O‘ZBEKISTON RESPUBLIKASI

OLIY TA’LIM, FAN VA INNOVATSIYALAR VAZIRLIGI

TERMIZ DAVLAT UNIVERSITETI

MAGISTRATURA BO‘LIMI

Qo‘lyozma huquqida

UDK 517.968.23

OLTIYEV BAXRIDDIN JO‘RAYEVICH

«SINGULYAR KOEFFITSIYENTLI BUZILUVCHAN GIPERBOLIK TIPDAGI TENGLAMA UCHUN BITSADZE- SAMARSKIY MASALASI»

70540101-Matematika (yo‘nalishlar bo‘yicha) mutaxassisligi bo‘yicha

Magistr akademik darajasini olish uchun yozilgan

DISSERTATSIYASI

Ilmiy rahbar: **PhD. N.D.BOLTAYEV**

T E R M I Z-2023

Magistrlik dissertatsiyasi mavzusi Termiz davlat universiteti rektorining 20___-yil ______dagi №________ sonli buyrug'i asosida tasdiqlangan.

Magistrlik dissertatsiyasi Termiz davlat universiteti "Algebra va geometriya" kafedrasida bajarilgan.

Magistrlik dissertatsiyasi elektron nusxasi Termiz davlat universitetining rasmiy veb sahifasiga joylashtirilgan.

Dissertatsiya manzilining QR-kodi:

Magistrlik dissertatsiyasi bilan Termiz davlat universitetining axborot-resurs markazida tanishish mumkin (__) raqam bilan ro'yxatga olingan.

Manzil: Termiz shahri Barkamol avlod ko'chasi 43-uy.)

Ilmiy rahbar: _____________ PhD. N.D.Boltayev

Kafedra mudiri: _____________ dots. S.T.Choriyeva

Magistratura bo'limi boshlig'i: _______ PhD. A.B.Narbayev

70540101-matematika mutaxassisligi bitiruvchisi Oltiyev Baxriddin Jo'rayevichning "Singulyar koeffitsiyentli buziluvchan giperbolik tipdagi tenglama uchun Bitsadze-Samarskiy masalasi" mavzusida magistrlik dissertatsiyasi

ANNOTATSIYASI

Tayanch so‘zlar: Umumlashgan Trikomi tenglamasi, singulyar koeffisientli Gellerstedt tenglamasi, Bitsadze-Samarskiy shartli masala, lokal va nolokal shartli masalalar, singulyar integral tenglama, Viner—Xopf integral tenglamasi, indeks, Fredgol’m integral tenglamasi.

Tadqiqot ob’yekti: Aralash tipdagi Trikomining umumlashgan tenglamasi va singulyar koeffisientli aralash tipdagi tenglamalar, ular uchun nostandart sohada qo‘yilgan lokal va nolokal shartli masalalar.

Ishning maqsadi: Umumlashgan Trikomi tenglamasining regulyar va umumlashgan yechimini topish, singulyar koeffisientli Gellerstedt tenglamasi uchun Trikomi masalasi yechimining yagonaligi va mavjudligini ko‘rsatish, elliptik qismi normal chiziq va koordinata o‘qlari bilan chegaralangan sohada singulyar koeffisientli Gellerstend tenglamasi uchun Bitsadze-Samarskiy shartli masala korrektligini isbotlash.

Tadqiqot metodlari: Dissertatsiyada ekstremum prinsipi usuli hamda regulyar va singulyar integral tenglamalar nazariyasi usullaridan foydalanilgan. Trikomining nostandart singulyar integral tenglamasi yechimini S.G.Mixlinning regulyarlashtirish usuli yordamida qurish.

Tadqiqot natijalarining nazariy va amaliy ahamiyati: Olingan natijalar nazariy ahamiyatga ega bo‘lib, undan matematik-fizika tenglamalari fanini o‘rganish davomida bitiruvchi kurs talabalari va magistrantlar uchun maxsus kurslar o‘qitishda hamda ushbu sohada ilmiy tadqiqot jarayonida foydalanish mumkin

Qo‘llanish sohasi: Oliy ta’lim va undan keyingi ta’lim jarayonlarida.

70540101-mathematics graduate Oltiyev Bakhriddin Jorayevich's master's thesis on the topic "Bitsadze-Samarsky problem for a destructible hyperbolic equation with singular coefficients"

ANNOTATION

Base words : Generalized Tricomi equation, Gellerstedt equation with singular coefficient, Bitsadze-Samarsky conditional problem, local and nonlocal conditional problems, singular integral equation, Wiener-Hopf integral equation, index, Fredgol's integral equation.

Research object : Generalized Tricomi equation of mixed type and equations of mixed type with singular coefficients, local and non-local conditional problems for them in non-standard domain.

Work Purpose : To find a regular and generalized solution of the generalized Tricomi equation, to show the uniqueness and existence of the solution of the Tricomi problem for the Gellerstedt equation with a singular coefficient, to prove the correctness of the Bitsadze-Samarskii conditional problem for the Gellerstand equation with a singular coefficient in the field whose elliptic part is bounded by the normal line and coordinate axes.

Research methods : The method of the extremum principle and the methods of the theory of regular and singular integral equations were used in the dissertation. Construction of the solution of Tricomi's non-standard singular integral equation using S.G. Mikhlin's regularization method.

Research of the results theoretical and practical Significance : The obtained results are of theoretical importance and can be used in the teaching of special courses for graduate students and undergraduates during the study of mathematical-physical equations and in the process of scientific research in this field.

Application field : Higher education and from him next education in processes.

MUNDARIJA

Kirish

Magistrlik dissertatsiyasi mavzusining asoslanishi va uning dolzarbligi.

Mamlakatimiz istiqlolga erishgan ilk kunlardanoq, davlatimiz tomonidan amalga oshirilayotgan bunyodkorlik ishlari vatanimiz mustaqilligi va ozodligi tufaylidir. Mustaqillik zamirida yuz berayotgan islohatlar sezilarli darajada insoniyat turmush tarzini oʻzgartirib yubordi. Soʻnggi yillarda yoshlarga yaratilgan imkoniyatlar har bir yigit qizni harakatda boʻlishga undaydi.

Hozirgi kunda Prezidentimiz Sh.Mirziyoyev tomonidan “Mening nazarimda, jamiyat hayotining tanasi iqtisodiyot boʻlsa, uning joni va ruhi – ma’naviyatdir. Biz yangi Oʻzbekistonni barpo etishda ana shu ikkita mustahkam ustunga, ya’ni, bozor tamoyillariga asoslangan kuchli iqtisodiyotga hamda ajdodlarimizning boy merosi, milliy va umuminsoniy qadriyatlarga asoslangan kuchli ma’naviyatga tayanamiz”[1-4]. Yoshlarga keng imkoniyatlar yaratib berilmoqda, yirik loyihalar ustida ishlanmoqda. Ularning bilim va istedodlarini shakllantirib, milliy ma’naviyatimizni uzoqlashib ketayotgani sezilib qolmoqda. Ular oʻzlari oʻqib kelgan xorijiy davlatlardagi tajribani oʻrganib, tajriba almashib kelishmoqda. Yangi Oʻzbekiston taraqqiyot strategiyasining maqsadi – aholining barcha qatlamlariga munosib hayot darajasini va turmush sharoitlarini yaratib berish, ishtimoiy himoya va bandlikni ta’minlash, daromadlar barqaror oʻsishiga erishish, jamiyatning madaniy darajasi, bag’rikenglik va mehribonlik fazilatlarini yanada mustahkamlashdan iborat [5].

