)^{1-2\beta}}-\int_{s+\varepsilon}^{1}\frac{\tau'(t)}{(t-s)^{1-2\beta}}\right)=$$

$$=\lim_{\varepsilon\to0}\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{x}\frac{b(s)ds}{(x-s)^{2\beta}}\left\{\tau(t)(s-t)^{2\beta-1}\Big|_{-1}^{s-\varepsilon}+(2\beta-1)\int_{-1}^{s-\varepsilon}\frac{\tau(t)dt}{(t-s)^{2-2\beta}}-\right.$$

$$\left.-\tau(t)(t-s)^{2\beta-1}\Big|_{s+\varepsilon}^{1}+(2\beta-1)\int_{s+\varepsilon}^{1}\frac{\tau(t)dt}{(t-s)^{2-2\beta}}\right\}=$$

$$=-\frac{\tau(-1)}{\Gamma(1-2\beta)}\int_{-1}^{x}\frac{b(s)(1+s)^{2\beta-1}}{(x-s)^{2\beta}}ds-\frac{\tau(1)}{\Gamma(1-2\beta)}\int_{-1}^{x}\frac{b(s)(1+s)^{2\beta-1}}{(x-s)^{2\beta}}ds+$$

$$+\lim_{\varepsilon\to0}\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{x}\frac{b(s)ds}{(x-s)^{2\beta}}\times$$

$$\times\left\{\tau(s-\varepsilon)\varepsilon^{2\beta-1}+\tau(s+\varepsilon)\varepsilon^{2\beta-1}+\frac{d}{ds}\int_{-1}^{s-\varepsilon}\tau(t)(s-t)^{2\beta-1}dt-\right.$$

$$\left.-\tau(s-\varepsilon)\varepsilon^{2\beta-1}-\frac{d}{ds}\int_{s+\varepsilon}^{1}\tau(t)(t-s)^{2\beta-1}dt-\tau(s+\varepsilon)\varepsilon^{2\beta-1}\right\}.$$

SHunday qilib ushbu tenglikni hosil qilamiz

$$I_1(x)=-\tau(-1)D_{-1,x}^{2\beta-1}b(x)(1+x)^{2\beta-1}-$$

$$-\tau(-1)D_{-1,x}^{2\beta-1}b(x)(1-x)^{2\beta-1}+$$

$$+\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{x}\frac{b(s)ds}{(x-s)^{2\beta}}\frac{d}{ds}\int_{-1}^{s}\frac{\tau(t)dt}{(s-t)^{1-2\beta}}-$$

$$-\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{x}\frac{b(s)ds}{(x-s)^{2\beta}}\frac{d}{ds}\int_{-1}^{s}\frac{\tau(t)dt}{(t-s)^{1-2\beta}},$$

yoki

$$I_1(x) = -\tau(-1)D_{-1,x}^{2\beta-1}b(s)(1+s)^{2\beta-1} -$$

$$-\tau(1)D_{-1,x}^{2\beta-1}b(s)(1-s)^{2\beta-1} +$$

$$+\frac{b(x)}{\Gamma(1-2\beta)}\int_{-1}^{x}\frac{ds}{(x-s)^{2\beta}}\frac{d}{ds}\int_{-1}^{s}\frac{\tau(t)dt}{(s-t)^{1-2\beta}} -$$

$$-\frac{b(x)}{\Gamma(1-2\beta)}\int_{-1}^{x}\frac{ds}{(x-s)^{2\beta}}\frac{d}{ds}\int_{s}^{1}\frac{\tau(t)dt}{(t-s)^{1-2\beta}} +$$

$$+\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{x}\frac{b(s)-b(x)}{(x-s)^{2\beta}}\frac{d}{ds}\int_{-1}^{s}\frac{\tau(t)dt}{(s-t)^{1-2\beta}} - \qquad (2.35)$$

$$-\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{x}\frac{b(s)-b(x)}{(x-s)^{2\beta}}\frac{d}{ds}\int_{s}^{1}\frac{\tau(t)dt}{(t-s)^{1-2\beta}}.$$

Ushbu belgilashlarni kiritib (2.35) tenglikni quyidagi ko‘rinishda yozib olamiz.

$$A_{-1,x} = \frac{1}{\Gamma(1-2\beta)}\int_{-1}^{x}\frac{b(s)-b(x)}{(x-s)^{2\beta}}\frac{d}{ds}\int_{-1}^{s}\frac{\tau(t)dt}{(s-t)^{1-2\beta}}, \qquad (2.36)$$

$$A_{x,1} = \frac{1}{\Gamma(1-2\beta)}\int_{-1}^{x}\frac{b(s)-b(x)}{(x-s)^{2\beta}}\frac{d}{ds}\int_{-1}^{s}\frac{\tau(t)dt}{(t-s)^{1-2\beta}}, \qquad (2.37)$$

(2.36) va (2.37) tengliklarga asosan

$$I_1(x) = -\tau(-1)D_{-1,x}^{2\beta-1}b(x)(1+x)^{2\beta-1} -$$

$$-\tau(1)D_{-1,x}^{2\beta-1}b(x)(1-x)^{2\beta-1} + \qquad (2.38)$$

$$+\Gamma(2\beta)b(x)D_{-1,x}^{2\beta-1}D_{-1,x}^{2\beta-1}\tau(x) +$$

$$+\Gamma(2\beta)b(x)D_{-1,x}^{2\beta-1}D_{x,1}^{1-2\beta}\tau(x) + A_{-1,x} - A_{x,1}.$$

Ushbu

$$D_{a,x}^{-\alpha}D_{t,b}^{\alpha}\Phi(t)=\cos\alpha\,\Phi(x)-\frac{\sin\pi\alpha}{\pi}\int_a^b\left(\frac{x-a}{t-a}\right)^{\alpha}\frac{\Phi(t)dt}{t-x},$$

$$D_{-1,x}^{2\beta-1}D_{x,1}^{1-2\beta}\tau(x)=\cos(1-2\beta)\pi\,\tau(x)-\frac{\sin(1-2\beta)\pi}{\pi}\int_{-1}^{1}\left(\frac{x+1}{t+1}\right)^{1-2\beta}\frac{\tau(t)}{t-x}dt,$$

formulalarga asosan $I_1(x)$ uchun yakuniy formulani hosil qilamiz.

$$I_1(x)=-\tau(-1)D_{-1,x}^{2\beta-1}b(x)(1+x)^{2\beta-1}-\tau(1)D_{-1,x}^{2\beta-1}b(x)(1-x)^{2\beta-1}+$$

$$+\Gamma(2\beta)b(x)\tau(x)+\Gamma(2\beta)b(x)\cos(1-2\beta)\pi\,\tau(x)-$$

$$-\Gamma(2\beta)\frac{\sin(1-2\beta)\pi}{\pi}b(x)\int_{-1}^{1}\left(\frac{x+1}{t+1}\right)^{1-2\beta}\frac{\tau(t)}{t-x}dt+ \qquad (2.39)$$

$$+A_{-1,x}-A_{x,1}.$$

Endi $I_2(x)$ ni hisoblaymiz

$$D_{-1,x}^{2\beta-1}\left(b(x)\int_{-1}^{1}\frac{\tau(t)dt}{(1-xt)^{2-\beta}}dt\right)=\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{x}\frac{b(s)ds}{(x-s)^{2\beta}}\int_{-1}^{1}\frac{\tau(t)dt}{(1-st)^{2-2\beta}}=$$

$$=\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{1}\tau(t)\int_{-1}^{x}\frac{b(s)-b(x)}{(x-s)^{2\beta}(1-st)^{2-2\beta}}ds+$$

$$+\frac{b(x)}{\Gamma(1-2\beta)}\int_{-1}^{1}\tau(t)dt\int_{-1}^{x}\frac{ds}{(x-s)^{2\beta}(1-st)^{2-2\beta}}=$$

Bu erda quyidagi shakl almashtirishlarni bajarib

$$s=-1+(1+x)\sigma,\quad ds=(1+x)d\sigma$$

$$(x-s)=(1+x)(1-\sigma)$$

$$1-st=1+t-t(1+x)\sigma=(1+t)\left(1-\frac{t(1+x)}{1+t}\sigma\right)$$

$$I_2(x)=\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{1}\tau(t)dt\int_{-1}^{x}\frac{b(s)-b(x)}{(x-s)^{2\beta}(1-st)^{2-2\beta}}ds+$$

$$+\frac{b(x)}{\Gamma(1-2\beta)}\int_{-1}^{1}\tau(t)dt\int_{0}^{1}(1+x)^{-2\beta}(1-\sigma)^{-2\beta}(1+t)^{2\beta-2}\left(1-\frac{t(1+x)}{1+t}\sigma\right)^{2\beta-2}(1+t)d\sigma=$$

$$=\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{1}\tau(t)dt\int_{-1}^{x}\frac{b(s)-b(x)}{(x-s)^{2\beta}(1-st)^{2-2\beta}}ds+$$

$$+\frac{b(x)}{\Gamma(1-2\beta)}\int_{-1}^{1}\tau(t)(1+x)^{1-2\beta}(1+t)^{2\beta-2}\int_{0}^{1}(1-\sigma)^{-2\beta}\left(1-\frac{t(1+x)}{1+t}\sigma\right)^{2\beta-2}d\sigma=$$

$$=\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{1}\tau(t)dt\int_{-1}^{x}\frac{b(s)-b(x)}{(x-s)^{2\beta}(1-st)^{2-2\beta}}ds+$$

$$+\frac{b(x)}{\Gamma(1-2\beta)}\int_{-1}^{1}\tau(t)(1+x)^{1-2\beta}(1+t)^{2\beta-2}\frac{\Gamma(1)\Gamma(1-2\beta)}{\Gamma(2-2\beta)}F\left(1,2-2\beta,2-2\beta;\frac{t(1+x)}{1+t}\right)dt=$$

$$=\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{1}\tau(t)dt\int_{-1}^{x}\frac{b(s)-b(x)}{(x-s)^{2\beta}(1-st)^{2-2\beta}}ds+$$

$$+\frac{b(x)}{(1-2\beta)\Gamma(1-2\beta)}\int_{-1}^{1}\tau(t)(1+x)^{1-2\beta}(1+t)^{2\beta-2}\left(1-\frac{t+tx}{1+t}\right)^{-1}.$$

SHunday qilib quyidagi munosabatga kelamiz

$$I_2(x)=\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{1}\tau(t)\int_{-1}^{x}\frac{b(s)-b(x)}{(x-s)^{2\beta}(1-st)^{2-2\beta}}ds+\frac{b(x)}{\Gamma(2-2\beta)}\int_{-1}^{1}\tau(t)\left(\frac{1+x}{1+t}\right)^{1-2\beta}\frac{dt}{1-tx}. \quad (2.39)$$

(2.38) va (2.39) formulalarga ko‘ra (2.30) tenglikni quyidagicha yozib olamiz

$$a(x)\tau(x) = -k_2 \frac{m+2}{2} \tau(1)\gamma D_{-1,x}^{2\beta-1}(1-x)^{2\beta-1} b(x) -$$

$$-k_2 \frac{m+2}{2} \tau(-1)\gamma D_{-1,x}^{2\beta-1}(1+x)^{2\beta-1} b(x) +$$

$$+k_2 \frac{m+2}{2} \gamma\tau(-1) D_{-1,x}^{2\beta-1} b(x)(1+x)^{2\beta-1} +$$

$$+k_2 \frac{m+2}{2} \gamma \tau(1) D_{-1,x}^{2\beta-1} b(x)(1-x)^{2\beta-1} - k_2 \frac{m+2}{2} \gamma \Gamma(2\beta) b(x)(1+\cos(1-2\beta)\pi)\tau(x) +$$

$$+k_2 \frac{m+2}{2} \gamma\Gamma(2\beta) \frac{\sin(1-2\beta)}{\pi} b(x) \int_{-1}^{1} \left(\frac{x+1}{t+1} \right)^{1-2\beta} \frac{\tau(t)}{t-x} dt -$$

$$-k_2 \frac{m+2}{2} \gamma (A_{-1,x} - A_{x,1}) + k_2 \frac{m+2}{2} (2\beta-1)\gamma \frac{b(x)}{\Gamma(2-2\beta)} \int_{-1}^{1} \left(\frac{1+x}{1+t} \right)^{1-2\beta} \frac{\tau(t)dt}{1-xt} + \qquad (2.40)$$

$$+k_2 \frac{m+2}{2} (2\beta-1)\gamma \frac{1}{\Gamma(1-2\beta)} \int_{-1}^{1} \tau(t) \int_{-1}^{x} \frac{b(s)-b(x)}{(x-s)^{2\beta}(1-st)^{2-2\beta}} ds +$$

$$-k_2(1-\beta)(m+2)\gamma D_{-1,x}^{2\beta-1}(1-x^2) b(x) \int_{-1}^{1} K_0(x,t)\tau(t)dt + \Phi_0(x).$$

Ushbu tenglikka asosan

$$(2\beta-1)\frac{1}{\Gamma(2-2\beta)} = -\frac{\Gamma(2\beta)}{\Gamma(2\beta)\Gamma(2-2\beta)} =$$

$$= -\frac{\Gamma(2\beta)}{\Gamma(1-(1-2\beta))\Gamma(1-2\beta)} = -\Gamma(2\beta)\frac{\sin(1-2\beta)\pi}{\pi},$$

(2.40) tenglikni quyidagicha yozib olamiz

$$\left[a(x)+k_2\frac{m+2}{2}\gamma\,\Gamma(2\beta)b(x)(1+\cos(1-2\beta)\pi)\right]\tau(x)=$$

$$=k_2\frac{m+2}{2}\gamma\,\Gamma(2\beta)\frac{\sin(1-2\beta\pi)}{\pi}b(x)\int_{-1}^{1}\left(\frac{x+1}{t+1}\right)^{1-2\beta}\left(\frac{1}{t-x}-\frac{1}{1-xt}\right)\tau(t)dt- \qquad (2.42)$$

$$-k_2\frac{m+2}{2}(A_{-1,x}-A_{x,1})+k_2\frac{m+2}{2}(2\beta-1)\gamma\frac{1}{\Gamma(2-2\beta)}\int_{-1}^{1}\tau(t)dt\int_{-1}^{x}\frac{b(s)-b(x)}{(x-s)^{2\beta}(1-st)^{2-2\beta}}ds+$$

$$+k_2(1-\beta)(m+2)\gamma\, D_{-1,x}^{2\beta-1}(1-x^2)b(x)\int_{-1}^{1}K_0(x,t)\tau(t)dt+F(x).$$

Endi $A_{1,x}$ ni hisoblash uchun bu integralni bo‘laklab integrallaymiz

$$A_{-1,x}=\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{x}\frac{b(s)-b(x)}{(x-s)^{2\beta}}\frac{d}{ds}\int_{-1}^{s}\frac{\tau(t)dt}{(s-t)^{1-2\beta}}$$

$$u=\frac{b(s)-b(x)}{(x-s)^{2\beta}},\ d\vartheta=\int_{-1}^{s}\frac{\tau(t)}{(s-t)^{1-2\beta}}dt,$$

$$du=2\beta\frac{b(s)-b(x)}{(x-s)^{1+2\beta}}+\frac{b'(s)}{(x-s)^{2\beta}},\ \vartheta=\int_{-1}^{s}\frac{\tau(t)dt}{(s-t)^{1-2\beta}}$$

$$A_{-1,x}=\frac{1}{\Gamma(1-2\beta)}\left[\frac{b(s)-b(x)}{(x-s)^{2\beta}}\int_{-1}^{s}\frac{\tau(t)dt}{(s-t)^{1-2\beta}}\right|_{-1}^{x}-$$

$$-\int_{-1}^{x}\left[2\beta\frac{b(s)-b(x)}{(x-s)^{1+2\beta}}+\frac{b'(s)}{(x-s)^{2\beta}}\right]\int_{-1}^{s}\frac{\tau(t)dt}{(s-t)^{1-2\beta}}ds\right]=$$

$$=-\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{x}\left[2\beta\frac{b(s)-b(x)}{(x-s)^{1+2\beta}}+\frac{b'(s)}{(x-s)^{2\beta}}\right]ds\int_{-1}^{s}\frac{\tau(t)dt}{(s-t)^{1-2\beta}}= \qquad (2.43)$$

$$=\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{x}\tau(t)dt\int_{t}^{x}\left[2\beta\frac{b(s)-b(x)}{(x-s)^{1+2\beta}}+\frac{b'(s)}{(x-s)^{2\beta}}\right]ds\frac{ds}{(s-t)^{1-2\beta}}.$$

$A_{x,1}$ ni hisoblaymiz

$$A_{x,1}(x)=\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{x}\frac{b(s)-b(x)}{(x-s)^{2\beta}}ds\frac{d}{ds}\int_{s}^{1}\frac{\tau(t)dt}{(t-s)^{1-2\beta}}.$$

$$u=\frac{b(s)-b(x)}{(x-s)^{2\beta}},\quad d\vartheta=\int_{s}^{1}\frac{\tau(t)dt}{(t-s)^{1-2\beta}},$$

$$du=2\beta\frac{b(s)-b(x)}{(x-s)^{1+2\beta}}+\frac{b'(s)}{(x-s)^{2\beta}},\vartheta=\int_{s}^{1}\frac{\tau(t)dt}{(t-s)^{1-2\beta}},$$

$$A_{x,1}=\frac{1}{\Gamma(1-2\beta)}\left[\frac{b(s)-b(x)}{(x-s)^{2\beta}}\int_{s}^{1}\frac{\tau(t)dt}{(t-s)^{1-2\beta}}\right|_{-1}^{x}-$$

$$-\int_{-1}^{x}\left[2\beta\frac{b(s)-b(x)}{(x-s)^{1+2\beta}}+\frac{b'(s)}{(x-s)^{2\beta}}\right]\int_{s}^{1}\frac{\tau(t)dt}{(t-s)^{1-2\beta}}\right]=$$

$$=-\frac{1}{\Gamma(1-2\beta)}\frac{b(-1)-b(x)}{(x+1)^{2\beta}}\int_{-1}^{1}\frac{\tau(t)dt}{(1+t)^{1-2\beta}}-\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{x}\left[2\beta\frac{b(s)-b(x)}{(x-s)^{1+2\beta}}+\frac{b'(s)}{(x-s)^{2\beta}}\right]ds\times$$

$$\times\int_{s}^{1}\frac{\tau(t)dt}{(t-s)^{1-2\beta}}=-\frac{1}{\Gamma(1-2\beta)}\frac{b(-1)-b(x)}{(1+x)^{2\beta}}\int_{-1}^{1}\frac{\tau(t)dt}{(1+t)^{1-2\beta}}- \qquad (2.44)$$

$$-\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{x}\tau(t)dt\int_{-1}^{t}\left[2\beta\frac{b(s)-b(x)}{(x-s)^{1+2\beta}}+\frac{b'(s)}{(x-s)^{2\beta}}\right]\frac{ds}{(t-s)^{1-2\beta}}-$$

$$-\frac{1}{\Gamma(1-2\beta)}\int_{x}^{1}\tau(t)dt\int_{-1}^{x}\left[2\beta\frac{b(s)-b(x)}{(x-s)^{1+2\beta}}+\frac{b'(s)}{(x-s)^{2\beta}}\right]\frac{ds}{(t-s)^{1-2\beta}}.$$

SHunday qilib

$$A_{-1,x}-A_{x,1}=-\frac{1}{\Gamma(1-2\beta)}\left[\int_{-1}^{x}\tau(t)dt\left[\int_{t}^{x}\left[2\beta\frac{b(s)-b(x)}{(x-s)^{1+2\beta}}+\frac{b'(s)}{(x-s)^{2\beta}}\right]\frac{ds}{(s-t)^{1-2\beta}}-\right.\right.$$

$$\left.\left.-\int_{-1}^{t}\left[2\beta\frac{b(s)-b(x)}{(x-s)^{1+2\beta}}+\frac{b'(s)}{(x-s)^{2\beta}}\right]\frac{ds}{(t-s)^{1-2\beta}}\right]-\int_{x}^{1}\tau(t)dt\int_{-1}^{x}\left[2\beta\frac{b(s)-b(x)}{(x-s)^{1+2\beta}}+\frac{b'(s)}{(x-s)^{2\beta}}\right]\frac{ds}{(t-s)^{1-2\beta}}\right]+ \qquad (2.45)$$

$$+\frac{1}{\Gamma(1-2\beta)}\frac{b(-1)-b(x)}{(1+x)^{2\beta}}\int_{-1}^{1}\frac{\tau(t)dt}{(1+t)^{1-2\beta}}=-\frac{1}{\Gamma(1-2\beta)}\int_{-1}^{1}\tau(t)dt\int_{-1}^{x}\left[2\beta\frac{b(s)-b(x)}{(x-s)^{1+2\beta}}+\frac{b'(s)}{(x-s)^{2\beta}}\right]ds\times$$

$$\times\frac{\sin g(s-t)}{|s-t|^{1-2\beta}}+\frac{1}{\Gamma(1-2\beta)}\frac{b(-1)-b(x)}{(1+x)^{2\beta}}\int_{-1}^{1}\frac{\tau(t)dt}{(1+t)^{1-2\beta}}.$$

(2.34), (2.43) va (2.44) formulalarga asosan (2.30) tenglikni ushbu ko‘rinishda yozib olamiz

$$A(x)\tau(x)+\frac{\mathrm{B}(x)}{\pi i}\int_{-1}^{1}\left(\frac{1+x}{1+t}\right)^{1-2\beta}\left(\frac{1}{t-x}-\frac{1}{1-xt}\right)\tau(t)dt=$$
$$=\int_{-1}^{1}K(x,t)\tau(t)dt+F(x), \qquad (2.46)$$

bu erda

$$A(x) = a(x) + \sin\beta\pi\, b(x),\ \mathrm{B}(x) = i\cos\beta\pi\, b(x), \tag{2.47}$$

$$K(x,t) = \frac{\cos\beta\pi}{\pi}\int_{-1}^{x}\left\{\left[2\beta\frac{b(s)-b(x)}{x-s} + b'(s)\right]\left(\frac{s-t}{|s-t|^{2-2\beta}} - \right.\right.$$

$$\left.\left. - (1-2\beta)\frac{b(s)-b(x)}{(1-st)^{2-2\beta}} + 2(1-\beta)\left(1-s^2\right)b(s)K_0(s,t)\right)\frac{ds}{(x-s)^{2\beta}} - \right. \tag{2.48}$$

$$- \frac{\cos\beta\pi\,(b(-1)-b(x))}{\pi(1+x)^{2\beta}(1+t)^{1-2\beta}},$$

$$F(x) = \Phi_0(x)/\gamma,$$

(2.46)- tenglama F. Trikomining Singulyar integral tenglamasidan iborat, bu tenglamani o‘rganamiz.

Ushbu tenglikni to‘g‘riligini tekshirish qiyin emas

$$\left(\frac{1+t}{1+x}\right)\left(\frac{1}{t-x} - \frac{1}{1-xt}\right) = \frac{1}{t-x} - \frac{t}{1-xt} = \frac{2\left(1-t^2\right)}{\left(1+t^2\right)\left(1+x^2\right)\left(\frac{2t}{1+t^2} - \frac{2x}{1+x^2}\right)}. \tag{2.49}$$

(2.49) tenglikka asosan (2.46) tenglamani quyidagi ko‘rinishda yozib olamiz

$$A(x)\tau(x) + \frac{\mathrm{B}(x)}{\pi i}\int_{-1}^{1}\left(\frac{1+t}{1+x}\right)^{2\beta-2}\frac{2(1-t^2)\tau(t)dt}{\left(1+x^2\right)\left(1+t^2\right)\left(\frac{2t}{1+t^2} - \frac{2x}{1+x^2}\right)} = \tag{2.50}$$

$$= \int_{-1}^{1}K(x,t)\tau(t)dt + F(x).$$

Endi (2.50) tenglamada ushbu almashtirishlarni bajaramiz

$$s = \frac{2t}{1+t^2},\ t = \frac{s}{1+\sqrt{1-s^2}};\ y = \frac{2x}{1+x^2},\ x = \frac{y}{1+\sqrt{1-y^2}},$$

va ushbu yangi noma'lum funksiyani kiritib

$$\rho(y)=(1+x)^{2\beta-2}\left(1+x^2\right)\tau(x) \tag{2.51}$$

(2.50) tenglamani ushbu ko'rinishda yozib olamiz

$$\overline{A}(y)\rho(y)+\frac{\overline{\mathrm{B}}(y)}{\pi i}\int_{-1}^{1}\frac{\rho(s)ds}{s-y}=\int_{-1}^{1}\widetilde{K}(y,s)\rho(s)ds+\widetilde{F}(y) \tag{2.52}$$

bu erda

$$\overline{A}(y)=A\left(\frac{y}{1+\sqrt{1-y^2}}\right),\ \overline{\mathrm{B}}(y)=\mathrm{B}\left(\frac{y}{1+\sqrt{1-y^2}}\right)$$

$$\widetilde{K}(x,y)=\frac{(1+x)^{2\beta-2}(1+x^2)K(x,t)}{2\sqrt{1-s^2}\,(1+t)^{2\beta-2}}$$

$$\widetilde{F}(y)=(1+x)^{2\beta-2}(1+x^2)F(x)$$

(2.52) tenglamani echishda Karleman-Vekua usulidan foydalanamiz va (2.52) tenglamaning o'ng tomonini vaqtinchalik ma'lum funksiya deb hisoblaymiz va uni

$$f(y)=\int_{-1}^{1}\widetilde{K}(y,s)\rho(s)ds+\widetilde{F}(y) \tag{2.53}$$

deb belgilab ushbu xarakteristik tenglamani hosil qilamiz.

$$\widetilde{A}(y)\rho(y)+\frac{\widetilde{\mathrm{B}}(y)}{\pi i}\int_{-1}^{1}\frac{\rho(s)ds}{s-y}=f(y), \tag{2.54}$$

bu erda

$$\widetilde{A}^2(y)-\widetilde{\mathrm{B}}^2(y)=a^2(x)+b^2(x)+2a(x)b(x)\sin\beta\pi\neq 0$$

bo'lganligi uchun (2.54) Singulyar integral tenglama normal tipdagi integral tenglamadir (2.54) integral tenglama yechimini $x=1$ nuqtada chegaralangan, $x=-1$ nuqtada esa cheksizlikka intilish mumkin bo'lgan $\rho(x)\in h$Gyolder sinfida izlaymiz. $h(1)$ sinfda (2.54) tenglamaning indeksini 0 ga teng ekanligini isbotlaymiz.

Haqiqatdan ham ushbu N.I.Musxeleshvili funksiyasini o‘rganamiz

$$G(z)=\frac{\widetilde{A}(z)-\widetilde{B}(z)}{\widetilde{A}(z)+\widetilde{B}(z)}=\frac{a(x)+\sin\beta\pi\, b(x)+i\cos\beta\pi\, b(x)}{a(x)+\sin\beta\pi\, b(x)+i\cos\beta\pi\, b(x)}$$

$G(z)$ ning logarifmini hisoblaymiz

$$\ln G(z)=\ln|G(z)|+i(\arg G(z)+2k\pi)=$$

$$=i[2\arg(a(x)+\sin\beta\pi\, b(x)+i\cos\beta\pi\, b(x))+2k\pi],$$

bu erda

$$a(x)>0, b(x)>0$$

bo‘lgani uchun

$$\ln G(z)=(2\varphi(x)+2k\pi)i$$

bu erda

$$\varphi(x)=arctg\,\frac{\cos\beta\pi\, b(x)}{a(x)+\sin\beta\pi\, b(x)}$$

N.I.Musxeleshvili formulasiga ko‘ra

$$\alpha_n+i\beta_n=\pm\frac{\ln(G(c_n))}{2\pi i},\quad n=0;1$$

bu erda «+» ishorasi $C_0=-1$ ga, ishorasi «-» $C_1=1$ ga mos keladi.

1. Ushbu hisoblashni bajaramiz

$$\alpha_0+i\beta_0=-\frac{\ln G(-1)}{2\pi i}=-\frac{\varphi(-1)}{\pi}-k,$$

bu erda $\varphi(-1)\in\left(0,\frac{\pi}{2}\right)$, ya’ni $\alpha_0=-\frac{\varphi(-1)}{\pi}-k$. λ_0 butun sonni shunday tanlaymizki $-1<\alpha_0+\lambda_0<0$, bo‘lsin, bundan, $\lambda_0=-k$.

2. Endi

$\alpha_1 + i\beta_1 = \dfrac{\ln G(1)}{2\pi i} = \dfrac{\varphi(1)}{\pi} + k$ ni hisoblaymiz.

Bu erda $\varphi(1) \in \left(0, \dfrac{\pi}{2}\right),$, ya'ni $\alpha_1 = \dfrac{\varphi(1)}{\pi} + k.$ Endi $\lambda_1$sonini shunday tanlaymizki $0 < \alpha_1 + \lambda_1 < 1$ tengsizlik bajarilsin, bu erdan, $\lambda_1 = -k$.

SHunday qilib, $h(1)$ sinfning indeksi

$\chi = -(\lambda_0 + \lambda_1) = -(k - k) = 0$ ga teng.

Shunday qilib, (2.54) Singulyar integral tenglama bir qiymatli yechimga ega. $h(1)$ sinfining kanonik funksiyasi

$$X(z) = (1+z)^{-\theta_0}(1-z)^{\theta_1}\omega_1(z)$$

bu erda

$$\theta_0 = \varphi(-1)/\pi, \ \ \theta_1 = \varphi(-1)/\pi, 0 < \theta_k < 1/2, k = 0{,}1, \omega_1(z),$$

chegaralangan noldan farqli funksiya.

(2.54) tenglamaga Karleman-Vekua usulini qo‘llab ushbu yechimga kelamiz.

$$\rho(y) = a^*(y) f(y) - \frac{b^*(y) z(y)}{\pi i} \int_{-1}^{1} \frac{f(s)ds}{z(s)(s-y)}, \tag{2.55}$$

bu erda

$$a^*(y) = \frac{\tilde{A}(y)}{\tilde{A}^2(y) - \tilde{B}^2(y)}, b^*(y) = \frac{\tilde{B}(y)}{\tilde{A}^2(y)\tilde{B}^2(y)}$$

$$z(y) = [\tilde{A}(y) + \tilde{B}(y)]X^+(y) = [\tilde{A}(y) - \tilde{B}(y)]X^-(y).$$

Endi (2.55) tenglikning o‘ng tomoniga $f(y)$ ning qiymatini (2.53) tenglikdan olib kelib qo‘ysak ushbu tenglamaga kelamiz

$$\rho(y) + \int_{-1}^{1} K_1(y,t)\rho(t)dt = F_1(y), \tag{2.56}$$

bu erda

$$K_1(y,t) = -a_1^*(y)\widetilde{K}(y,t) + \frac{b^*(y)z(y)}{\pi i}\int_{-1}^{1}\frac{\widetilde{K}(s,t)ds}{z(s)(s-y)},$$

$$F_1(y) = a^*(y)\widetilde{F}(y) - \frac{b^*(y)z(y)}{\pi i}\int_{-1}^{1}\frac{\widetilde{F}(s)ds}{z(s)(s-y)}.$$

Shunday qilib, ta’riflangan masala yechimi ekvivalent ravishda Fredgolmning ikkinchi tur integral tenglamasiga olib kelindi. Bu tenglamaning bir qiymatli yechimga ega ekanligi masala yechimi yagonaligidan kelib chiqadi.