Oʻzbekistonda ta’lim tizimini isloh qilishning dasturiy hujjatlarida takidlanganidek, mamlakatimiz ta’lim tizimi hodimlari oldiga raqobatbardosh kadrlar tayyorlash, ta’lim tarbiya jarayonini jahon andozalari darajasiga yetkazishni asosiy vazifa qilib qoʻygan[6]. Shu ma’noda olib qaraganda, yoshlarning yangi avlodi istiqbol masalalarini kun tartibiga dadil qoʻyadigan va uni yecha oladigan, siyosiy hamda ijtimoiy – iqtisodiy hayotda oʻziga mustaqil yoʻl topa oladigan qobiliyatga ega boʻlishi kerak.

Jahon miqyosida, differensial tenglamalar nazariyasi sohasida olib borilayotgan tadqiqotlarda, giperbolik-parabolik va elliptik-giperbolik aralash tipdagi tenglamalar uchun lokal va nolokal chegaraviy masalalar yechimini topish yetakchi o‘rinlardan birini egallaydi. Bu aralash tipdagi tenglamalar uchun nolokal chegaraviy masalalar nazariyasi texnika va tabiatda, jumladan, gaz dinamikasida, gaz-neft havzalari holatini o‘rganishda, aerodinamikada, murakkab tuzilishga ega ob’ektlarda issiqlik va massa almashinishini o‘rganishda, gidrodinamikada, yer osti suvlarini filtrlash, g‘ovak muhit bilan o‘ralgan kanallarda suyuqlik harakatini o‘rganishda, elektrodinamikada, o‘tkazgichlarda elektr tebranishlari va boshqa hodisalarning matematik modelini qurishda, shuningdek, boshqa qator sohalarda o‘z tatbiqlari bilan muhim ahamiyatga ega.

Dunyoda aralash tipdagi tenglamalar sohasida o‘rganilayotgan masalalar ko‘lami sezilarli darajada kengaymoqda va singulyar koeffitsientli tenglamalar uchun nolokal masalalarni tadqiq etish jadal rivojlanmoqda. Xususan, aralash tipdagi tenglamalar uchun chegaraviy masalalarni aniqlash, eng sodda buzuluvchan elliptik tenglama uchun normal sohada Dirixle va Xolmgren masalasi yechimini beruvchi formulalarni oshkor ko‘rinishda topish, aralash tipdagi eng sodda tenglama uchun ekstremum prinsipini ta’riflash, singulyar koeffitsientli tenglamalar uchun kichik hadlar oldidagi koeffitsientlar qabul qiladigan qiymatlarga qarab korrekt masalalarni qo‘yishga katta e’tibor qaratilmoqda.

Hozirgi vaqtda O‘zbekistonda ilmiy va amaliy tatbiqlarga ega bo‘lgan fundamental fanlarga e’tibor yanada kuchaymoqda. Singulyar koeffitsientli aralash tipdagi tenglamalar uchun chegaraviy masalalarga keltiriluvchi amaliy va nazariy muammolarni yechishning samarali usullarini topish, A.V.Bitsadzening ekstremum prinsiplarini ta’riflash sohadagi muhim vazifalardan biri hisoblanadi. Shuningdek, matematik fanlarning ustuvor yo‘nalishlari bo‘yicha, xususan, matematik fizika va xususiy hosilali differensial tenglamalar hamda matematik modellashtirish bo‘yicha xalqaro standartlar darajasida olib borilayotgan ilmiy-tadqiqotlar mamlakatimizdagi matematik ilmiy tadqiqotlarnirivojlantirishga sezilarli hissa qo‘shadi. Bu borada, singulyar koeffitsientli giperbolo-parabolik va elliptiko-giperbolik tenglamalarga qo‘yilgan Bitsadze-Samarskiy tipidagi nolokal shartli masalalar korrektligini isbotlash muhim ilmiy-amaliy ahamiyat kasb etadi.

O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PF-4947-son “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha harakatlar strategiyasi to‘g‘risida”gi Farmoni, 2017-yil 17-fevraldagi PQ-2789-son “Fanlar akademiyasi faoliyati, ilmiy tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora tadbirlari to‘g‘risida”, 2017-yil 20-apreldagi PQ-2909-son “Oliy ta’lim tizimini yanada rivojlantirish chora-tadbirlari to‘g‘risida”, 2018-yil 27-apreldagi PQ-3682-son “Innovatsion g‘oyalar, texnologiyalar va loyihalarni amaliyotga joriy qilish tizimini yanada takomillashtirish chora-tadbirlari to‘g‘risida”, 2020-yil 7-maydagi PQ-4708-son “Matematika sohasida ta’lim sifatini oshirish va ilmiy tadqiqot ishlarini rivojlantirish to‘g‘risida” gi qarorlari hamda fundamental fanlarga tegishli boshqa normativ huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda muayyan darajada xizmat qiladi.

Tadqiqotning ob’yekti va predmeti: singulyar koeffitsientli giperbolik va Gellerstedt tenglamalari uchun nolokal masalalarni tadqiq etishdan iborat. Singulyar koeffitsientli giperbolik tipdagi tenglama uchun Bitsadze-Samarskiy tipidagi shartli masalalar.

Tadqiqotning maqsadi va vazifalari: giperbolik tipdagi singulyar koeffitsientli tenglamalar uchun nolokal Bitsadze-Samarskiy shartli masalalarning korrekt qo‘yilganini isbotlashdan iborat.

– A.V.Bitsadzening ekstremum prinsiplarini ta’riflash va isbotlash;

– singulyar integral tenglamani regulyarizatsiyalash;

– Viner–Xopf integral tenglamasini keltirib chiqarish va tadqiq etish;

– singulyar koeffitsientli elliptik-parabolik, giperbolik-parabolik va elliptik-giperbolik tenglamalarga Bitsadze-Samarskiy tipidagi nolokal shartli masalalarning korrektligini isbotlash.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

– singulyar koeffitsientli giperbolik tipdagi tenglama uchun Bitsadze-Samarskiy shartli masalasining aralash sohaning parabolik qismidagi yechimi topilgan;

– singulyar koeffitsientli aralash giperbolik-parabolik tipdagi tenglama yechimining yagonaligi ekstremum prinsipi, yechimning mavjudligi Fredgolm integral tenglamalar nazariyasi asosida isbotlangan;

– singulyar koeffitsientli Gellerstedt tenglamasi uchun Bitsadze-Samarskiy sharti parallel xarakteristikada berilgan masalalar yechimlarining mavjudligi va yagonaligi haqidagi teoremalar isbotlangan;

– kompleks o‘zgaruvchili funksiyalar nazariyasining qoldiqlar nazariyasidan foydalanib, Viner-Xopf integral tenglamasi indeksining nolga tengligi isbotlangan.