II BOB YUZASIDAN XULOSALAR.

Soha ichida buziladigan singulyar koeffitsientli elliptik turdagi tenglama uchun umumiy ulash shartli masalaning qo‘yilishi , yechish metodikasi va bayoni deb ataluvchi II bobda soha ichida buziladigan singulyar koeffitsientli elliptik turdagi tenglama uchun umumiy ulashish shartli masalaning qo‘yilishi va yagonaligini ko`rsatishning metodikasi**,** masalasi: D_1 sohada (1) tenglamaning $C(\overline{D}_1)$ I $C^2(D_1)$ sinfga tegishli va ushbu

$$u(x,0)=\tau(x), \quad x\in\bar{I}$$

$$u\big|_{\Gamma}=\varphi_0(s),$$

shartlarni qanoatlantiruvchi regulyar yechimi, Shakli o‘zgargan Koshi masalasi, D_2 sohada (1) tenglamaning $C(\overline{D}_2)$ I $C^2(D_2)$ sinfga tegishli va ushbu

$$u(x,0)=\tau(x), \quad x\in\bar{I}$$

$$\lim_{y\to -0}(-y)^{\beta_0}\frac{\partial u}{\partial y}=\nu(x), \quad x\in I$$

shartlarni qanoatlantiruvchi yechimi topish.

T masalasi yechimining yagonaligi, T masalasi yechimining mavjudligini soddalik uchun Γ chiziq (2.1) tenglamaning normal chizig‘i

$$\sigma_0: x^2+\frac{4}{(m+2)^2}y^{m+2}=1$$

bilan ustma-ust tushgan holda o‘rgaib chiqdik.

III **BOB. SINGULYAR KOEFFITSIENTLI TENGLAMANING BIR SINFI UCHUN GELLERSTEDT VA BUZILISH CHIZIG'IDA UMUMIY ULANISH SHARTLI MASALA YECHIMINING ASOSIY NATIJALARI BAYONI.**

3.1§. **Singulyar integral tenglamalar sistemasini keltirib chiqarish.**

Ushbu $$sign y|y|^m U_{xx} + U_{yy} - \frac{m}{2y}U_y = 0$$

(3.1)

aralash turdagi tenglamani $z = x + iy$ kompleks tekisligining $\operatorname{Im} z > 0$ yarim tekisligida, shu yarim tekisligida yotuvchi va uchlari $A(-1,0)$, $B(1,0)$ nuqtalarda bo'lgan σ_0: $x^2 + \frac{4}{(m+2)^2}y^{m+2} = 1$ normal chiziq, $Imz < 0$ yarim tekislikda esa (3.1) tenglamaning AC va BC xarakteristikalar bilan chegaralangan bir bog'lamli chekli D sohada o'rganamiz[10,11,12,14,15,16].

Ushbu belgilashlarni kiritamiz. C_0 bu AC xarakteristika bilan O(0,0) nuqtadan chiquvchi xarakteristikaning kesishish nuqtasi bo'lsin, C_1 esa BC xarakteristikaning O nuqtadan chiquvchi xarakteristikaning kesishish nuqtasi .

$$D^+ = D \cap \{y > 0\}. \qquad D^- = D \cap \{y < 0\}$$

$\theta(x_0)$- orqali $M(x_0,0)$ nuqta orqali chiquvchi

$$x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = x_0$$

$$OC_1 : x - \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 0$$

xarakteristikaning kesishish nuqtalarining affiksini belgilaymiz.

Bu tenglamalarni qo'shib

$$2x = x_0 \ , \qquad x = \frac{x_0}{2}$$

tenglamalarni, ayirib esa

$$\frac{4}{m+2}(-4)^{\frac{m+2}{2}}=x_0\ ,\quad y=-\left(\frac{m+2}{4}x_0\right)\frac{2}{m+2}$$

formulaga kelamiz, ya'ni

$$\theta(x_0)=\frac{x_0}{2}-i\left(\frac{m+2}{4}x_0\right)^{\frac{2}{m+2}} \tag{3.2}$$

affikcni hosil qilamiz.

Ushbu ish Trikomi masalasining bir umumlashmasiga bag'ishlangan.

BF-masalasi. D aralash sohada (1.1) tenglamaning ushbu shartlarni qanoatlantiruvchi

1. $U(x,y)\in C(\overline{D})$

2. $U(x,y)\in C^2(D^+)$ va D^+ da (3.1) tenglamaning regulyar yechimi.

3. $U(x,y)$ D^- xarakteristik uchburchakda (3.1) tenglamaning R_1 – sinfga tegishli umumlashgan yechimi.

4. $U(x,y)$ funksiya ushbu shartlarni qanoatlantiradi.

$$U\Big|_{\sigma_0}=\varphi(x),\ x\in[-1,1] \tag{3.3}$$

$$U\Big|_{AC_0}=\psi(x),\ \ x\in[-1,0] \tag{3.4}$$

$$U[\theta(x)]-U[\theta(1-x)]=\psi_1(x),\ x\in[0,1] \tag{3.5}$$

$$U(x,0)-U(1-x,0)=f(x)\ ,\quad x\in[0,1] \tag{3.6}$$

$$\tau(x)-\tau(1-x)=f(x),\ \ x\in[-1,1] \tag{3.7}$$

Bu yerda $\varphi(x),\psi_0(x),\psi_1(x),f(x)$ yetarli darajadagi silliq funksiyalar.

$$f(1-x)=-f(x)\ ,\ \ \psi_1(1-x)=-\psi_1(x)$$

Agar $U(x,0)=\tau(x)$ deb belgilash kiritsak (3.6) Frankl sharti ushbu ko'rinishni oladi.

$$\tau(x)-\tau(1-x)=f(x) \qquad (1.6^*)$$

(1.6^*) sharti $x\rightarrow 1-\mathrm{X}$ desak

$$\tau(1-\mathrm{X})-\tau(\mathrm{X})=f(1-\mathrm{X})$$

bu yerda (31.5) va (3.6) shartlar mos ravishda OC_1 xarakteristika OB buzilish chizig‘ida berilgan Frankl shartlaridir.

BF-masalasi yechimining yagonaligi (3.1) – tenglamaning D^- sohada shakli o‘zgargan Koshi shartlari:

$$U(x,0)=\tau(x), x\in J : \lim_{y\to -0}(-y)^{\frac{-m}{2}}\frac{\partial U}{\partial y}=\nu(x)$$

Qanoatlantiruvchi yechimini beruvchi Dalamber formulasi ushbu ko‘rinishda bo‘ladi [5,6,21].

$$U(x;y)=\frac{1}{2}\left[\tau\left(x-\frac{2}{m+2}(-y)^{\frac{m+2}{2}}\right)+\tau\left(x+\frac{2}{m+2}(-y)^{\frac{m+2}{2}}\right)\right]- \tag{3.8}$$
$$-\frac{(-y)^{\frac{m+2}{2}}}{m+2}\int_{-1}^{1}\nu\left(x+\frac{2t}{m+2}(-y)^{\frac{m+2}{2}}\right)dt$$

Dalamber formulasidan (3.4) shartga asosan $U(x;y)\Big|_{AC}$ ni topamiz, ya’ni

$$1\ AC_0 : x-\frac{2}{m+2}(-y)^{\frac{m+2}{2}}=-1$$

bu yerda

$$\frac{2}{m+2}(-y)^{\frac{m+2}{2}}=1+x; (-y)^{\frac{m+2}{2}}=\frac{2(1+x)}{m+2}$$

$$y=-\left[\frac{2(1+x)}{m+2}\right]^{\frac{2}{m+2}}, x\in\left(-1,-\frac{1}{2}\right)$$

$$U(x;y)\Big|_{AC_0}=\frac{1}{2}[\tau[x-(1+x_1)]+\tau[x+(1+x)]]- \tag{3.9}$$
$$-\frac{1+x}{2}\int_{-1}^{1}\nu[x+(1+x)t]dt=\psi(x),\quad \mathrm{x}\in\left(-1,-\frac{1}{2}\right)$$

(3.9) tenglikda

$$x=1+2\mathrm{X},\quad \mathrm{X}=\frac{x-1}{2},\quad 1+\mathrm{X}=\frac{x+1}{2}$$

almashtirish bajaramiz [19], ya'ni $x\in\left[-1,-\frac{1}{2}\right]$ bo'lganda $x\in[-1,0]$ bo'ladi.

$$\frac{1}{2}[\tau(-1)+\tau(x)]-\frac{x+1}{4}\int_{-1}^{1}\nu\left[\frac{x-1}{2}+\frac{x+1}{2}\right]dt=\psi\left(\frac{x-1}{2}\right),\quad x\in[-1,0] \tag{3.10}$$

(3.10) dagi integralda

$$z=\frac{x-1}{2}+\frac{x+1}{2}t,\quad t=-1,z=-1$$

$$\partial z=\frac{2}{1+x}dt \qquad t=1,z=x$$

almashtirish bajaramiz.

$$\frac{1}{2}[\tau(-1)+\tau(x)]-\frac{x+1}{4}\int_{-1}^{x}\nu[z]\frac{2}{1+x}dt=\psi\left(\frac{x-1}{2}\right) \qquad x\in[-1,0]$$

$$\tau(-1)+\tau(x)-\int_{-1}^{x}\nu(z)=2x\left(\frac{x-1}{2}\right)$$

Bu yerda hosila olib $\tau'(x)-\nu(x)=\psi'\left(\frac{x-1}{2}\right)$ yoki $\nu(x)=\tau'(x)+\psi'\left(\frac{x-1}{2}\right)$

tenglikka kelamiz. Shunday qilib

$$\nu(x)=\tau'(x)+\psi'\left(\frac{x-1}{2}\right),\qquad x\in(-1,0) \tag{3.11}$$

tenglik noma'lum $\tau(x)$ va $\nu(x)$ o'rtasidagi birinchi funksional munosabatdir.

Endi ikkinchi funksional munosabatni topamiz. Buning uchun Dalamber formulasida $x\to\frac{x_0}{2},\quad y\to-\left(\frac{m+2}{4}x_0\right)^{\frac{2}{m+2}}$ ga almashtiramiz, ya'ni

$$U[\theta(x_0)]=\frac{1}{2}\left[\tau\left(\frac{x}{2}-\frac{2}{m+2}\cdot\frac{m+2}{4}x\right)+\tau\left(\frac{x}{2}+\frac{2}{m+2}\cdot\frac{m+2}{4}x\right)\right]-$$

$$-\frac{\frac{m+2}{4}x}{m+2}\int_{-1}^{1}\nu\left[\frac{x}{2}+\frac{2x}{m+2}\cdot\frac{m+2}{4}x\right]dt=$$

$$=\frac{1}{2}[\tau(0)+\tau(x)]-\frac{x}{4}\int_{-1}^{1}\nu\left(\frac{x}{2}+\frac{x}{2}t\right)dt\ . \tag{3.12}$$

Endi $U[\theta(1-x)]$ ni hisoblaymiz. Dalamber formulasida ushbu almashtirishni bajaramiz.

$$x \to \frac{1-x_0}{2}\ , \qquad y \to -\left(\frac{m+2}{4}(1-x_0)\right)^{\frac{2}{m+2}}$$

$$U[\theta(1-x)] = \frac{1}{2}\tau\left[\frac{1-x}{2} - \frac{2}{m+2}.\frac{m+2}{4}(1-x)\right] + \frac{1}{2}\tau\left[\frac{1-x}{2} + \frac{2}{m+2}.\frac{m+2}{4}(1-x)\right] -$$

(3.13)

$$-\frac{1-x}{4}\int_{-1}^{1}\nu\left(\frac{1-x}{2}+\frac{1-x}{2}t\right)dt = \frac{1}{2}\tau(0) + \frac{1}{2}\tau(1-x) - \frac{1-x}{4}\int_{-1}^{1}\nu\left(\frac{1-x}{2}+\frac{1-x}{2}t\right)dt.$$

Endi (3.12) va (3.13) ifodalarni (3.5) chegaraviy shartga qo‘yib ushbu tenglikka kelamiz.

$$\frac{1}{2}\tau(x) - \frac{x}{4}\int_{-1}^{1}\nu\left(\frac{x}{2}+\frac{x}{2}t\right)dt - \frac{1}{2}\tau(1-x) + \frac{1-x}{2}\int_{-1}^{1}\nu\left(\frac{1-x}{2}+\frac{1-x}{2}t\right)dt = \psi_1(x)\ \ , x \in (0,1)$$

(3.14)

(3.14) – tenglikning birinchi integralida

$$z = \frac{x}{2} + \frac{x}{2}t\,, \qquad t=-1\ , z=0$$

$$dz = \frac{x}{2}dt\,, \qquad t=1\ , z=x$$

ikkinchi integralida

$$z = \frac{1-x}{2} + \frac{1-x}{2}t\,, \qquad t=-1\ , z=0$$

$$dz = \frac{1-x}{2}dt\ , t=1\ , \quad z=1-x\ .$$

Almashtirishlarni bajaramiz. Natijada (3.14)- tenglik ushbu ko‘rinishni oladi

$$\tau(x) - \frac{x}{2}\int_0^x \nu(z)\cdot\frac{2}{x}dz - \tau(1-x) + \frac{1-x}{2}\int_0^{1-x}\nu(z)\frac{2}{1-x}dz = \psi_1(x) \qquad x \in (0,1)$$

ya’ni

$$\tau(x) - \int_0^x \nu(z)\cdot dz - \tau(1-x) + \int_0^{1-x}\nu(z)dz = \psi_1(x) \tag{3.15}$$

(3.15)-tenglikni differensiallab ushbu tenglikka kelamiz

$$\tau'(x)-\nu(x)+\tau'(1-x)-\nu(1-x)=\psi_1'(x) \qquad (3.16)$$

yoki

$$\tau'(x)+\tau'(1-x)=\nu(x)+\nu(1-x)+\psi_1'(x)$$

(1.6^*) shartdan $\tau'(x)+\tau'(1-x)=f_1'(x)$, bu tenglikka asosan oxirgi tenglamani ushbu ko‘rinishda yozib olamiz

$$\nu(x)+\nu(1-x)=f'(x)-\psi_1'(x) \quad ,x\in[0,1] \qquad (3.17)$$

(3.17) tenglik $[0,1]$ oraliqdagi ikkinchi funksional munosabatdir[10.

3.2-§. Trikomi integral tenglamasini regulyarizatsiyalash.

Teorema1. BF masalasining yechimi $\psi_0(x)\equiv 0$ $,\varphi(x)\equiv 0$ $,\psi_1(x)\equiv 0$ $, f(x)\equiv 0,$ shartlar bajarilganda yopiq $\overline{D}^+$ sohada $U(x,y)$ aynan nolga tengdir , ya’ni $U(x,y)\equiv 0$.

Isbot. Faraz qilaylik $U(x,y)$ funksiya 1-teorema shartlarini qanoatlantirsin va $U(x,y)\not\equiv 0$ bo‘lsin , u holda bu funksiya $\overline{D}^+$ o‘zining musbat maksimum va manfiy minimum qiymatlariga erishadi. Xopf prinsipiga ko‘ra $U(x,y)$ funksiya o‘zining eng katta musbat (EKM) qiymatini soha ichida qabul qila olmaydi. $U(x,y)$ funksiya o‘zining EKM qiymatini AB kesmaning $(x_0,0)$ nuqtada qabul qilsin.