Tadqiqotning asosiy masalalari va farazlari. Eng sodda buzuluvchan aralash tipdagi tenglama uchun dastlabki fundamental tadqiqotlarni o‘tgan asrning yigirmanchi yillarida italiyalik mashhur matematik Franchesko Trikomi [7] amalga oshirgan. Bu ishdan keyin aralash tipdagi tenglamalar uchun chegaraviy masalalar nazariyasi jadal rivojlanish bosqichiga o‘tdi. Mazkur yo‘nalishda muhim natijalar quyidagi mualliflar tomonidan olingan: Ye.Xolmgren [8] eng sodda buzuluvchan elliptik tenglama uchun normal sohada Dirixle va Xolmgren masalasi yechimini beruvchi formulalarni oshkor ko‘rinishda topgan; S.Gellerstedt[9] eng sodda buzuluvchan elliptik tipdagi tenglama uchun potensiallar nazariyasini yaratgan va natijalari asosida Dirixle va Xolmgren [8] masalalarida Grin funksiyasini topgan; S.G.Mixlin [10,11] o‘z ishlarida Karlemanning singulyar integral tenglamasini yechish usulini hamda Liuvillning funksiyalarni analitik davom ettirish haqidagi teoremasidan foydalanib F.Trikomining singulyar tenglamasini yechish usulini yaratgan; A.V.Bitsadze

[12,13] aralash tipdagi eng sodda tenglama uchun ekstremum prinsipini ta’riflagan;

O‘tgan asrning 70-yillari boshiga kelib, aralash tipdagi tenglamalar uchun chegaraviy masalalar nazariyasining barcha muammolari hal etilgandek tuyuldi va bu sohaning keyingi rivoji uchun tubdan yangi qo‘yilgan masalalarga zaruriyat tug‘ildi. Bu yangi masala A.V.Bitsadze va A.A.Samarskiyning [14] hamkorlikda ta’riflagan va o‘rgangan masalasi bo‘ldi. Bu masala aralash tipdagi tenglamalar uchun chegaraviy masalalarning keyingi rivojida yangi bir turki bo‘ldi. Mazkur ish keyin Bitsadze-Samarskiy shartli masalalari nazariyasining paydo bo‘lishi va rivojlanishiga asos bo‘ldi.

Hozirgi vaqtda Bitsadze-Samarskiy shartiga o‘xshash shartli masalalar nolokal masalalar deb nomlanadi.

Mazkur dissertatsiya singulyar koeffitsientli buziluvchan giperbolik tipdagi tenglama uchun Bitsadze- Samarskiy, ya’ni Gellerstedt tenglamalari uchun nolokal masalalarni tadqiq etishga bag‘ishlangan.

Tadqiqot mavzusi bo‘yicha adabiyotlar sharhi (tahlili) Buzuluvchan va aralash tipdagi tenglamalar uchun chegaraviy masalalar nazariyasida e’tiborga loyiq natijalarni I.L.Karol [15], K.N.Babenko[16], S.P.Soldatov [17], Ye.I.Moiseev[18,19], M.A.Sadibekov[20], T.Sh.Kalmenovlar [21], o‘zbek matematiklaridan M.S.Saloxiddinov[22], T.D.Jo‘raev[23,24], M. Mirsaburov[25,26], B. Islomov, A.K.O‘rinov[27] va boshqalar olishgan, shu sababli yuqoridagi mualliflar tomonidan chop etilgan maqola va monografiyalaridan foydalanildi.

Xususan [28] adabiyotdagi K. I. Babenko tomonidan shakli o‘zgargan Koshi masalasi uchun kiritilgan umumlashgan yechimlar sinfinidan foydalanilgan.

[29] adabiyotdagi torning erkin tebranishi

$$u_{tt} = a^2 u_{xx}$$

tenglamasidan foydalanib, xarakteristik tenglamalar tuzilgan.

[14,26,30,31] adabiyotlardan

$$u[\theta_0(x)] = \mu_1 u[\theta^*{}_1(p_1(x))] + \mu_2 u[\theta^*{}_2(p_2(x))] + \rho(x), \ x \in \bar{I}, \quad \mu_1 > 0, \mu_2 > 0$$

tenglamada izlanayotgan $u(x, y)$ funksiyaning qiymatlarini AC, E_1B_1 va E_2B_2 xarakteristikalari bilan bog‘laydigan Bitsadze-Samarskiy sharti hisoblashda foydalanilgan.

Tadqiqotda qo‘llanilgan metodikaning tavsifi. Dissertatsiyada ta’riflangan masalalar yechimini yagonaligini isbotlashda A.V.Bitsadzening ekstremum prinsipiga o‘xshash prinsiplardan va ketma-ket yaqinlashish iterasiya usulidan hamda masalalar yechimini mavjudligini isbotlashda singulyar integral tenglamalar nazariyasidan foydalangan holda Viner-Xopf, Fredgolm integral tenglamalari, shuningdek, oddiy differensial tenglamalar nazariyasidan ikkinchi tartibli tenglamalarni yechish usulidan foydalanilgan.

Tadqiqot natijalarining nazariy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati singulyar koeffitsientli aralash tipdagi tenglamalar uchun lokal va nolokal masalalar nazariyasining keyingi rivojida foydalanish mumkinligi, hamda nostandart singulyar integral tenglamalar nazariyasini yanada rivojlantirishi bilan izohlanadi.

Tadqiqot natijalarining amaliy ahamiyati singulyar koeffitsientli xususiy hosilali differensial tenglamalarga olib kelinadigan amaliy masalalarni yechishga tadqiq etilishi hamda nostandart singulyar integral tenglamalarni yechishda qo‘llanishi bilan izohlanadi.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish, uchta bob, xulosa va adabiyotlar ro‘yxatidan iborat. Dissertatsiyaning hajmi 54 betdan iborat.

I BOB. SINGULYAR KOEFFITSIENTLI BUZILUVCHAN GIPERBOLIK TIPDAGI TENGLAMA UCHUN MASALALARNI O‘RGANISHNING NAZARIY ASOSLARI

1.1-§ Shakli o‘zgargan Koshi masalasi

Ushbu

$$-(-y)^m u_{xx} + u_{yy} + \left(\alpha_0 /(-y)^{1-m/2}\right) u_x + (\beta_0 / y) u_y = 0 \qquad (1.1)$$

tenglamani $z = x + iy$, $\operatorname{Im} z < 0$ kompleks yarim tekisligining chekli bir bog‘lamli D sohasida o‘rganamiz. D soha (1.1) tenglamaning

$$AC : x - \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = -1, \qquad BC : x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 1 \ ,$$

harakteristikalari, hamda $y = 0$ o‘qining AB kesmasi bilan chegaralangan, bu yerda $A(-1,0)$, $B(1,0)$.

(1.1) tenglamada m, α_0 va β_0-o‘zgarmas sonlar bo‘lib, ular ushbu shartlarni qanoatlantiradi:

$$m > 0 \ , \ -m/2 < \beta_0 < (m+4)/2 \ , \ \left|\alpha_0\right| < (m+2)/2 \ .$$

(1.1) tenglama yechimining tuzilishi va differensial xossalari uning kichik hadlari oldidagi α_0 va β_0 koeffitsientlariga qat’iy bog‘liqdir.

Bu bog‘liqlikni oydinlashtirish maqsadida, α_0 va β_0 parametrlar tekisligida

$$A_0 D_0 : \beta_0 - \alpha_0 = (m+4)/2, \quad D_0 B_0 : \beta_0 + \alpha_0 = (m+4)/2 \ ,$$

$$B_0C_0 : \beta_0 - \alpha_0 = -m/2, \quad A_0C_0 : \beta_0 + \alpha_0 = -m/2 \;,$$

$A_0 D_0 B_0 C_0$ kvadratni kiritamiz va $P(\alpha_0, \beta_0)$ nuqtaning bu kvadratda o‘zgarishiga qarab, (1.1) tenglama uchun masalalar qo‘yamiz.

(1.1) tenglamaning regulyar yechimi deganda, D sohada ikkinchi tartibli uzluksiz hosilalarga ega bo‘lgan va bu tenglamani qanoatlantiradigan $u(x, y)$ funksiya tushuniladi.

1. $P(\alpha_0, \beta_0) \in \Delta A_0B_0C_0 \cup A_0C_0 \cup B_0C_0 \cup \{C_0\}$ bo‘lsin.

Shakli o‘zgargan Koshi masalasi. D sohada (1.1) tenglamaning ushbu

$$u(x,0) = \tau(x) \;, \qquad x \in \bar{J} \;, \tag{1.2}$$

$$\lim_{y \to -0} (-y)^{\beta_0} \frac{\partial u}{\partial y} = \nu(x) \;, \quad x \in J \;, \tag{1.3}$$

boshlang‘ich shartlarni qanoatlantiruvchi $u(x, y) \in C(\bar{D}) \cap C^2(D)$ regulyar yechimi topilsin, bu yerda $\tau(x) \in C(\bar{J}) \cap C^2(J)$, $\nu(x) \in C^2(J)$-berilgan funksiyalar, $J = (-1,1)$ $y = 0$ o‘qining intervali.

Shakli o‘zgargan Koshi masalasini Riman metodi yordamida yechamiz. (1.1) tenglama quyidagi xarakteristik koordinatalarga nisbatan:

$$\xi = x - \frac{2}{m+2}(-y)^{\frac{m+2}{2}} \;, \quad \eta = x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}} \;, \tag{1.4}$$

$$\eta - \xi = \frac{4}{m+2}(-y)^{\frac{m+2}{2}} \geq 0 \;, \;\; \eta \geq \xi$$

ushbu

$$L(u) = u_{\xi\eta} + \frac{\beta}{\eta - \xi} u_\xi - \frac{\alpha}{\eta - \xi} u_\eta = 0 \;, \tag{1.5}$$

Eyler-Puasson-Darbu tenglamasiga o‘tadi, bu yerda

$$\alpha = \frac{m+2(\beta_0+\alpha_0)}{2(m+2)}\ ,\quad \beta = \frac{m+2(\beta_0-\alpha_0)}{2(m+2)}$$

(1.4) akslantirishda $x0y$ tekislikdagi D soha $\xi 0\eta$ tekislikdagi $\Delta = A_1, B_1, C_1$ uchburchakka akslanadi: bu uchburchakning uchlari $A_1 = A_1(-1,-1)$, $B_1 = B_1(1,1)$ va $C_1 = C_1(-1,1)$ nuqtalarda bo‘lib, tomonlari $\xi=-1$, $-1\le\eta\le 1$; $\eta=1$, $-1\le\xi\le 1$; $\eta-\xi=0$ kesmalardan iborat, bu almashtirishda (1.2) va (1.3) shartlar ushbu

$$\lim_{\eta-\xi\to 0} u(\xi,\eta) = \tau(\xi)\ ,\qquad -1\le\xi\le 1\ , \tag{1.6}$$

$$\lim_{\eta-\xi\to 0}\left(\frac{m+2}{4}(\eta-\xi)\right)^{\alpha+\beta}\left(\frac{\partial u}{\partial\xi}-\frac{\partial u}{\partial\eta}\right) = \nu(\xi)\ ,\quad -1<\xi<1 \tag{1.7}$$

ko‘rinishda bo‘ladi. (1.5) tenglama bilan birgalikda unga qo‘shma

$$M(\xi) = \frac{\partial^2\vartheta}{\partial\xi\partial\eta} - \frac{\partial}{\partial\xi}\left(\frac{\beta}{\eta-\xi}\vartheta\right) + \frac{\partial}{\partial\eta}\left(\frac{\alpha}{\eta-\xi}\vartheta\right) = 0 \tag{1.8}$$

tenglamani ham o‘rganamiz.

Giperbolik tenglamalar nazariyasida $R(\xi,\eta;\xi_0,\eta_0)$-Riman funksiyasi fundamental ahamiyatga ega va u (1.5) tenglama uchun quyidagicha aniqlanadi:

1) $R(\xi,\eta;\xi_0,\eta_0)$-$(\xi\ ,\eta)$ o‘zgaruvchi bo‘yicha (1.8) qo‘shma tenglamaning yechimi bo‘ladi;

2) $\xi=\xi_0$, $\eta=\eta_0$ xarakteristikalarda $R(\xi,\eta;\xi_0,\eta_0)$ funksiya ushbu

$$R(\xi_0,\eta;\xi_0,\eta_0) = \exp\left(\int_{\eta_0}^{\eta}\frac{\beta dt}{t-\xi_0}\right) = \exp\left(\beta\ln\frac{\eta-\xi_0}{\eta_0-\xi_0}\right) = \left(\frac{\eta-\xi_0}{\eta_0-\xi_0}\right)^{\beta} \tag{1.9}$$

$$R(\xi,\eta_0;\xi_0,\eta_0) = \exp\left(-\int_{\xi_0}^{\xi}\frac{\alpha dt}{\eta_0-t}\right) = \exp\left(\alpha\ln\frac{\eta_0-\xi}{\eta_0-\xi_0}\right) = \left(\frac{\eta_0-\xi}{\eta_0-\xi_0}\right)^{\alpha} \tag{1.10}$$

qiymatlarni qabul qiladi, bu yerda $\eta\le\eta_0$, $\xi\ge\xi_0$, $\eta\ge\xi$.