Bu yerda ushbu 3 ta holni ko‘rib o‘tamiz $x_0\in(-1,0)$, $x_0\in(0,1)$, $x_0=0.$

1. Faraz qilaylik $x_0\in(-1,0)$ bo‘lsin. U holda (1.11) ga mos bir jinsli $\psi'\left(\frac{x-1}{2}\right)\equiv 0$ shartga ko‘ra x_0 nuqtada $\tau'(x_0)=0$ bo‘lgani uchun $\nu(x_0)>0$, lekin bu

tenglik bizga yaxshi ma’lum bo‘lgan Zarembo Jiro prinsipiga ziddir.

2. Faraz qilaylik $x_0 \in (0,1)$ (1.6^*) ga mos bir jinsli shartga ko‘ra bu ekstremum $1-x_0$ nuqtada ham erishiladi . Demak bu nuqtalarda $\nu(x_0)<0$, $\nu(1-x_0)<0$ bundan esa $\nu(x_0)+\nu(1-x_0)<0$, lekin bu tenglik (3.15) ga mos bir jinsli shartga zid , bu yerda $\nu(x_0)+\nu(1-x_0)=0$ demak $x_0 \notin (0,1)$.

3. Endi $x_0=0$ bo‘lsin, bu yerda (1.6^*) shartga $x_0=0$ desak $\tau(0)=\tau(1)=0$.

Shunday qilib U(x,y) funksiya o‘zining musbat maksimumiga $\overline{D}^+$ sohaning hech bir nuqtasida erishmayapti bu esa Veershtrass teoremasiga zid.

Yuqoridagi kabi U(x,y) funksiya $\overline{D}^+$ sohaning hech bir nuqtasida o‘zining manfiy minimumga erishmasligini ham ko‘rsatish mumkin. Teorema isbot bo‘ldi.

Natija. Agar BF masalasi yechimi mavjud bo‘lsa u yagonadir.

Haqiqatdan ham bir jinsli shartlar asosida biz 1-teoremaga ko‘ra $\overline{D}^+$ sohada $U(x,y)\equiv 0$ ekanligini isbotladik . Demak AB da $U(x,0)=0$, $\lim_{y\to 0}(-y)^{\beta_0}\frac{\partial u}{\partial y}=0$ bu bir jinsli shartlar asosida , shakli o‘zgargan Koshi masalasini Dalamber formulasi yordamida D^- sohada tiklab D^-da $U(x,y)\equiv 0$ ekanligiga ishonch hosil qilamiz .Shunday qilib BF masalasi bittadan ortiq yechimga ega bo‘la olmaydi.

3.3-§. $\nu_1(x)$ va $\nu_0(x)$ noma`lum funksiyalar o`rtasidagi birinchi va ikkinchi funksional munosabatlar asosiy natijalari bayoni.

BF masalasi yechimining mavjudligi. Shunday qilib, bizda $\upsilon(x)$va $\tau'(x)$ lar o‘rtasida ushbu ikkita munosabat bor.

$$\upsilon(x)=\tau'(x)+\psi_0'\left(\frac{x-1}{2}\right) \qquad ,x\in(-1,0) \qquad (3.18)$$

$$\upsilon(x)+\upsilon(1-x)=f'(x)-\psi_1(x) \qquad , \quad x\in[0,1) \qquad (3.19)$$

(3.18) va (3.19) – munosabatlar aralash sohaning giperbolik qismidan olingan. Endi elliptik sohadan olingan $\nu(x) va\, \tau(x)$ orasidagi munosabatlarni topamiz.

Buning uchun D^{+} sohada Dirixle masalasi:

$$U(x,0)=\tau(x) \qquad ,x\in \bar{J} : U(x,y)\Big|_{\tau_0}=\varphi(x) \quad , \quad x\in \bar{J} \tag{3.20}$$

yechimini beruvchi formuladan foydalanamiz.

$$U(x,y)=\frac{1}{2\pi}\int_{-1}^{1}\upsilon(t)\left\{\ln\left[(x-t)^2+\frac{4y^{m+2}}{(m+2)^2}\right]-\ln\left[(1-xt)^2+\frac{4t^2y^{m+2}}{(m+2)^2}\right]\right\}dt+$$

$$+\frac{1}{2\pi}(1-R^2)\int_{-1}^{1}\frac{\varphi(t)}{\sqrt{1-t^2}}\left(r^{-2}+r_1^{-2}\right)dt \ . \tag{3.21}$$

Bu yerda

$$R^2=x^2+\frac{4y^{m+2}}{(m+2)^2}$$

$$\left.\begin{matrix} r^2 \\ r_1^2 \end{matrix}\right\}=(x-t)^2+\frac{4}{(m+2)^2}\left(y^{\frac{m+2}{2}} \ \mu\, \eta^{\frac{m+2}{2}}\right)^2$$

(3.21) tekislikda y=o deb, ushbu munosabatga kelamiz

$$\tau(t)=\frac{1}{2\pi}\int_{-1}^{1}\upsilon(t)\left\{\ln(x-t)^2-\ln(1-xt)^2\right\}dt+$$

$$+\frac{(1-x^2)}{\pi}\int_{-1}^{1}\frac{\varphi(t)dt}{\sqrt{1-t^2}\left(x^2+2xt+1\right)} \quad , \quad x\in\bar{J}. \tag{3.22}$$

bu tenglikdan hosila xisoblab, ushbu munosabatni hosil qilamiz

$$\tau'(x)=-\frac{1}{\pi}\int_{-1}^{1}\left(\frac{1}{t-x}-\frac{t}{1-xt}\right)\nu(t)dt+\Phi(x),$$

Bu yerda

$$\Phi(x)=\frac{1}{\pi}\frac{\partial}{\partial x}\left[(1-x^2)\int_{-1}^{1}\frac{\varphi(x)dt}{\sqrt{1-t^2}\left(x^2-2xt+1\right)}\right]$$

ma'lum funksiya. (3.22) munosabat D^{+} sohadan buzilish chizig'i y=0 ga ko'chirilgan ikkinchi funksional munosabatdir. Shuni ta'kidlash joizki bu yerda $x\in J$. Endi (1.6^{*}) dan x bo'yicha hosila olib ushbu tenglikka kelamiz.

$$\tau'(x)+\tau'(1-x)=f'(x) \quad , \quad x\in(0,1) \qquad (3.23)$$

Dastlab (-1,0) oraliqda (3.11) va (3.22) tengliklardan foydalanib $\tau'(x)$ ni yo‘qotamiz , ya’ni (3.22) tenglikdan $\tau'(x)$ ni (3.11) tenglikka qo‘yib ushbu munosabatni hosil qilamiz.

$$\nu(x)=-\frac{1}{\pi}\int_{-1}^{1}\left(\frac{1}{t-x}-\frac{t}{1-xt}\right)\nu(t)dt+\Phi(x)+\psi_1(x) \quad , \; x\in(-1,0) \qquad (3.24)$$

Ushbu ayniyatni to‘g‘riligini bevosita tekshirib ko‘rish mumkin

$$\left(\frac{1}{t-x}-\frac{t}{1-xt}\right)=\left(\frac{1+t}{1+x}\right)\left(\frac{1}{t-x}-\frac{t}{1-xt}\right) \qquad (3.27)$$

$$((1-xt)-t^2+xt)(1+x)=((1-xt)-t+x)(1+t)$$

$$(1-t^2)(1+x)=(1-xt-t+x)(1+t)$$

tenglikni har ikkala tomonini soddalashtirsak haqiqatdan ham to‘g‘ri 0=0 ko‘rinishga keladi . Endi (3.27) ayniyatdan foydalanib (3.24) tenglikni ushbu ko‘rinishda yozib olamiz [7,8].

$$\nu(x)=-\frac{1}{\pi}\int_{-1}^{1}\left(\frac{1+t}{1+x}\right)\left(\frac{1}{t-x}-\frac{t}{1-xt}\right)\nu(t)dt+\varphi_1[x] \quad , \qquad \varphi_1(x)=\phi(x)+\psi_1(x) \qquad x\in(-1,0)$$

(3.28)

(3.28) dagi integralni (-1,0) va (0,1) oraliqdagi integrallarga ajratib ushbu ko‘rinishda yozib olamiz .

$$\nu(x)=-\frac{1}{\pi}\int_{-1}^{0}\left(\frac{1+t}{1+x}\right)\left(\frac{1}{t-x}-\frac{t}{1-xt}\right)\nu(t)dt-\frac{1}{\pi}\int_{0}^{1}\left(\frac{1+t}{1+x}\right)\left(\frac{1}{t-x}-\frac{t}{1-xt}\right)\nu(t)dt+\varphi_1[x],$$

$$x\in(-1,0) \qquad (3.29)$$

(3.29) ning birinchi integralida $t=\frac{s-1}{2}$, $S\in[-1,1]$ (3.29) ning ikkinchi integralida $t=\frac{1+S}{2}$, $S\in[-1,1]$ almashtirishlar bajarib uni ushbu ko‘rinishda yozib olamiz

$$\nu(x)=-\frac{1}{\pi}\int_{-1}^{1}\left(\frac{1+\frac{s-1}{2}}{1+x}\right)\left(\frac{1}{\frac{s-1}{2}-x}-\frac{1}{1-x\frac{s-1}{2}}\right)\nu\left(\frac{s-1}{2}\right)d\,\frac{s}{2}-$$

$$-\frac{1}{\pi}\int_{-1}^{1}\left(\frac{1+\frac{s+1}{2}}{1+x}\right)\left(\frac{1}{\frac{s+1}{2}-x}-\frac{1}{1-x\frac{s+1}{2}}\right)\nu\left(\frac{s+1}{2}\right)d\,\frac{s}{2}+\varphi_1[x]\quad ,x\in(-1,0)\qquad .$$

(3.30)

Endi (3.30) formulada $\mathrm{X}=\frac{x-1}{2}$, $\quad x\in[-1,1]$

$$\nu\left(\frac{x-1}{2}\right)=-\frac{1}{\pi}\int_{-1}^{1}\left(\frac{1+\frac{s-1}{2}}{1+\frac{x-1}{2}}\right)\left(\frac{1}{\frac{s-1}{2}-\frac{x-1}{2}}-\frac{1}{1-\frac{x-1}{2}\cdot\frac{s-1}{2}}\right)\nu\left(\frac{s-1}{2}\right)d\,\frac{s}{2}-$$

$$-\frac{1}{\pi}\int_{-1}^{1}\left(\frac{1+\frac{s+1}{2}}{1+\frac{x-1}{2}}\right)\left(\frac{1}{\frac{s+1}{2}-\frac{x-1}{2}}-\frac{1}{1-\frac{x-1}{2}\cdot\frac{s+1}{2}}\right)\nu\left(\frac{s+1}{2}\right)d\,\frac{s}{2}+\varphi_1[x]$$

(3.31)

Yoki $\nu\left(\frac{x-1}{2}\right)=\nu_o(x),\ \nu\left(\frac{x+1}{2}\right)=\nu_1(x)$ bilan belgilab ushbu tenglikni hosil qilamiz.

$$\nu_0(x)=-\frac{1}{2\pi}\int_{-1}^{1}\left(\frac{1+s}{1+x}\right)\left(\frac{2}{s-x}-\frac{4}{4-(x-1)(s+1)}\right)\nu_0(s)ds-$$

$$-\frac{1}{2\pi}\int_{-1}^{1}\left(\frac{3+s}{1+x}\right)\left(\frac{2}{s-x+2}-\frac{1}{4-(x-1)(s+1)}\right)\nu_1(s)ds+\varphi_1[x]\ ,s=-1\quad ,x\in(0,1)$$

(3.32)

(3.32) – tenglamani ushbu ko‘rinishda yozib olamiz.

$$\nu_0(x)+\frac{1}{2\pi}\int_{-1}^{1}\left(\frac{1+s}{1+x}\right)\left(\frac{2}{s-x}-\frac{4}{4-(x-1)(s+1)}\right)\nu_0(s)ds=$$

$$=-\frac{1}{2\pi}\int_{-1}^{1}\left(\frac{3+s}{1+x}\right)\left(\frac{2}{s-x+2}\right)\nu_1(s)ds+R_1[\nu_1]+\varphi_1[x]\qquad .$$

(3.33)

Buerda $\quad R_1[\nu_1]=\frac{1}{2\pi}\int_{-1}^{1}\left(\frac{3+s}{1+x}\right)\left(\frac{1}{4-(x-1)(s+1)}\right)\nu_1(s)ds$ -regulyar operator.

(3.33) Singulyar tenglama, ushbu bilan xarakterlanadiki, tenglamaning o‘ng tomoni S = -1, x = 1 nuqtada birinchi tartibli maxsuslikka ega va Fredgol’m operatori emas, ya’ni nofredgol’m operatordir [13]. (3.33) tenglama $\nu_0(x)$ va $\nu_1(x)$ noma’lum funksiyalar uchun birinchi tenglamadir.

Endi $\nu_0(x)$ va $\nu_1(x)$ o‘rtasidagi ikkinchi funksional munosabatni topamiz. Buning uchun (0,1) oraliq uchun o‘rinli bo‘lgan

$$\tau'(x)+\tau'(1-x)=f'(x)\ , \quad x\in(0,1)$$

(3.34)

Tenglikdan foydalanamiz. (3.22) tenglikka asosan

$$-\frac{1}{\pi}\int_{-1}^{1}\left(\frac{1+t}{1+x}\right)\left(\frac{1}{t-x}-\frac{1}{1-xt}\right)\upsilon(t)dt+\phi(x)-$$

$$-\frac{1}{\pi}\int_{-1}^{1}\left(\frac{1+t}{2-x}\right)\left(\frac{1}{t-1+x}-\frac{1}{1-(1-x)t}\right)\upsilon(t)dt+\phi(1-x)=f'(x) \qquad (3.35)$$

(3.35) tenglikda (-1, 1) oraliqdagi integrallarni (-1, 0) va (o,1) oraliqdagi ikkita integralga ajratib yozamiz.

$$-\frac{1}{\pi}\int_{-1}^{0}\left(\frac{1+t}{1+x}\right)\left(\frac{1}{t-x}-\frac{1}{1-xt}\right)\upsilon(t)dt-$$

$$-\frac{1}{\pi}\int_{0}^{1}\left(\frac{1+t}{1+x}\right)\left(\frac{1}{t-x}-\frac{1}{1-xt}\right)\upsilon(t)dt\ , \chi=1\ , t=1$$

$$-\frac{1}{\pi}\int_{-1}^{0}\left(\frac{1+t}{2-x}\right)\left(\frac{1}{t-(1-x)}-\frac{1}{1-(1-x)t}\right)\upsilon(t)dt-$$

$$-\frac{1}{\pi}\int_{0}^{1}\left(\frac{1+t}{2-x}\right)\left(\frac{1}{t-(1-x)}-\frac{1}{1-(1-x)t}\right)\upsilon(t)dt=$$

$$=f'(x)-\phi(x)-\phi(1-x) \qquad , \qquad x\in(0,1)$$

(3.36)

(3.36) – tenglamaning to‘rtinchi integralida t = 1 – S almashtirish bajaramiz $dt=-ds$ va (3.17) – tenglikka asosan $\nu(1-x)=-\nu(x)+f'(x)-\psi_1'(x)$ (3.36) – tenglikni ushbu ko‘rinishda yozib olamiz.

$$-\frac{1}{\pi}\int_{-1}^{0}\left(\frac{1+t}{1+x}\right)\left(\frac{1}{t-x}-\frac{1}{1-xt}\right)\upsilon(t)dt-$$

$$-\frac{1}{\pi}\int_{0}^{1}\left(\frac{1+t}{1+x}\right)\left(\frac{1}{t-x}-\frac{1}{1-xt}\right)\upsilon(t)dt-$$

$$-\frac{1}{\pi}\int_{-1}^{0}\left(\frac{1+t}{2-x}\right)\left(\frac{1}{t-(1-x)}-\frac{1}{1-(1-x)t}\right)\upsilon(t)dt-$$

$$-\frac{1}{\pi}\int_{0}^{1}\left(\frac{2-s}{2-x}\right)\left(\frac{1}{1-s-(1-x)}-\frac{1}{1-(1-x)(1-s)}\right)\upsilon(s)ds=\phi_2(x) \quad , \quad x\in(0,1).$$

(3.37)

Bu yerda

$$\phi_2(x)=\frac{1}{\pi}\int_{0}^{1}\left(\frac{1-s}{1-x}\right)\left(\frac{1}{1-s-(1-x)}-\frac{1}{1-(1-x)(1-s)}\right).$$

$$\left(f'(s)-\psi'(s)\right)ds+f'(x)-\phi(x)-\phi(1-x) \quad , \quad x\in(0,1).$$

Endi (-1, 0) oraliq bo‘yicha olingan integrallarda $t=\frac{s-1}{2}$ va (o,1) oraliq bo‘yicha olingan integrallarda $t=\frac{s-1}{2}$ integral o‘zgaruvchilarini almashtiramiz.

$$-\frac{1}{\pi}\int_{-1}^{1}\left(\frac{1+\frac{s-1}{2}}{1+x}\right)\left(\frac{1}{\frac{s-1}{2}-x}-\frac{1}{1-x\frac{s-1}{2}}\right)\nu\left(\frac{s-1}{2}\right)d\,\frac{s}{2}-$$

$$-\frac{1}{\pi}\int_{-1}^{1}\left(\frac{1+\frac{s+1}{2}}{1+x}\right)\left(\frac{1}{\frac{s+1}{2}-x}-\frac{1}{1-x\frac{s+1}{2}}\right)\nu\left(\frac{s+1}{2}\right)d\,\frac{s}{2}-$$

$$-\frac{1}{\pi}\int_{-1}^{1}\left(\frac{1+\frac{s-1}{2}}{2-x}\right)\left(\frac{1}{\frac{s-1}{2}-(1-x)}-\frac{1}{1-(1-x)\frac{s-1}{2}}\right)\nu\left(\frac{s-1}{2}\right)d\,\frac{s}{2}-$$

$$-\frac{1}{\pi}\int_{-1}^{1}\left(\frac{1+\frac{s+1}{2}}{2-x}\right)\left(\frac{1}{\frac{s+1}{2}-(1-x)}-\frac{1}{1-(1-x)\frac{s+1}{2}}\right)\nu\left(\frac{s+1}{2}\right)d\,\frac{s}{2}=$$

$$= f'(x) - \phi(x) - \phi(1-x), \quad x \in (0,1) \quad , X = \frac{x+1}{2} \quad , x \in [-1,1] \quad .$$

(3.38)

Endi (3.38) – tenglikda $x = \dfrac{x+1}{2}$ $, x \in [-1,1]$ almashtirish bajaramiz.

$$-\frac{1}{2\pi}\int_{-1}^{1}\left(\frac{1+\frac{s-1}{2}}{1+\frac{x+1}{2}}\right)\left(\frac{1}{\frac{s-1}{2}-\frac{x+1}{2}} - \frac{1}{1-\frac{x+1}{2}\cdot\frac{s-1}{2}}\right)\nu_0(s)ds$$

$$-\frac{1}{2\pi}\int_{-1}^{1}\left(\frac{1+\frac{s+1}{2}}{1+\frac{x+1}{2}}\right)\left(\frac{1}{\frac{s+1}{2}-\frac{x+1}{2}} - \frac{1}{1-\frac{x+1}{2}\cdot\frac{s+1}{2}}\right)\nu_1(s)ds -$$

$$-\frac{1}{2\pi}\int_{-1}^{1}\left(\frac{1+\frac{s-1}{2}}{2-\frac{x+1}{2}}\right)\left(\frac{1}{\frac{s-1}{2}-\left(1-\frac{x+1}{2}\right)} - \frac{1}{1-\left(1-\frac{x+1}{2}\right)\cdot\frac{s-1}{2}}\right)\nu_0(s)ds -$$

$$-\frac{1}{2\pi}\int_{-1}^{1}\left(\frac{2-\frac{s+1}{2}}{2-\frac{x+1}{2}}\right)\left(\frac{1}{1-\frac{s+1}{2}-1+\frac{x+1}{2}} - \frac{1}{1-\left(1-\frac{x+1}{2}\right)\cdot\left(1-\frac{s+1}{2}\right)}\right)\nu_1(s)ds = \phi_1(x) \ ,$$

$$x \in (-1,1). \qquad (3.36^*)$$

Bu yerda

$\phi_1(x) = f'(x) - \phi(x) - \phi(1-x).$

$$-\frac{1}{2\pi}\int_{-1}^{1}\left(\frac{1+s}{3+x}\right)\left(\frac{2}{s-x} - \frac{4}{4-(x+1)(s-1)}\right)\nu_0(s)ds -$$

$$-\frac{1}{2\pi}\int_{-1}^{1}\left(\frac{3+s}{3+x}\right)\left(\frac{2}{s-x} - \frac{1}{4-(x+1)(s+1)}\right)\nu_1(s)ds -$$

$$-\frac{1}{2\pi}\int_{-1}^{1}\left(\frac{1+s}{3-x}\right)\left(\frac{2}{s-1-(1-x)} - \frac{4}{4-(1-x)(s-1)}\right)\nu_0(s)ds -$$

$$-\frac{1}{2\pi}\int_{-1}^{1}\left(\frac{3-s}{3-x}\right)\left(\frac{2}{(1-s)-(1-x)} - \frac{1}{4-(1-x)(1-s)}\right)\nu_1(s)ds = \phi_1(x) \ , \quad x \in (-1,1). \qquad (3.38^*)$$

Endi (3.38^*) – tenglamani ushbu ko‘rinishda yozib olamiz.

$$-\frac{1}{2\pi}\int_{-1}^{1}\left(\frac{1+s}{3+x}\right)\left(\frac{2}{s-x-2}-\frac{4}{4-(x+1)(s-1)}\right)\nu_0(s)ds-$$

$$-\frac{1}{2\pi}\int_{-1}^{1}\left(\frac{3+s}{3+x}\right)\left(\frac{2}{s-x}-\frac{1}{4-(x+1)(s+1)}\right)\nu_1(s)ds-$$

$$-\frac{1}{2\pi}\int_{-1}^{1}\left(\frac{1+s}{3-x}\right)\left(\frac{2}{s+x-2}-\frac{4}{4-(1-x)(s-1)}\right)\nu_0(s)ds+$$

$$+\frac{1}{2\pi}\int_{-1}^{1}\left(\frac{3-s}{3-x}\right)\left(\frac{2}{s-x}-\frac{1}{4-(1-x)(1-s)}\right)\nu_1(s)ds=\phi_2(x)\ ,\ \ x\in(-1,1). \qquad (3.39)$$

Ushbu soddalashtirishni bajaramiz.

$$\frac{3-s}{3-x}-\frac{3+s}{3+x}=\frac{(3-s)(3+x)-(3+s)(3-x)}{9-x^2}=\frac{9+3x-3s-sx-[9-3x+3s-sx]}{9-x^2}=$$

$$=\frac{6x-6s}{9-x^2}=\frac{6(x-s)}{9-x^2}\quad . \qquad (3.40)$$

Endi (3.39) – tenglamani ushbu ko‘rinishda yozib olamiz.

$$\frac{1}{\pi}\int_{-1}^{1}\left[\frac{3-s}{3-x}-\frac{3+s}{3+x}\right]\frac{\nu_1(s)ds}{s-x}+$$

$$+\frac{2}{\pi}\int_{-1}^{1}\left[\left(\frac{3-s}{3-x}\right)\cdot\frac{1}{4-(1-x)(1-s)}+\left(\frac{3+s}{3+x}\right)\cdot\frac{1}{4-(1+x)(1+s)}\right]\nu_1(s)ds$$

$$-\frac{1}{\pi}\int_{-1}^{1}\left[\left(\frac{1+s}{3+x}\right)\cdot\frac{1}{s-x-2}+\left(\frac{1+s}{3-x}\right)\cdot\frac{1}{s+x-2}\right]\nu_0(s)ds+$$

$$+\frac{2}{\pi}\int_{-1}^{1}\left[\left(\frac{1+s}{3+x}\right)\cdot\frac{1}{4+(1+x)(1-s)}+\left(\frac{1+s}{3-x}\right)\cdot\frac{1}{4+(1-x)(1-s)}\right]\nu_0(s)ds=\phi_2(x)\,,\ x\in(-1,1) \quad (3.41)$$

(3.40) – tenglikka asosan (3.41) – tenglamaning chap tomonidagi 2,3 integral operatorlar M(x,s) tekisligining (-1,-1),(1,1) va (-1,1), ajralgan nuqtalarida birinchi tartibli maxsuslikka ega. 1 va 4 integral operatorlar regulyar operatorlardir. (3.41) tenglama $\nu_0(x)$ va $\nu_1(x)$ noma’lum funksiyalarga nisbatan ikkinchi tenglamadir.

(3.33) – tenglamani regulyarizatsiyalash. (3.33) – tenglamani ushbu ko‘rinishda yozib olamiz.

$$\nu_0(x) = -\frac{1}{\pi}\int_{-1}^{1}\left(\frac{1+s}{1+x}\right)\left(\frac{1}{s-x} - \frac{2}{4-(1-x)(1-s)}\right)\nu_0(s)ds = R_2(\nu_1),\; x\in(-1,1), \qquad (3.42)$$

bu yerda

$$R_2(\nu_1) = -\frac{1}{2\pi}\int_{-1}^{1}\left(\frac{3+s}{1+x}\right)\frac{2}{s-x+2}\nu_1(s)ds + R_1[\nu_1] + \phi(x)$$

(3.42) – tenglamada $\rho(x) = (1+x)\nu_0(x)$, $g(x) = (1+x)R_2(\nu_1)$ almashtirish bajarib

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$$\rho(x) + \frac{1}{\pi}\int_{-1}^{1}\left(\frac{1}{1-x} - \frac{2}{4-(1-x)(1-t)}\right)\rho(t)dt = g(x) \qquad (3.43)$$

ko‘rinishda yozib olamiz. z – kompleks tekisligining ixtiyoriy nuqtasi bo‘lsin. Karleman metodiga asosan[8]

$$\phi(z) = \frac{1}{2\pi i}\int_{-1}^{1}\left(\frac{1}{t-z} - \frac{2}{4-(1-z)(1-t)}\right)\rho(t)dt,\; \phi(\infty) = 0 \qquad (3.44)$$

funksiyani kiritib olamiz. (3.44) – ifodadan ko‘rinib turibdiki $\phi(x)$ funksiyamiz yuqori va quyi yarim tekislikda golomorf funksiyadir. $\phi^+(x)$, $\phi^-(x)$-orqali mos ravishda $\phi(x)$ funksiyaning y = 0 dagi qiymatlarini ya’ni $\phi^+(x) = \lim\limits_{\substack{z\to x \\ Jm>0}}\phi(z)$,

$\phi^-(x) = \lim\limits_{\substack{z\to x \\ Jm<0}}\phi(z)$ larni belgilaymiz. $\phi\left(\frac{3+z}{z-1}\right)$- ni hisoblaymiz.

$$\phi\left(\frac{3+z}{z-1}\right) = \frac{1}{2\pi i}\int_{-1}^{1}\left(\frac{1}{t-\frac{3+z}{z-1}} - \frac{2}{4-\left(1-\frac{3+z}{z-1}\right)(1-t)}\right)\rho(t)dt =$$

$$= \frac{1}{2\pi i}\int_{-1}^{1}\left(\frac{z-1}{t(z-1)-3-z} - \frac{2}{4-\frac{z-1-3-z}{z-1}(1-t)}\right)\rho(t)dt =$$

$$= \frac{1}{2\pi i}\int_{-1}^{1}\left(\frac{z-1}{t(z-1)-4-(z-1)} - \frac{2}{4+\frac{4}{z-1}(1-t)}\right)\rho(t)dt =$$

$$=\frac{1}{2\pi i}\int_{-1}^{1}\left(\frac{z-1}{(z-1)(t-1)-4}-\frac{2(z-1)}{4z-4+4-4t}\right)\rho(t)dt=$$

$$=\frac{1}{2\pi i}\int_{-1}^{1}\left(\frac{z-1}{(z-1)(t-1)-4}-\frac{2(z-1)}{4(z-t)}\right)\rho(t)dt=$$

$$=\frac{1}{2\pi i}\int_{-1}^{1}\frac{z-1}{2}\left(\frac{1}{t-z}-\frac{2}{4-(1-z)(1-t)}\right)\rho(t)dt=\frac{z-1}{2}\phi(z)$$

$\omega=\frac{3+z}{z-1}$ kasr chiziqli almashtirish.

$$\omega=\frac{3+x+iy}{x+iy-1}=\frac{3+x+iy}{x-1+iy}\cdot\frac{x-1-iy}{x-1-iy}=\frac{3x-3-3iy+x^2-x-xyi+xyi-iy+y^2}{(x-1)^2+y^2}=$$

$$=\frac{3(x-1)+x(x-1)+y^2}{(x-1)^2+y^2}-\frac{4y}{(x-1)^2+y^2}i=$$

$$=\frac{(x-1)(3+x)+y^2}{(x-1)^2+y^2}-\frac{4y}{(x-1)^2+y^2}i=u+iv\,.$$

Bu yerdan ko‘rinib turibdiki yuqori yarim tekislik quyi yarim tekislikka va aksincha quyi yarim tekislik yuqori yarim tekislikka aniqlanadi. y=0 da

$$\omega=\frac{3+x}{x-1}=\frac{x-1+4}{x-1}=1+\frac{4}{x-1}$$

$J=(-1,1)\to(-\infty,-1)=\Delta$ ga akslanadi.