(1.5) tenglama uchun Riman funksiyasi ushbu

$$R(\xi,\eta;\xi_0,\eta_0)=\frac{(\eta-\xi)^{\alpha+\beta}}{(\eta_0-\xi)^{\beta}(\eta-\xi_0)^{\alpha}}F(\beta,\alpha,1;\sigma)\ , \quad (1.11)$$

ko‘rinishga ega, bu yerda $F(\cdots)$ Gaussning gipergeometrik funksiyasi va

$$\sigma=\frac{(\xi-\xi_0)(\eta-\eta_0)}{(\xi-\eta_0)(\eta-\xi_0)}\ .$$

Δ_ε orqali $P_1P:\ \xi=\xi_0,\quad PP_2:\eta=\eta_0$ va $\eta=\xi+\varepsilon$ to‘g‘ri chiziqning P_1P_2 kesmasi bilan chegaralangan uchburchakni belgilaymiz, bu yerda

$$P=P(\xi_0,\eta_0)\ ,\ \ P_1=P_1(\xi_0,\xi_0+\xi)\ ,\ P_2=P_2(\eta_0-\varepsilon,\eta_0)$$

$R(\xi,\eta;\xi_0,\eta_0)$ va $u(\xi,\eta)$ funksiyalar uchun ushbu

$$2(R(\xi,\eta;\xi_0,\eta_0)Lu(\xi,\eta)-u(\xi,\eta)MR(\xi,\eta;\xi_0,\eta_0))=$$

$$=\frac{\partial}{\partial\xi}\left(R\frac{\partial u}{\partial\eta}-u\frac{\partial R}{\partial\eta}+\frac{2\beta}{\eta-\xi}uR\right)+\frac{\partial}{\partial\eta}\left(R\frac{\partial u}{\partial\xi}-u\frac{\partial R}{\partial\xi}-\frac{2\alpha}{\eta-\xi}uR\right) \quad (1.12)$$

ayniyat o‘rinli, (1.12) tenglikni Δ_ε soha bo‘yicha integrallab, keyin esa Gauss-Ostrogradskiy formulasini qo‘llab va $Lu=0$, $MR=0$ ayniyatlarni hisobga olib, ushbu

$$0=\int\limits_{\partial\Delta_\varepsilon}\left(R\frac{\partial u}{\partial\eta}-u\frac{\partial R}{\partial\eta}+\frac{2\beta}{\eta-\xi}uR\right)d\eta-\left(R\frac{\partial u}{\partial\xi}-u\frac{\partial R}{\partial\xi}-\frac{2\alpha}{\eta-\xi}uR\right)d\xi$$

tenglikka kelamiz, bu yerda $\partial\Delta_\varepsilon=P_1P_2\cup P_2P\cup PP_1$ - Δ_ε soha chegarasi. Endi P_2P da: $d\eta=0$, PP_1 da: $d\xi=0$ ekanligini e’tiborga olib, oxirgi tenglikni ushbu:

$$0=\int\limits_{P_1P_2}\left\{\left(R\frac{\partial u}{\partial\eta}-u\frac{\partial R}{\partial\eta}+\frac{2\beta}{\eta-\xi}uR\right)d\eta-\left(R\frac{\partial u}{\partial\xi}-u\frac{\partial R}{\partial\xi}-\frac{2\alpha}{\eta-\xi}uR\right)d\xi\right\}-$$

$$-\int\limits_{P_2P}\left(R\frac{\partial u}{\partial\xi}-u\frac{\partial R}{\partial\xi}-\frac{2\alpha}{\eta-\xi}uR\right)d\xi+\int\limits_{PP_1}\left(R\frac{\partial u}{\partial\eta}-u\frac{\partial R}{\partial\eta}+\frac{2\beta}{\eta-\xi}uR\right)d\eta \quad (1.13)$$

ko‘rinishda yozib olamiz.

(1.13) tenglikning oxirgi ikki integralini, aniqrog‘i $u(\xi,\eta)$ ning hosilalari qatnashgan hadlarini bo‘laklab integrallab, ushbu

$$\int_{P_2P}\left(R\frac{\partial u}{\partial \xi}-u\frac{\partial R}{\partial \xi}-\frac{\alpha R}{\eta_0-\xi}uR\right)d\xi=u(P)R(P,P)-u(P_2)R(P_2,P)-2\int_{P_2P}u\left(\frac{\partial R}{\partial \xi}+\frac{2\alpha}{\eta_0-\xi}\right)d\xi \tag{1.14}$$

$$\int_{PP_1}\left(R\frac{\partial u}{\partial \eta}-u\frac{\partial R}{\partial \xi}+\frac{2\beta}{\eta-\xi_0}uR\right)d\eta=u(P_1)R(P_1,P)-u(P)R(P,P) \tag{1.15}$$

ifodalarni hosil qilamiz. (1.9) va (1.10) tengliklarga ko‘ra, (1.14) va (1.15) tengliklarning o‘ng tomonidagi integrallar nolga, $R(P,P)=1$ ga tengdir.

Shunday qilib, yuqorida aytilganlarni hisobga olib, (1.13) tenglikni ushbu:

$$u(\xi_0,\eta_0)=\frac{(uR)_{P_1}+(uR)_{P_2}}{2}+\int_{\xi_0}^{\eta_0-\varepsilon}\left\{\left[\frac{\alpha+\beta}{\eta-\xi}R+\frac{1}{2}\left(\frac{\partial R}{\partial \xi}-\frac{\partial R}{\partial \eta}\right)\right]u\right\}\Bigg|_{\eta=\xi+\varepsilon}d\xi+$$
$$+\frac{1}{2}\int_{\xi_0}^{\eta_0-\varepsilon}\left[\left(\frac{\partial u}{\partial \eta}-\frac{\partial u}{\partial \xi}\right)R\right]\Bigg|_{\eta=\xi+\varepsilon}d\xi \tag{1.16}$$

ko‘rinishda yozib olamiz.

(1.16) munosabatga Riman formulasi deyiladi.

Endi $P(\alpha_0,\beta_0)$ nuqtaning $A_0C_0B_0D_0$ kvadratda joylashishiga qarab, (1.1) tenglama uchun (1.2) va (1.3) boshlang‘ich shartlarni qanoatlantiruvchi shakli o‘zgargan Koshi masalasi yechimini beruvchi formulalarni keltirib chiqaramiz.