Shunday qilib (3.44) tenglik bilan aniqlangan $\phi(z)$ funksiya

$$\phi\left(\frac{3+z}{z-1}\right)=\frac{z-1}{2}\phi(z) \qquad (3.45)$$

xossaga ega. (3.44) funksiya uchun Soxoskiy – Plemele formulalari quyidagiga bo‘lindi [10].

$$\phi^{+}(x)=\frac{\phi(x)}{2}+\frac{1}{2\pi i}\int_{-1}^{1}\left(\frac{1}{t-x}-\frac{2}{4-(1-x)(1-t)}\right)\rho(t)dt \qquad (3.46)$$

$$\phi^{-}(x)=-\frac{\phi(x)}{2}+\frac{1}{2\pi i}\int_{-1}^{1}\left(\frac{1}{t-x}-\frac{2}{4-(1-x)(1-t)}\right)\rho(t)dt \qquad (3.47)$$

$$\phi^{+}(x)-\phi^{-}(x)=\rho(x)$$

$$\phi^{+}(x)+\phi^{-}(x)=\frac{1}{\pi i}\int_{-1}^{1}\left(\frac{1}{t-x}-\frac{2}{4-(1-x)(1-t)}\right)\rho(t)dt .$$

Soxoskiy–Plemele formulasiga ko‘ra (3.43) – tenglamani ushbu ko‘rinishda yozib olamiz[16].

$$\phi^{+}(x)-\phi^{-}(x)+i\left(\phi^{+}(x)+\phi^{-}(x)\right)=g(x)$$

$$(1+i)\phi^{+}(x)-(1-i)\phi^{-}(x)=g(x),\ x\in(-1,1)$$

$$\phi^{+}(x)-\frac{(1-i)}{(1+i)}\phi^{-}(x)=\frac{1}{(1+i)}g(x).$$

(3.48)

Endi (3.48) – shartni x ni $\frac{3+x}{x-1}$ bilan almashtiramiz. Bu yerda $x\in(-\infty,-1)$ natijada (3.48) – munosabat ushbu ko‘rinishni oladi.

$$(1+i)\phi^{+}\left(\frac{3+x}{x-1}\right)-(1-i)\phi^{-}\left(\frac{3+x}{x-1}\right)=g\left(\frac{3+x}{x-1}\right)\quad,\quad x\in(-\infty,-1)$$

Endi (3.45) ga asosan

$$(1+i)\frac{x-1}{2}\phi^{-}(x)-(1-i)\frac{x-1}{2}\phi^{+}(x)=g\left(\frac{3+x}{x-1}\right)$$

$$\phi^{+}(x)-\frac{(1+i)}{(1-i)}\phi^{-}(x)=-\frac{2}{x-1}\cdot\frac{1}{1-i}g\left(\frac{3+x}{x-1}\right)\quad,x\in(-\infty,-1) \qquad (3.49)$$

$$\frac{1-i}{1+i}=\frac{e^{-\frac{\pi}{4}i}}{e^{\frac{\pi}{4}i}}=e^{-\frac{\pi}{2}i}=-i$$

$$\frac{1-i}{1+i}=\frac{(1-i)}{1-i^{2}}=\frac{1-2i-1}{2}=-i=e^{-\frac{\pi}{2}i}.$$

Endi (3.48) va (3.49) tengliklarni birlashtiramiz, ya’ni

$$\phi^{+}(x)-\tau(x)\phi^{-}(x)=h(x)\quad,\quad x\in(-\infty,+\infty) \qquad (3.50)$$

$$G(x)=\begin{cases}-i, & x\in(-1,1)=J\\ i, & x\in(-\infty,-1)=\Delta\end{cases}\quad,\ 1,\ \ x\notin J\cup\Delta \qquad (3.51)$$

$$h(x)=\begin{cases}\dfrac{g(x)}{1+x}, & x\in(-1,1)=J\\ -\dfrac{2}{x-1}\cdot\dfrac{1}{1-i}\,g\left(\dfrac{3+x}{x-1}\right), & x\in(-\infty,-1)=\Delta\end{cases},\ 0,\quad x\notin J\cup\Delta. \qquad (3.52)$$

Shunday qilib, (3.43) integral tenglamaning yechimini topish masalasi, kompleks o‘zgaruvchilar nazariyasining ushbu masalasiga olib kelindi: yuqori va quyi yarim tekisliklarda golomorf bo‘lgan va haqiqiy o‘qda (3.50) shartni qanoatlantiruvchi $\phi(\infty)=0$ golomorf funksiya topilsin[14].

Bu masala yechimini oshkor topish mumkin. Dastlab berilgan masalaga mos bir jinsli masalani yechamiz ya’ni z kompleks tekisligida shunday $X(z)$ funksiya topamizki, u $J\cup\Delta$ oraliqdan tashqarida golomorf bo‘lsin va $J\cup\Delta$ ushbu

$$X^{+}(x)=G(x)X^{-}(x) \qquad (3.53)$$

shartli qanoatlantirilsin.

Unda o‘rganilayotgan masalaning xususiy yechimlaridan biri ushbu ko‘rinishda bo‘ladi.

$$X(x)=\exp\left\{\frac{1}{2\pi i}\int_{-1}^{1}\left(\frac{1}{t-z}-\frac{z-1}{4-(z-1)(t-1)}\right)\ln\tau(t)dt=\right.$$

$$=\exp\left\{\frac{1}{2\pi i}\int_{-1}^{1}\left(\frac{1}{t-z}-\frac{z-1}{4-(z-1)(t-1)}\right)\ln e^{-\frac{\pi}{2}i}dt=\right.$$

$$=\exp\left\{-\frac{1}{4}\int_{-1}^{1}\left(\frac{1}{t-z}-\frac{z-1}{4-(z-1)(t-1)}\right)dt=\cdot\right.$$

Bu yerda $X\left(\frac{3+x}{x-1}\right)=X(z)$

$$X\left(\frac{3+z}{z-1}\right)=\exp\left\{\frac{1}{2\pi i}\int_{-1}^{1}\left(\frac{1}{t-\frac{3+z}{z-1}}-\frac{\frac{3+z}{z-1}-1}{4-\left(\frac{3+z}{z-1}-1\right)(t-1)}\right)\ln\tau(t)dt=\right.$$

$$=\exp\left\{\frac{1}{2\pi i}\int_{-1}^{1}\left(\frac{z-1}{t(z-1)-3-z}-\frac{z+3-z+1}{4(z-1)-(3+z-z+1)(t-1)}\right)\ln(t)dt=\right.$$

$$= \exp\left\{\frac{1}{2\pi i}\int_{-1}^{1}\left(\frac{z-1}{t(z-1)-(z-1)-4}-\frac{4}{4(z-1)-4(t-1)}\right)\ln\tau(t)dt = \right.$$

$$= \exp\left\{\frac{1}{2\pi i}\int_{-1}^{1}\left(\frac{z-1}{(z-1)(t-1)-4}-\frac{1}{z-t}\right)\ln\tau(t)dt = \right.$$

$$= \exp\left\{\frac{1}{2\pi i}\int_{-1}^{1}\left(\frac{1}{t-z}-\frac{z-1}{4-(z-1)(t-1)}\right)\ln\tau(t)dt = X(z)\right..$$

Endi $X(z)$ ni hisoblaymiz

$$X(x) = \exp\left\{-\frac{1}{4}\left[\ln(t-z)\Big|_{-1}^{1}+\ln\left(4-(z-1)(t-1)\Big|_{-1}^{1}\right)\right]\right\} =$$

$$= \exp\left\{-\frac{1}{4}\left[\ln(1-z)-\ln(-1-z)+\ln 4-\ln(4+2(z-1))\right]\right\} =$$

$$= \exp\left\{-\frac{1}{4}\left[\ln(1-z)-\ln(-1-z)+\ln 4-\ln(2(z+1))\right]\right\} =$$

$$= \exp\left\{-\frac{1}{4}\left[\ln(1-z)-\ln(-1-z)+\ln 2-\ln(z+1)\right]\right\}$$

bu yerda

$$X^{+}(x) = \exp\left\{-\frac{1}{4}\left[\ln(1-x)-\ln(1+x)-(-\pi i)+\ln 2-\ln(x+1)\right]\right\} =$$

$$= \exp\left\{-\frac{1}{4}\left[\ln\frac{1-x}{(1+x)^2}+\pi i\right]\right\} = \exp\left\{\ln\left(\frac{(1+x)^2}{1-x}\right)^{\frac{1}{4}}-\frac{\pi i}{4}\right\} = \sqrt[4]{\frac{(1+x)^2}{2(1-x)}}\cdot e^{-\frac{\pi i}{4}}$$

$$X^{-}(x) = \exp\left\{-\frac{1}{4}\left[\ln(1-x)-\ln(1+x)-(\pi i)+\ln 2-\ln(x+1)\right]\right\} =$$

$$= \exp\left\{-\frac{1}{4}\left[\ln\frac{2(1-x)}{(1+x)^2}-\pi i\right]\right\} = \sqrt[4]{\frac{(1+x)^2}{2(1-x)}}\cdot e^{\frac{\pi i}{4}}.$$

Shunday qilib (3.53) masala yechildi ya’ni $G(x) = \dfrac{X^{+}(x)}{X^{-}(x)}$ ko‘rinishni oldi.

Endi bir jinsli bo‘lmagan (3.50) tenglamaga qaytaylik

$$\phi^{+}(x)-\frac{X^{+}(x)}{X^{-}(x)}\phi^{-}(x)=h(x)$$

yoki

$$\frac{\phi^{+}(x)}{X^{+}(x)}-\frac{\phi^{-}(x)}{X^{-}(x)}=\frac{h(x)}{X^{-}(x)}.$$

Endi Liuvillning analitik davom ettirish haqidagi teoremasidan foydalanib, umumiy yechimni ushbu ko‘rinishda yozib olamiz[29].

$$\frac{\phi(z)}{X(z)}=\frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{h(t)}{X^{+}(t)}\cdot\frac{dt}{t-z}+\frac{c_0}{1-z}+\frac{c_1}{1+t}$$

(3.54)

(3.54) yechimni ushbu ko‘rinishda yozib olamiz

$$\phi(z)=\frac{X(z)}{2\pi i}\int_{-1}^{1}\frac{h(t)}{X^{+}(t)}\cdot\frac{dt}{t-z}+\frac{X(z)}{2\pi i}\int_{J}\frac{h(t)dt}{X^{+}(t)-z}+\left(\frac{c_0}{1-z}+\frac{c_1}{1+t}\right)X(z) \qquad (3.55)$$

$$\phi(z)=\frac{X(z)}{2\pi i}\int_{-1}^{1}\frac{g(t)}{1+i}\cdot\frac{1}{X^{+}(t)}\cdot\frac{dt}{t-z}+\frac{X(z)}{2\pi i}\int_{J}\left(-\frac{2}{t-1}\frac{1}{1-i}g\left(\frac{3+t}{t-1}\right)\right)\cdot\frac{1}{X^{+}(t)}\left(\frac{1}{t-z}\right)dt+$$
$$+\left(\frac{c_0}{1-z}+\frac{c_1}{1+t}\right)X(z).$$

(3.56)

Ikkinchi integralda $\xi=\frac{3+t}{t-1}$ $\quad$ $\xi t-\xi=3+t$, $\quad t\to-\infty,\xi\to1$, $\quad t\to-1,\xi\to-1$

$$t(1-\xi)=-\xi-3\ ,\quad t=\frac{3+\xi}{\xi-1}$$

almashtirish bajaramiz

$$dt=\frac{\xi-1-(\xi+3)}{(\xi-1)^2}d\xi=\frac{-4}{(\xi-1)^2}d\xi\ ,\qquad t-1=\frac{\xi+3}{\xi-1}-1=\frac{4}{\xi-1}$$

$$\frac{1}{t-z}=\frac{1}{\frac{\xi+3}{\xi-1}-z}=\frac{\xi-1}{\xi+3-z(\xi-1)}=\frac{\xi-1}{\xi-1-z(\xi-1)+4}=\frac{\xi-1}{(1-z)(\xi-1)+4}=\frac{\xi-1}{4-(z-1)(\xi-1)}$$

shunday qilib (3.55) tenglikni ushbu ko‘rinishda yozib olamiz

$$\phi(z)=\frac{X(z)}{2\pi i(1+i)}\int_{-1}^{1}\frac{g(t)}{X^{+}(t)}\cdot\frac{dt}{t-z}+\frac{X(z)}{2\pi i}\int_{1}^{-1}\left(\frac{\xi-1}{-2}\frac{1}{1-i}g(\xi)\right)\cdot\frac{1}{X^{+}\left(\frac{\xi+3}{t-1}\right)}\times$$

$$\times\frac{\xi-1}{4-(\xi-1)(z-1)}\cdot\frac{-4}{(\xi-1)^{2}}d\xi+\left(\frac{c_0}{1-z}+\frac{c_1}{1+z}\right)X(z_1)$$

yoki

$$\phi(z)=\frac{X(z)}{2\pi i(1+i)}\int_{-1}^{1}\frac{g(t)dt}{X^{+}(t)(t-z)}-\frac{X(z)}{2\pi i(1-i)}\int_{-1}^{1}\frac{1}{X^{-}(z)}\cdot\frac{2}{4-(\xi-1)(t-1)}g(t)dt+$$

$$\times+\left(\frac{c_0}{1-z}+\frac{c_1}{1+z}\right)X(z)$$

endi Soxoskiy – Plemele formulasiga ko‘ra[8]

$$\phi^{+}(x)=\frac{X^{+}(x)}{2(1+i)}\cdot\frac{g(x)}{X^{+}(x)}+\frac{X^{+}(x)}{2\pi i(1+i)}\int_{-1}^{1}\frac{g(t)dt}{X^{+}(t)(t-x)}+\left(\frac{c_0}{1-x}+\frac{c_1}{1+x}\right)X^{+}(x)$$

$$\phi^{-}(x)=-\frac{X^{-}(x)}{2(1+i)}\cdot\frac{g(x)}{X^{+}(x)}+\frac{X^{-}(x)}{2\pi i(1+i)}\int_{-1}^{1}\frac{g(t)dt}{X^{+}(t)(t-x)}+\left(\frac{c_0}{1-x}+\frac{c_1}{1+x}\right)X^{-}(x)$$

$$\phi(x)=\phi^{+}(x)-\phi^{-}(x)$$

$$\rho(x)=\left(1-\frac{X^{-}(x)}{X^{+}(x)}\right)\cdot\frac{g(x)}{2(1+i)}+\left(X^{+}(x)-X^{-}(x)\right)\frac{1}{2\pi i(1+i)}\int_{-1}^{1}\frac{g(t)dt}{X^{+}(t)(t-x)}+$$

$$+\frac{\left(X^{+}(x)-X^{-}(x)\right)}{2\pi i(1-i)}\int_{-1}^{1}\frac{1}{X^{-}(t)}\cdot\frac{2}{4-(t-1)(x-1)}g(t)dt+m$$

$$\rho(x)=\left(1+\frac{1}{-i}\right)\cdot\frac{g(x)}{2(1+i)}+X^{+}(x)\left(1-\frac{X^{-}(x)}{X^{+}(x)}\right)\cdot\frac{1}{2\pi i(1+i)}\int_{-1}^{1}\frac{g(t)dt}{X^{+}(t)(t-x)}+$$

$$+X^{-}(x)\left(1-\frac{X^{+}(x)}{X^{-}(x)}\right)\frac{1}{2\pi i(1-i)}\int_{-1}^{1}\frac{1}{X^{-}(t)}\cdot\frac{2}{4-(t-1)(x-1)}g(t)dt+m$$

yoki

$$\rho(x)=(1+i)\cdot\frac{g(x)}{2(1+i)}+X^{+}(x)\left(1-\frac{1}{-i}\right)\cdot\frac{1}{2\pi i}\cdot\frac{1}{1+i}\int_{-1}^{1}\frac{g(t)dt}{X^{+}(t)(t-x)}+$$

$$+X^{-}(x)(1-i)\frac{1}{2\pi i}\cdot\frac{1}{1-i}\int_{-1}^{1}\frac{1}{X^{-}(t)}\cdot\frac{2}{4-(t-1)(x-1)}g(t)dt+m$$

yoki

$$\rho(x)=\frac{g(x)}{2}+X^{+}(x)\left(\frac{1-i}{1+i}\right)\cdot\frac{1}{2\pi i}\cdot\int_{-1}^{1}\frac{g(t)dt}{X^{+}(t)(t-x)}+$$

$$+X^{-}(x)(1+i)\frac{1}{2\pi i(1-i)}\int_{-1}^{1}\frac{1}{X^{-}(t)}\cdot\frac{2}{4-(t-1)(x-1)}g(t)dt+m$$

$$\frac{1-i}{1+i}=\frac{(1-i)^2}{1-i^2}=\frac{1-2i+i^2}{2}=-i$$

yoki

$$\rho(x)=\frac{g(x)}{2}-\frac{1}{2\pi}\cdot\int_{-1}^{1}\frac{X^{+}(x)}{X^{+}(x)}\cdot\frac{g(t)dt}{(t-x)}+\frac{1}{2\pi}\int_{-1}^{1}\frac{X^{-}(x)}{X^{-}(t)}\cdot\frac{2}{4-(t-1)(x-1)}g(t)dt$$

yoki

$$\rho(x)=\frac{g(x)}{2}-\frac{1}{2\pi}\cdot\int_{-1}^{1}\sqrt[4]{\frac{(1+x)^2}{1-x}\cdot\frac{1-t}{(1+x)^2}}\left(\frac{1}{t-x}-\frac{2}{4-(t-1)(x-1)}\right)g(t)dt+$$

$$+\left(\frac{c_0}{1-x}+\frac{c_1}{1+x}\right)\left(X^{+}(x)-X^{-}(x)\right).$$

Biz yechimni $x=-1$ chegaralangan va $x=1$ nuqtada birdan kichik maxsuslikka ega bo‘lgan funksiyalar sinfida izlaganimiz uchun $c_0=0, c_1=0$ deb olamiz. Shunday qilib (3.43) tenglamaning yechimi quyidagi ko‘rinishda bo‘ladi.

$$\rho(x)=\frac{g(x)}{2}-\frac{1}{2\pi}\cdot\int_{-1}^{1}\sqrt[4]{\frac{(1+x)^2(1-t)}{(1-x)(1+x)^2}}\cdot\left(\frac{1}{t-x}-\frac{2}{4-(t-1)(x-1)}\right)g(t)dt$$

(3.57)

3.4-§. Viner- Xopf integral tenglamasini keltirib chiqarish bo’yicha asosiy natijalari bayoni.

Endi (3.57) formulada $\rho(x)=(1+x)\nu_0(x), g(x)=(1+x)R_2[\nu_1]$ belgilashlardan foydalanib (3.57) ni ushbu ko‘rinishda yozib olamiz.

$$(1+x)\nu_0(x)=\frac{(1+x)R_2[\nu_1]}{2}-\frac{1}{2\pi}\cdot\int_{-1}^{1}\sqrt[4]{\frac{(1+x)^2(1-t)}{(1-x)(1+t)^2}}\cdot\left(\frac{1}{t-x}-\frac{2}{4-(t-1)(x-1)}\right)(1+x)R_2(\nu_1)dt$$

Yoki

$$\nu_0(x)=\frac{R_2[\nu_1]}{2}-\frac{1}{2\pi}\cdot\int_{-1}^{1}\sqrt[4]{\frac{(1+t)^2(1-t)}{(1-x)(1+x)^2}}\cdot\left(\frac{1}{t-x}-\frac{2}{4-(t-1)(x-1)}\right)R_2(\nu_1)dt \qquad (3.58)$$

shunday qilib biz $\nu_0(x)$ ni $\nu_1(x)$ orqali ifodalab oldik .

Endi

$$R_2[\nu_1]=-\frac{1}{2\pi}\cdot\int_{-1}^{1}\left(\frac{3+s}{1+x}\right)\frac{2}{s-x+2}\nu_1(s)ds+R_1[\nu_1]+\phi(x).$$

tenglikni (3.58) – yechimga qo‘yib, ushbu ifodani hosil qilamiz.

$$\nu_0(x)=\frac{1}{2}\left[-\frac{1}{2\pi}\cdot\int_{-1}^{1}\left(\frac{3+s}{1+x}\right)\frac{2}{s-x+2}\nu_1(s)ds+R_1[\nu_1]+\phi(x)\right]-$$

$$-\frac{1}{2\pi}\int_{-1}^{1}\sqrt[4]{\frac{(1+t)^2(1-t)}{(1-x)(1+x)^2}}\cdot\left(\frac{1}{t-x}-\frac{2}{4-(t-1)(x-1)}\right)\times$$

$$\times\left[-\frac{1}{2\pi}\cdot\int_{-1}^{1}\left(\frac{3+s}{1+t}\right)\frac{2}{s-t+2}\nu_1(s)ds+R_1[\nu_1]+\phi(t)\right]dt$$

yoki

$$\nu_0(x)=-\frac{1}{2\pi}\cdot\int_{-1}^{1}\left(\frac{3+s}{1+x}\right)\frac{\nu_1(s)ds}{s-x+2}+\frac{R_1[\nu_1]}{2}+\frac{\phi(x)}{2}+$$

$$-\frac{1}{2\pi}\int_{-1}^{1}\sqrt[4]{\frac{(1+t)^2(1-t)}{(1-x)(1+x)^2}}\cdot\left(\frac{1}{t-x}-\frac{2}{4-(t-1)(x-1)}\right)dt \qquad (3.59)$$

$$\int_{-1}^{1}\left(\frac{3+s}{1+x}\right)\frac{\nu_1(s)ds}{s-x+2}-\frac{1}{2\pi}\int_{-1}^{1}\sqrt[4]{\frac{(1+t)^2(1-t)}{(1-x)(1+x)^2}}\cdot\left(\frac{1}{t-x}-\frac{2}{4-(t-1)(x-1)}\right)(R_1[\nu_1]+\phi(t))dt$$

$$\frac{3+s}{1+x}-1+1=\frac{3+s-1-x}{1+x}+1=\frac{s-x+2}{1+x}+1$$

$$\frac{3+s}{1+t}-1+1=\frac{s-t+2}{1+t}+1 \ .$$

Tengliklardan foydalanib (3.59) – tenglikni ushbu ko‘rinishda yozib olamiz

$$\nu_0(x)=-\frac{1}{2\pi}\cdot\int_{-1}^{1}\left(\frac{3+s}{1+x}\right)\frac{\nu_1(s)ds}{s-x+2}+\frac{R_1[\nu_1]}{2}+\frac{\phi(x)}{2}+$$

$$-\frac{1}{2\pi}\int_{-1}^{1}\sqrt[4]{\frac{(1+t)^2(1-t)}{(1-x)(1+x)^2}}\cdot\left(\frac{1}{t-x}-\frac{2}{4-(t-1)(x-1)}\right)dt\times$$

$$\times\int_{-1}^{1}\left(\frac{3+s}{1+x}\right)\frac{\nu_1(s)ds}{s-x+2}-\frac{1}{2\pi}\int_{-1}^{1}\sqrt[4]{\frac{(1+t)^2(1-t)}{(1-x)(1+x)^2}}\cdot\left(\frac{1}{t-x}-\frac{2}{4-(t-1)(x-1)}\right)R_1[\nu_1]dt$$

$$-\frac{1}{2\pi}\int_{-1}^{1}\sqrt[4]{\frac{(1+t)^2(1-t)}{(1-x)(1+x)^2}}\cdot\left(\frac{1}{t-x}-\frac{2}{4-(t-1)(x-1)}\right)\phi(t)dt$$

yoki

$$\nu_0(x)=-\frac{1}{2\pi}\cdot\int_{-1}^{1}\frac{\nu_1(s)ds}{s-x+2}+$$

$$-\frac{1}{2\pi^2}\int_{-1}^{1}\sqrt[4]{\frac{(1+t)^2(1-t)}{(1-x)(1+x)^2}}\cdot\left(\frac{1}{t-x}-\frac{2}{4-(t-1)(x-1)}\right)dt\int_{-1}^{1}\frac{\nu_1(s)ds}{s-x+2}+R_3[\nu_1]+\phi_1(x), \tag{3.60}$$

bu yerda

$$R_3[\nu_1]=-\frac{1}{2\pi}\cdot\int_{-1}^{1}\frac{\nu_1(s)ds}{1+x}+$$

$$+\frac{1}{2\pi^2}\int_{-1}^{1}\sqrt[4]{\frac{(1+t)^2(1-t)}{(1-x)(1+x)^2}}\cdot\left(\frac{1}{t-x}-\frac{2}{4-(t-1)(x-1)}\right)\int_{-1}^{1}\frac{\nu_1(s)ds}{1+t}+\frac{R_1[\nu_1]}{2}+$$

$$+\frac{1}{2\pi}\int_{-1}^{1}\sqrt[4]{\frac{(1+t)^2(1-t)}{(1-x)(1+x)^2}}\cdot\left(\frac{1}{t-x}-\frac{2}{4-(t-1)(x-1)}\right)R_1[\nu_1]dt$$

-regulyar operatorlar

$$\phi_1(x)=\frac{\phi(x)}{2}-\frac{1}{2\pi}\int_{-1}^{1}\sqrt[4]{\frac{(1+t)^2(1-t)}{(1-x)(1+x)^2}}\cdot\left(\frac{1}{t-x}-\frac{2}{4-(t-1)(x-1)}\right)\phi(t)dt\,.$$

-ma’lum funksiya

(3.60) – tenglamani ushbu ko‘rinishda yozib olamiz

$$\nu_0(x)=-\frac{1}{2\pi}\cdot\int_{-1}^{1}\frac{\nu_1(s)ds}{s-x+2}+\frac{1}{2\pi^2}\int_{-1}^{1}\nu_1(s)ds\times$$

$$\times\int_{-1}^{1}\sqrt[4]{\frac{(1+t)^2(1-t)}{(1-x)(1+x)^2}}\cdot\left(\frac{1}{t-x}-\frac{2}{4-(t-1)(x-1)}\right)\frac{dt}{s-t+2}+R_3[\nu_1]+\phi_1(x),\qquad x\in(-1,1) \tag{3.61}$$

(3.61) – tenglikdagi ichki integralni hisoblaymiz

$$J(x)=\int_{-1}^{1}\frac{(1+t)^{2\alpha}(1-t)^{\alpha}}{(1+x)^{2\alpha}(1-x)^{\alpha}}\left(\frac{1}{t-x}-\frac{2}{4-(t-1)(x-1)}\right)\frac{dt}{s-t+2}\quad. \tag{3.62}$$

Dastlab (3.62) dagi ratsional ko‘paytuvchini oddiy kasrlarga almashtiramiz.

$$\left(\frac{1}{t-x}-\frac{2}{4-(t-1)(x-1)}\right)\frac{1}{s-t+2}=\frac{1}{t-x}-\frac{1}{s-t+2}-$$

$$-\frac{2}{4-(t-1)(x-1)}\cdot\frac{1}{s-t+2}=\left(\frac{1}{t-x}+\frac{1}{s-t+2}\right)\frac{1}{s-x+2}$$

$$-\left(\frac{2(x-1)}{4-(t-1)(x-1)}-\frac{2}{(s+1)-(t-1)}\right)\frac{2}{2(x-1)(s+1)-8}$$

$$2(x-1)(s+1)-2(x-1)(t-1)-8+2(t-1)(x-1)=2(x-1)(s+1)-8 \tag{3.63}$$

(3.63) – tenglikka asosan (3.62) – ifodani ushbu ko‘rinishda yozib olamiz.

$$J(x)=\frac{1}{(1+x)^{2\alpha}(1-x)^{\alpha}}\cdot\left[\frac{1}{s-x+2}\int_{-1}^{1}(1+t)^{2\alpha}(1-t)^{\alpha}\left(\frac{1}{t-x}+\frac{1}{s-t+2}\right)+\right.$$

$$\left.+\frac{2}{2(4-(x-1)(s+1))}\int_{-1}^{1}(1+t)^{2\alpha}(1-t)^{\alpha}\left(\frac{2(x-1)}{4-(t-1)(x-1)}-\frac{2}{s-t+2}\right)dt\right]= \qquad (3.64)$$

$$=\frac{1}{(1+x)^{2\alpha}(1-x)^{\alpha}}\left[\frac{1}{s-x+2}(J_1(x)+J_2(x))+\frac{1}{4-(x-1)(s+1)}(2(x-1)J_3(x)-2J_2(s))\right]$$

Bu yerda

$$J_1(x)=\int_{-1}^{1}\frac{(1+t)^{2\alpha}(1-t)^{\alpha}}{t-x}dt \qquad (3.65)$$

$$J_2(s)=\int_{-1}^{1}\frac{(1+t)^{2\alpha}(1-t)^{\alpha}}{s-t+2}dt \qquad (3.66)$$

$$J_3(x)=\int_{-1}^{1}\frac{(1+t)^{2\alpha}(1-t)^{\alpha}}{4-(t-1)(x-1)}dt \qquad (3.67)$$

1. $J_1(x)$ ni hisoblash uchun [29] kitobning 125 betdagi (4.66) formulada:

$$\int_{-1}^{1}\frac{(1+t)^{\alpha-1}(1-t)^{\beta-1}}{t-x}dt=\frac{\pi ctg\beta\pi}{(1+x)^{1-\alpha}(1-x)^{1-\beta}}-\frac{2^{\beta-1}B(\overline{\alpha},\overline{\beta}-1)}{(1+x)^{1-\overline{\alpha}}}F\left(\overline{\alpha},1-\overline{\beta},2-\overline{\beta};\frac{1-x}{2}\right) \qquad (3.68)$$

formulada

$$\overline{\alpha}-1=2\alpha,\ \overline{\alpha}=1+2\alpha=1+2\cdot\frac{1}{4}=1+\frac{1}{2}=\frac{3}{2}$$

$$\overline{\beta}-1=\alpha,\ \overline{\beta}=1+\alpha=1+\frac{1}{4}=\frac{5}{2}$$

qiymatlarni qo‘yib

$$J_1(x)=\int_{-1}^{1}\frac{(1+t)^{2\alpha}(1-t)^{\alpha}}{t-x}dt=\frac{\pi ctg\frac{5}{4}\pi}{(1+x)^{-2\alpha}(1-x)^{-\alpha}}-\frac{2^{\frac{1}{4}}B\left(\frac{3}{2},\frac{1}{4}\right)}{(1+x)^{-2\overline{\alpha}}}F\left(\frac{3}{2},-\frac{1}{4},2-\frac{5}{4};\frac{1-x}{2}\right)= \qquad (3.69)$$

$$=\pi(1+x)^{2\alpha}(1+x)^{\alpha}-2^{\frac{1}{4}}B\left(\frac{3}{2},\frac{1}{4}\right)(1+x)^{2\alpha}F\left(\frac{3}{2},-\frac{1}{4},\frac{3}{4};\frac{1-x}{2}\right)$$

2. Endi $J_3(x)$ ni hisoblaymiz. Buning uchun [29] kitobning (4.68) formulasida:

$$J_3(x)=\int_{-1}^{1}\frac{(1+t)^{\bar{\alpha}-1}(1-t)^{\bar{\beta}-1}}{1-(ax-b)(at-b)}dt=\frac{2^{\bar{\alpha}+\bar{\beta}-1}B(\bar{\alpha},\bar{\beta})}{1-(ax-b)c}F\left(\bar{\beta},1,\bar{\alpha}+\bar{\beta};\frac{2a(b-ax)}{1-(ax-b)c}\right)$$

$c=0$, $a=\frac{1}{2}$, $b=\frac{1}{2}$ desak $4\int_{-1}^{1}\frac{(1+t)^{\bar{\alpha}-1}(1-t)^{\bar{\beta}-1}}{4-(x-1)(t-1)}dt=\frac{2^{\bar{\alpha}+\bar{\beta}-1}B(\bar{\alpha},\bar{\beta})}{1}F\left(\bar{\beta},1,\bar{\alpha}+\bar{\beta};\frac{1-x}{2}\right)$

formulaga ega bo‘lamiz.

$$\bar{\alpha}-1=2\alpha=\frac{1}{2}\ ,\ \bar{\alpha}=1+\frac{1}{2}=\frac{3}{2}$$

$$\bar{\beta}-1=\alpha=\frac{1}{4}\ ,\ \bar{\beta}=1+\frac{1}{4}=\frac{5}{4}.$$

Shunday qilib

$$J_3(x)=\int_{-1}^{1}\frac{(1+t)^{2\alpha}(1-t)^{\alpha}}{4-(x-1)(t-1)}dt=\frac{2^{\frac{3}{2}+\frac{5}{4}-1}}{4}B\left(\frac{3}{2},\frac{5}{4}\right)F\left(\frac{5}{4},1,\frac{3}{2}+\frac{5}{4};\frac{1-x}{2}\right)=\frac{2^{\frac{7}{4}}}{4}B\left(\frac{3}{2},\frac{5}{4}\right)\times$$

$$\times F\left(\frac{5}{4},1,\frac{11}{4};\frac{1-x}{2}\right)=\frac{1}{2^{\frac{1}{4}}}B\left(\frac{3}{2},\frac{5}{4}\right)F\left(\frac{5}{4},1,\frac{11}{4};\frac{1-x}{2}\right). \qquad (3.70)$$