A. $P(\alpha_0,\beta_0)\in\Delta A_0B_0C_0$ bo‘lsin. Bu holda $\alpha>0$, $\beta>0$, $\alpha+\beta<1$. (1.7) boshlang‘ich shartga asosan, (1.21) Bols formulasini hisobga olgan holda, ushbu:

$$\lim_{\varepsilon\to 0}\left(\frac{\partial u}{\partial\eta}-\frac{\partial u}{\partial\xi}\right)R(\xi,\eta;\xi_0,\eta_0)\Bigg|_{\eta=\xi+\varepsilon}=-\left(\frac{4}{m+2}\right)^{\alpha+\beta}\frac{\Gamma(1-\alpha-\beta)}{\Gamma(1-\alpha)\Gamma(1-\beta)}\left(\eta_0-\xi\right)^{-\beta}(\xi-\xi_0)^{-\alpha}\nu(\xi)$$

(1.17)

tenglikni to‘g‘riligini tekshirish qiyin emas.

Endi quyidagi limitni hisoblaymiz:

$$I=\lim_{\varepsilon\to 0}\left[\frac{\alpha+\beta}{\eta-\xi}R+\frac{1}{2}\left(\frac{\partial R}{\partial\xi}-\frac{\partial R}{\partial\eta}\right)\right]\Bigg|_{\eta=\xi+\varepsilon}u(\xi,\xi+\varepsilon)=$$

$$=\lim_{\varepsilon\to 0}\frac{(\eta-\xi)^{\alpha+\beta}}{2(\eta_0-\xi)^{\beta}(\eta-\xi_0)^{\alpha}}\left[\frac{\beta}{\eta_0-\xi}F(\beta,\alpha,1;\sigma)+\frac{\partial F(\beta,\alpha,1;\sigma)}{\partial\sigma}\frac{\partial\sigma}{\partial\xi}+\right.$$

$$\left.+\frac{\alpha}{\eta-\xi_0}F(\beta,\alpha,1;\sigma)+\frac{\partial F(\beta,\alpha,1;\sigma)}{\partial\sigma}\frac{\partial\sigma}{\partial\eta}\right]\Big|_{\eta=\xi+\varepsilon}u(\xi,\xi+\varepsilon)$$

bu yerdan ushbu:

$$1-\sigma=\frac{(\eta_0-\xi_0)(\eta-\xi)}{(\eta_0-\xi)(\eta-\xi_0)}\ ,\quad \sigma_\xi=\frac{\eta-\eta_0}{\eta-\xi_0}\cdot\frac{\xi_0-\eta_0}{(\xi-\eta_0)^2}\ ,$$

$$\sigma_\eta=\frac{\xi-\xi_0}{\xi-\eta_0}\cdot\frac{\eta_0-\xi_0}{(\eta-\xi_0)^2}$$

tengliklarni va (1.46) , (1.19) formulalarni hisobga olib, ushbu

$$I=\lim_{\varepsilon\to 0}\frac{(\eta-\xi)^{\alpha+\beta}}{2(\eta_0-\xi)^{\beta}(\eta-\xi_0)^{\alpha}}\left\{\frac{\beta}{\eta_0-\xi}F(\beta,\alpha,1;\sigma)+\frac{\alpha}{\eta-\xi_0}F(\beta,\alpha,1;\sigma)+\ \alpha\beta\left(\frac{(\eta_0-\xi)(\eta-\xi_0)}{(\eta_0-\xi_0)(\eta-\xi)}\right)^{\alpha+\beta}\times\right.$$

$$\left.\times\left[\frac{(\eta-\eta_0)(\xi_0-\eta_0)}{(\eta-\xi_0)(\xi-\eta_0)^2}+\frac{(\xi-\xi_0)(\xi_0-\eta_0)}{(\xi-\eta_0)(\eta-\xi_0)^2}\right]F(1-\alpha,1-\beta,2;\sigma)\right\}\Bigg|_{\eta=\xi+\varepsilon}u(\xi,\xi+\varepsilon)$$

(1.18)

tenglikni hosil qilamiz. Oxirgi tenglikda $\varepsilon\to 0$ da limitga o‘tib,

$$I=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\frac{(\eta_0-\xi_0)^{1-\alpha-\beta}}{(\eta_0-\xi)^{1-\alpha}(\xi-\xi_0)^{1-\beta}}\tau(\xi)\qquad(1.19)$$

natijaga kelamiz.

Shunday qilib, (1.16) formuladan (1.17) va (1.19) tengliklarga ko‘ra, ushbu

$$u(\xi_0,\eta_0)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\int_{\xi_0}^{\eta_0}\frac{(\eta_0-\xi_0)^{1-\alpha-\beta}\tau(\xi)d\xi}{(\eta_0-\xi)^{1-\alpha}(\xi-\xi_0)^{1-\beta}}-$$
$$-\left(\frac{4}{m+2}\right)^{\alpha+\beta}\frac{\Gamma(1-\alpha-\beta)}{2\Gamma(1-\alpha)\Gamma(1-\beta)}\int_{\xi_0}^{\eta_0}\frac{\nu(\xi)d\xi}{(\eta_0-\xi)^{\beta}(\xi-\xi_0)^{\alpha}} \quad (1.20)$$

tenglikka ega bo‘lamiz. Bu yerda $\xi=\xi_0+(\eta_0-\xi_0)\frac{1+\sigma}{2}$ almashtirish bajarib va eski x,y o‘zgaruvchilarga o‘tib, ushbu

$$u(x,y)=\gamma_1\int_{-2}^{1}\tau\left(x+\frac{2t}{m+2}(-y)^{\frac{m+2}{2}}\right)(1-t)^{\alpha-1}(1+t)^{\beta-1}dt+$$
$$+\gamma_2(-y)^{1-\beta_0}\int_{-1}^{1}\nu\left(x+\frac{2t}{m+2}(-y)^{\frac{m+2}{2}}\right)(1-t)^{-\beta}(1+t)^{-\alpha}dt \quad (1.21)$$

formulaga ega bo‘lamiz, bu yerda

$$\gamma_1=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}2^{1-\alpha-\beta},\quad \gamma_2=-\frac{\Gamma(2-\alpha-\beta)2^{\alpha+\beta-1}}{(1-\beta_0)\Gamma(1-\alpha)\Gamma(1-\beta)}$$

(1.21) formula shakli o‘zgargan Koshi masalasining yechimini beruvchi Darbu formulasi deyiladi.

(1.21) formulaning tuzilishidan ko‘rinib turibdiki, agar $\tau(x)$, $\nu(x)\in C^2((x_1,x_2))$ sinflarga tegishli bo‘lib, ular (x_1,x_2) intervalning chap chegarasi x_1 nuqtada mos ravishda β va $1-\alpha$ dan kichik, o‘ng chegarasi x_2 nuqtada mos ravishda α va $1-\beta$ dan kichik tartibda cheksizlikka aylansa, u holda, $u(x,y)$ funksiya $[x_1,x_2]$ kesma va

$$x-\frac{2}{m+2}(-y)^{\frac{m+2}{2}}=x_1,\quad x+\frac{2}{m+2}(-y)^{\frac{m+2}{2}}=x_2$$

xarakteristikalar bilan chegaralangan D sohada ikkinchi tartibli uzluksiz hosilalarga ega bo‘ladi. Bevosita hisoblashlar yordamida (1.21) formula (1.1) tenglamaning yechimi bo‘lishini va bu yechim (1.2), (1.3) boshlang‘ich shartlarni qanoatlantirishini tekshirib ko‘rish qiyin emas. (1.21) formulani hosil qilish usulining o‘zidan (1.1)-(1.3) shakli o‘zgargan Koshi masalasi yechimi yagona (Riman funksiyasi Volterra integral tenglamasinining yechimidan iborat) va u boshlang‘ich shartlarga uzluksiz bog‘liq ekanligi kelib chiqadi.

Agar $\tau(x)$ va $v(x)$ funksiyalar (x_1, x_2) intervalda uzluksiz bo‘lsa, (1.21) ifodaga (1.1) tenglamaning umumlashgan yechimi deyiladi. Umumlashgan $u(x,y)$ yechim u yoki bu aniq bir silliqlikka ega bo‘lishi uchun, $\tau(x)$ va $v(x)$ funksiyalarning o‘zi ma’lum bir silliqliklarga ega bo‘lishi zarur. Keyinchalik umumlashgan yechimni $A_1 B_1 C_1$ xarakteristik uchburchakda o‘rganamiz, bu xarakteristik uchburchak $\xi = -1$ xarakteristikaning $A_1 C_1$ kesmasi, $\eta = 1$ xarakteristikaning $C_1 B_1$ kesmasi va buzilish chizig‘i $\eta = \xi$ ning AB kesmasi bilan chegaralangan.

1.2-§ (1.1) tenglamaning R_1 sinfga tegishli umumlashgan yechimlari

Shakli o‘zgargan Koshi masalasi uchun K. I. Babenko [28] tomonidan kiritilgan quyidagi umumlashgan yechimlar sinfini kiritamiz.

Ta’rif. (1.1) tenglamaning (1.21) umumlashgan yechimi R_1 sinfga tegishli deyiladi, agarda $\tau(t)$ funksiya $-1 \le t < 1$ oraliqda $\alpha_1 > 1 - \beta$, $v(t)$ funksiya esa $-1 \le t < 1$ oraliqda $\alpha_2 > \alpha$ ko‘rsatkich bilan Gyolder shartini qanoatlantirsa.

1.1. Lemma. *Agar (1.1) tenglamaning $u(x,y)$ -umumlashgan yechimi R_1* sinfga tegishli bo‘lsa, u holda u_x *va* u_y *lar* ABC *uchburchakda uzluksiz,* $(-y)^{\beta_0} u_y$ *esa* $y = 0$ *buzilish chizig‘igacha uzluksiz va ushbu*

$$\lim_{y \to -0} (-y)^{\beta_0} \frac{\partial u}{\partial y} = v(x), \quad -1 < x < 1$$

tenglik o'rinlidir.

Isbot. $\tau(x)\in C^{(0,\alpha_1)}[-1,1]$ va $\nu(x)\in C^{(0,\alpha_2)}[-1,1)$ bo'lgani uchun ularni quyidagi ko'rinishda ifodalash mumkin:

$$\begin{aligned}\tau(t)&=\tau(-1)+\int_{-1}^{t}(t-s)^{-\beta+\varepsilon}\varphi(s)ds,\\ \nu(t)&=\nu(-1)+\int_{-1}^{t}(t-s)^{\alpha-1+\varepsilon}\psi(s)ds,\end{aligned}\tag{1.22}$$

bu yerda $\varepsilon>0$-yetarli kichik son, $\varphi(s)$ va $\psi(s)$ esa $-1\le s\le 1$ kesmada uzluksiz.

(1.22) ni (1.20) formulaga qo'yib, ushbu:

$$u(\xi,\eta)=\tau(-1)-\left(\frac{4}{m+2}\right)^{\alpha+\beta}\frac{(\eta-\xi)^{1-\alpha-\beta}}{2(1-\alpha-\beta)}\nu(-1)+$$

$$+\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\int_{\xi}^{\eta}\frac{(\eta-\xi)^{1-\alpha-\beta}dt}{(\eta-t)^{1-\alpha}(t-\xi)^{1-\beta}}\int_{-1}^{t}(t-s)^{-\beta+\varepsilon}\varphi(s)ds-$$

$$-\left(\frac{4}{m+2}\right)^{\alpha+\beta}\frac{\Gamma(1-\alpha-\beta)}{2\Gamma(1-\alpha)\Gamma(1-\beta)}\int_{\xi}^{\eta}\frac{dt}{(\eta-t)^{\beta}(t-\xi)^{\alpha}}\int_{-1}^{t}(t-s)^{\alpha-1+\varepsilon}\psi(s)ds$$

tenglikka ega bo'lamiz. Bu yerda integrallash tartibini o'zgartirib, $u(\xi,\eta)$ ni quyidagi ko'rinishda yozib olamiz.

$$u(\xi,\eta)=\tau(-1)-\left(\frac{4}{m+2}\right)^{\alpha+\beta}\frac{(\eta-\xi)^{1-\alpha-\beta}}{2(1-\alpha-\beta)}\nu(-1)+$$

$$+\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\int_{-1}^{\xi}\varphi(s)ds\int_{\xi}^{\eta}\frac{(\eta-\xi)^{1-\alpha-\beta}(t-s)^{-\beta+\varepsilon}dt}{(\eta-t)^{1-\alpha}(t-\xi)^{1-\beta}}+$$

$$+\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\int_{\xi}^{\eta}\varphi(s)ds\int_{s}^{\eta}\frac{(\eta-\xi)^{1-\alpha-\beta}(t-s)^{-\beta+\varepsilon}dt}{(\eta-t)^{1-\alpha}(t-\xi)^{1-\beta}}-$$

$$-\left(\frac{4}{m+2}\right)^{\alpha+\beta}\frac{\Gamma(1-\alpha-\beta)}{2\Gamma(1-\alpha)\Gamma(1-\beta)}\int_{-1}^{\xi}\psi(s)ds\int_{\xi}^{\eta}\frac{(t-s)^{\alpha-1+\varepsilon}dt}{(\eta-t)^{\beta}(t-\xi)^{\alpha}}-$$

$$-\left(\frac{4}{m+2}\right)^{\alpha+\beta}\frac{\Gamma(1-\alpha-\beta)}{2\Gamma(1-\alpha)\Gamma(1-\beta)}\int_{\xi}^{\eta}\psi(s)ds\int_{s}^{\eta}\frac{(t-s)^{\alpha-1+\varepsilon}dt}{(\eta-t)^{\beta}(t-\xi)^{\alpha}} \qquad (1.23)$$