3. Endi $J_2(x)$ hisoblaymiz

$$J_2(x)=\int_{-1}^{1}\frac{(1+t)^{2\alpha}(1-t)^{\alpha}}{s-t+2}dt$$

buning uchun [29] kitobning (4.70) formulasida:

$$\int_{-1}^{1}\frac{(1+t)^{\bar{\alpha}-1}(1-t)^{\bar{\beta}-1}}{bs-at+1}dt=\frac{\pi}{\sin\bar{\beta}\pi}\cdot\frac{b^{\bar{\beta}-1}a^{1-\bar{\alpha}-\bar{\beta}}}{(1+a-bs)^{1-\bar{\alpha}}}(1+s)^{1-\bar{\beta}}+$$

$$+\frac{B(\bar{\alpha},\bar{\beta}-1)}{2^{2-\bar{\alpha}-\bar{\beta}}}F\left(2-\bar{\alpha}-\bar{\beta},1,2-\bar{\beta};\frac{b(1+s)}{2a}\right) \qquad (3.71)$$

(1.71) formulada $a=\frac{1}{2}$, $b=\frac{1}{2}$ deb olib, ushbu tenglikka kelamiz

$$2\int_{-1}^{1}\frac{(1+t)^{\bar{\alpha}-1}(1-t)^{\bar{\beta}-1}}{s-t+1}dt=\frac{\pi}{\sin\bar{\beta}}\cdot\frac{\left(\frac{1}{2}\right)^{\bar{\beta}-1}\left(\frac{1}{2}\right)^{1-\bar{\alpha}-\bar{\beta}}}{\frac{(2+1-s)^{1-\bar{\alpha}}}{2^{1-\alpha}}}(1+s)^{1-\bar{\beta}}+$$

$$+\frac{B(\bar{\alpha},\bar{\beta}-1)}{2^{2-\bar{\alpha}-\bar{\beta}}}F\left(2-\bar{\alpha}-\bar{\beta},1,2-\bar{\beta};-\frac{1+x}{2}\right)$$

$$\overline{\alpha}-1=2\alpha=\frac{1}{2}\ ,\ \overline{\alpha}=1+\frac{1}{2}=\frac{3}{2}$$

$$\overline{\beta}-1=\alpha=\frac{1}{4}\ ,\ \overline{\beta}=1+\frac{1}{4}=\frac{5}{2}$$

demak

$$J_2(s)=\int_{-1}^{1}\frac{(1+t)^{2\alpha}(1-t)^{\alpha}}{s-t+2}dt=\frac{\pi}{2\sin\frac{\pi}{4}}\cdot\frac{2^{\overline{\alpha}+1-\overline{\alpha}}}{(3-s)^{-\frac{1}{2}}}(1+s)^{\frac{1}{4}}+$$

$$+\frac{B\left(\frac{3}{2},\frac{1}{4}\right)}{2^{-\frac{3}{4}}}F\left(-\frac{3}{4},1,\frac{3}{4};-\frac{1+s}{2}\right)= \tag{3.72}$$

$$=-\frac{2\pi}{2\sqrt{2}}\cdot\frac{2}{(3-s)^{-\frac{1}{2}}}(1+s)^{\frac{1}{4}}-\frac{2^{\frac{3}{4}}}{2}B\left(\frac{3}{2},\frac{1}{4}\right)(1+x)^{2\alpha}F\left(-\frac{3}{4},1,\frac{3}{4};-\frac{1+x}{2}\right)=$$

$$=2^{\frac{1}{2}}(1+s)^{\frac{1}{4}}(3-s)^{\frac{1}{2}}+2^{\frac{3}{4}}B\left(\frac{3}{2},\frac{1}{4}\right)(1+x)^{2\alpha}F\left(-\frac{3}{4},1,\frac{5}{4};-\frac{1+s}{2}\right).$$

Shunday qilib (3.69), (3.71), (3.72) – tenglikka asosan (3.64) – ifodani quyidagicha yozib olamiz:

$$J(s)=\frac{1}{(1+x)^{2\alpha}(1-x)^{\alpha}}\cdot\frac{1}{s-t+2}\Big[\pi(1+x)^{2\alpha}(1+x)^{\alpha}-$$

$$-2^{\frac{1}{4}}B\left(\frac{3}{2},\frac{1}{4}\right)(1+x)^{2\alpha}F\left(\frac{3}{2},-\frac{1}{4},\frac{3}{4};\frac{1-x}{2}\right)+$$

$$+2^{\frac{1}{2}}(1+s)^{\frac{1}{4}}(3-s)^{\frac{1}{2}}+2^{\frac{3}{4}}B\left(\frac{3}{2},\frac{1}{4}\right)(1+x)^{2\alpha}F\left(-\frac{3}{4},1,\frac{5}{4};-\frac{1+s}{2}\right)\Big]+$$

$$+\frac{1}{4-(x-1)(s+1)}\Big[2^{\frac{3}{4}}(x-1)B\left(\frac{3}{2},\frac{5}{4}\right)F\left(\frac{5}{4},1,\frac{11}{4};\frac{1-x}{2}\right)+$$

$$+2^{\frac{3}{2}}(1+s)^{\frac{1}{4}}(3-s)^{\frac{1}{2}}-2^{\frac{3}{4}}B\left(\frac{3}{2},\frac{1}{4}\right)F\left(-\frac{3}{4},1,\frac{5}{4};-\frac{1+s}{2}\right)\Big]$$

Endi (3.73) ifodani (3.61) – tenglikka qo‘yib ush bu tenglikka ega bo‘lamiz

$$\nu_0(x) = -\frac{1}{2\pi}\cdot\int_{-1}^{1}\frac{\nu_1(s)ds}{s-x+2}+$$

$$+\frac{\pi}{2\pi^2}\int_{-1}^{1}\frac{\nu_1(s)ds}{s-x+2}+\frac{2^{\frac{3}{4}}B\left(\frac{3}{2},\frac{1}{4}\right)}{2\pi^2(1+x)^{2\alpha}(1-x)^{\alpha}}\int_{-1}^{1}\left[\left(\frac{1+x}{2}\right)^{\frac{1}{2}}F\left(\frac{3}{2},-\frac{1}{4},\frac{3}{4};\frac{1-x}{2}\right)-\right.$$

$$\left.-F\left(-\frac{3}{4},1,\frac{5}{4};-\frac{1+s}{2}\right)\right]\frac{\nu_1(s)ds}{s-x+2}-\frac{2^{\frac{1}{2}}}{2\pi^2}\int_{-1}^{1}\left(\frac{3-s}{1+x}\right)^{2\alpha}\left(\frac{1+s}{1-x}\right)^{\alpha}\frac{\nu_1(s)ds}{s-x+2}-$$

$$-\frac{1}{2\pi^2(1+x)^{2\alpha}(1-x)^{\alpha}}\int_{-1}^{1}\left[2^{\frac{3}{4}}(x-1)B\left(\frac{3}{2},\frac{5}{4}\right)F\left(\frac{5}{4},1,\frac{11}{4};\frac{1-x}{2}\right)+\right. \tag{3.73}$$

$$\left.+2^{\frac{3}{2}}(1+s)^{\frac{1}{4}}(3-s)^{\frac{1}{2}}-2^{\frac{3}{4}}B\left(\frac{3}{4},\frac{1}{4}\right)F\left(-\frac{3}{4},1,\frac{5}{4};-\frac{1+s}{2}\right)\right]\times$$

$$\times\frac{\nu_1(s)ds}{4-(x-1)(s+1)}+R_3[\nu_1]+\phi_1(x)$$

yoki

$$\nu_0(x) = -\frac{2^{\frac{3}{4}}B\left(\frac{3}{2},\frac{1}{4}\right)}{2\pi^2(1-x)^{\alpha}(1+x)^{2\alpha}}\int_{-1}^{1}\left[\left(\frac{1+x}{2}\right)^{\frac{1}{2}}F\left(\frac{3}{2},-\frac{1}{4},\frac{3}{4};\frac{1-x}{2}\right)-\right.$$

$$\left.-F\left(-\frac{3}{4},1,\frac{5}{4};-\frac{1+s}{2}\right)\right]\frac{\nu_1(s)ds}{s-x+2}-\frac{2^{\frac{1}{2}}}{2\pi^2}\int_{-1}^{1}\left(\frac{3-s}{1+x}\right)^{2\alpha}\left(\frac{1+s}{1-x}\right)^{\alpha}\frac{\nu_1(s)ds}{s-x+2}+$$

$$+B_0(x,s)+R_3[\nu_1]+\Phi_1(x) \tag{3.74}$$

$B_0(x,s)\in[-1,1]\times[-1,1]$ da uzluksiz funksiya.

(3.74) tenglamaning xarakteristik qismini ajratib olamiz.

$$\nu_0(x) = -\frac{2^{\frac{1}{2}}\cdot 2^{2\alpha}}{2\pi^2}\int_{-1}^{1}\left(\frac{1+s}{1-x}\right)^{\alpha}\frac{\nu_1(s)ds}{s-x+2}+R_4[\nu_1]+B_0(x,s)+\Phi_1(x) \tag{3.75}$$

Bu yerda

$$R_4[\nu_1] = R_3[\nu_1] - \frac{2^{\frac{1}{2}}}{2\pi^2}\int_{-1}^{1}\left[\left(\frac{3-s}{1+x}\right)^{2\alpha} - 2^{2\alpha}\right]\left(\frac{1+s}{1-x}\right)^{\alpha}\frac{\nu_1(s)ds}{s-x+2}$$

(3.75) munosabatda x=-x bilan almashtirib, $\nu_0(x) = \nu_0(-x) = \tilde{\nu}_0(x)$ belgilashga asosan

$$\tilde{\nu}_0(x) = -\int_{-1}^{1} m\left(\frac{1+s}{1+x}\right)\frac{\nu_1(s)ds}{1+s} + R_5[\nu_1] + \Phi_3(x) \qquad (3.76)$$

bunda

$$m(y) = \frac{2^{\frac{1}{2}}\cdot 2^{2\alpha}}{2\pi^2}\left(\frac{y^{-\alpha}}{1+y}\right),\ y = \frac{1+s}{1+x},\ R_5[\nu_1] = R_4[\nu_1] + B_0(x,s).$$

(3.76) munosabat $\tilde{\nu}_0(x)$ va $\nu_1(x)$ funksiyalar orasidagi birinchi funksional munosabat deyiladi.

Xuddi shuningdek, $\tilde{\nu}_0(x)$ va $\nu_1(x)$ funksiyalar orasidagi ikkinchi funksional munosabatni xam xosil qilish mumkin.

Natijada Viner-Xopf integral tenglamasi xosil kilinadi. Viner-Xopf integral tenglamasi xam siingulyar integral tenglama deyiladi. Bu tenglama Fure almashtirishi yordamida Koshi yadroli integral tenglama kabi Rimanning chegaraviy masalasiga keltiriladi va uning yechimi kvadraturada integrallanuvchi funksiya bo‘ladi.

III BOB YUZASIDAN XULOSALAR.

Uchta paragrafdan iborat bo'lgan III bobda singulyar koeffitsientli tenglamaning bir sinfi uchun Gellerstedt va buzilish chizig'ida umumiy ulanish shartli masala yechimining asosiy natijalari bayoni, singulyar integral tenglamalar sistemasini keltirib chiqarish, $\nu_1(x)$ va $\nu_0(x)$ noma`lum funksiyalar orasidagi birinchi va ikkinchi funksional munosabatlar keltirilgan.

Singulyar integral tenglamalar sistemasini keltirib chiqarish. Ushbu

$sign y|y|^m U_{xx} + U_{yy} - \frac{m}{2y} U_y = 0$ aralash turdagi tenglamani $z = x + iy$ kompleks

tekisligining $\operatorname{Im} z > 0$ yarim tekisligida, shu yarim tekisligida yotuvchi va

uchlari $A(-1,0)$, $B(1,0)$ nuqtalarda bo'lgan σ_0: $x^2 + \frac{4}{(m+2)^2} y^{m+2} = 1$ normal

chiziq, $Imz < 0$ yarim tekislikda esa (3.1) tenglamaning AC va BC xarakteristikalar bilan chegaralangan bir bog'lamli chekli D sohada o'rganamiz.

BF-masalasi**.** D aralash sohada (1.1) tenglamaning ushbu shartlarni qanoatlantiruvchi

1. $U(x,y) \in C(\overline{D})$

2. $U(x,y) \in C^2(D^+)$ va D^+ da (3.1) tenglamaning regulyar yechimi.

3. $U(x,y)$ D^- xarakteristik uchburchakda (3.1) tenglamaning $R_1 -$ sinfga tegishli umumlashgan yechimi, Trikomi integral tenglamasini regulyarizatsiyalash, $\nu_1(x)$ va $\nu_0(x)$ noma`lum funksiyalar o`rtasidagi birinchi va ikkinchi funksional munosabatlar asosiy natijalari bayoni keltirilgan.

Xulosa

Ushbu magistrlik dissertatsiyasi ishi “Aralash tipdagi tenglama uchun Gellerstedt va buzilish chizig‘ida umumiy ulanish shartli masalalar” mavzusiga bag’ishlangan.

Magistrlik dissertatsiyasi kirish qismi, 3 bob, 10 paragraf hamda 44 adabiyotni o’z ichiga olgan foydalanilgan adabiyotlar ro‘yxatidan iborat. Belgilashlar ikki raqamli bo‘lib, ular orasi nuqta bilan ajratilgan. Birinchi raqam paragraf nomerini bildiradi, ikkinchi son esa tartib nomerini bildiradi. Masalan, 2.1-teorema yozuvi-ikkinchi paragrafning 1- teoremasi ekanligini bildiradi, yoki (2.8) belgilash 2- paragrafdagi 8- formula ekanligini anglatadi.

Kirish qismida masalaning dolzarbligi, maqsad va vazifalari, tadqiqot ob`ekti va predmeti, ilmiy yangiligi va farazlari hamda bajarilgan ishlar tasnifi keltirilgan, shuningdek, tadqiq etilayotgan masalalarning holati haqida so’z yuritilib, asosiy tushunchalar qisqacha keltirilgan.

Dissertatsiya ishining birinchi bobida asosiy tushunchalar, ta’riflar keltirib o’tilgan. Shuningdek, aralash tipdagi tenglama uchun Gellerstedt va buzilish chizig‘ini umumiy ulanish shartli masalani o’rganishning nazariy asoslari haqida so`z yuritilgan.

II bobda soha ichida buziladigan singulyar koeffitsientli elliptik turdagi tenglama uchun umumiy ulashish shartli masalaning qo‘yilishi va yagonaligini ko`rsatishning metodikasi hamda T masalasi yechimining mavjudligi bayoni keltirilgan.

Uchta paragrafdan iborat bo’lgan III bobda singulyar koeffitsientli tenglamaning bir sinfi uchun Gellerstedt va buzilish chizig‘ida umumiy ulanish shartli masala yechimining asosiy natijalari bayoni, singulyar integral tenglamalar sistemasini keltirib chiqarish, $\nu_1(x)$ va $\nu_0(x)$ noma`lum funksiyalar orasidagi birinchi va ikkinchi funksional munosabatlar keltirilgan.

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