(1.23) ifodaning birinchi va uchinchi integrallarining ichki integrallarida $t=\eta-(\eta-\xi)\sigma$ ko‘rinishda, ikkinchi va to‘rtinchi integrallarning ichki integrallarida $t=\eta-(\eta-s)\sigma$ shaklda integral o‘zgaruvchilarini almashtirib va gipergeometrik funksiyalarning integral ifodasidan foydalanib, $u(\xi,\eta)$ ni quyidagi ko‘rinishda yozib olamiz:

$$u(\xi,\eta)=\tau(-1)-\left(\frac{4}{m+2}\right)^{\alpha+\beta-1}\frac{(\eta-\xi)^{1-\alpha-\beta}}{1-\beta_0}\nu(-1)+$$

$$+\int_{-1}^{\xi}\varphi_{11}(s)(\eta-s)^{-\beta+\varepsilon}F\left(\alpha,\beta-\varepsilon,\alpha+\beta;\frac{\eta-\xi}{\eta-s}\right)ds+$$

$$+\int_{\xi}^{\eta}\varphi_{22}(s)(\eta-s)^{\alpha-\beta+\varepsilon}(\eta-\xi)^{-\alpha}F\left(\alpha,1-\beta,1+\alpha-\beta+\varepsilon;\frac{\eta-s}{\eta-\xi}\right)ds+ \qquad (1.24)$$

$$+\int_{-1}^{\xi}\psi_{11}(s)(\eta-\xi)^{1-\alpha-\beta}(\eta-s)^{\alpha-1+\varepsilon}F\left(1-\beta,1-\alpha-\varepsilon,2-\alpha-\beta;\frac{\eta-\xi}{\eta-s}\right)ds+$$

$$+\int_{\xi}^{\eta}\psi_{22}(s)(\eta-\xi)^{-\alpha}(\eta-s)^{\alpha-\beta+\varepsilon}\times F\left(1-\beta,\alpha,1-\beta+\alpha+\varepsilon;\frac{\eta-s}{\eta-\xi}\right)ds$$

bu yerda

$$\varphi_{11}(s)=\varphi(s)\,,$$

$$\varphi_{22}(s) = \frac{\Gamma(\alpha+\beta)\Gamma(1-\beta+\varepsilon)}{\Gamma(\beta)\Gamma(1+\alpha-\beta+\varepsilon)}\varphi(s)\ ,$$

$$\psi_{11}(s) = -\left(\frac{4}{m+2}\right)^{\alpha+\beta}\frac{\Gamma(1-\alpha-\beta)}{2\Gamma(2-\alpha-\beta)}\psi(s)\ ,$$

$$\psi_{22}(s) = -\left(\frac{4}{m+2}\right)^{\alpha+\beta}\frac{\Gamma(1-\alpha-\beta)\Gamma(\alpha+\varepsilon)}{2\Gamma(1-\alpha)\Gamma(1-\beta+\alpha+\varepsilon)}\psi(s) \qquad (1.25)$$

(1.24) tenglikning o‘ng tomonidagi uchinchi integral ostidagi ifodani (1.21) Bols formulasiga ko‘ra quyidagi ko‘rinishda yozib olamiz:

$$(1-z)^{1-\alpha-\beta}F(1-\beta,1-\alpha-\varepsilon,2-\alpha-\beta;1-z) = \frac{\Gamma(\alpha)\Gamma(\beta-\varepsilon)}{\Gamma(1-\varepsilon)\Gamma(\alpha+\beta-1)}$$

$$\left[F(\beta-\varepsilon,\alpha,1-\varepsilon;z) - \frac{\Gamma(1-\varepsilon)\Gamma(1-\alpha-\beta)}{\Gamma(1-\beta)\Gamma(1-\alpha-\varepsilon)}F(\beta-\varepsilon,\alpha,\alpha+\beta;1-z)\right] \qquad (1.26)$$

bu yerda $1-z = \dfrac{\eta-\xi}{\eta-s}$. Endi (1.24) formula (1.26) tenglikka ko‘ra, ushbu:

$$u(\xi,\eta) = \tau(-1) - \left(\frac{4}{m+2}\right)^{\alpha+\beta-1}\frac{(\eta-\xi)^{1-\alpha-\beta}}{1-\beta_0}\nu(-1) +$$

$$+\int_{-1}^{\xi}\varphi_1(s)(\eta-s)^{-\beta+\varepsilon}F\left(\alpha,\beta-\varepsilon,\alpha+\beta;\frac{\eta-\xi}{\eta-s}\right)ds +$$

$$+\int_{\xi}^{\eta}\varphi_2(s)(\eta-\xi)^{-\alpha}(\eta-s)^{\alpha-\beta+\varepsilon}F\left(\alpha,1-\beta,1+\alpha-\beta+\varepsilon:\frac{\eta-s}{\eta-\xi}\right)ds +$$

$$+\int_{-1}^{\xi}\psi_1(s)(\eta-s)^{-\beta+\varepsilon}F\left(\beta-\varepsilon,\alpha,1-\varepsilon;\frac{\xi-s}{\eta-s}\right)ds, \qquad (1.27)$$

ko‘rinishni oladi, bu yerda

$$\varphi_1(s)=\varphi_{11}(s)-\frac{\Gamma(1-\alpha-\varepsilon)\Gamma(\beta-\varepsilon)\Gamma(\alpha)}{\Gamma(1-\beta)\Gamma(1-\alpha-\beta)\Gamma(\alpha+\beta-1)}\psi_{11}(s),$$

$$\varphi_2(s)=\varphi_{22}(s)+\psi_{22}(s),$$

$$\psi_1(s)=\frac{\Gamma(\beta-\varepsilon)\Gamma(\alpha)}{\Gamma(1-\varepsilon)\Gamma(\alpha+\beta-1)}\psi_{11}(s)$$

(1.26) formuladan ko‘rinib turibdiki, u_ξ va u_η hosilalar ABC uchburchakda mavjud va ular uchun

$$\frac{\partial u}{\partial \xi}=O\left((\eta-\xi)^{-\alpha-\beta}\right), \qquad \frac{\partial u}{\partial \eta}=O\left((\eta-\xi)^{-\alpha-\beta}\right)$$

tengliklar o‘rinli.

(1.27) formuladagi birinchi va ikkinchi integrallardan olingan birinchi tartibli hosilalar $c(\eta-\zeta)^{-\beta+\varepsilon}$ miqdor bilan chegaralangan. Yuqorida keltirilgan mulohazalarga asosan, ushbu:

$$\left(\frac{m+2}{4}\right)^{\alpha+\beta}(\eta-\xi)^{\alpha+\beta}\left(\frac{\partial u}{\partial \xi}-\frac{\partial u}{\partial \eta}\right)=$$

$$=v(-1)+2\left(\frac{m+2}{4}\right)^{\alpha+\beta}\frac{\alpha(\beta-\varepsilon)}{1-\varepsilon}\int_{-1}^{\xi}\psi_1(s)(\eta-s)^{\alpha-1+\varepsilon}\times \qquad (1.28)$$

$$\times F\left(1-\beta,1-\alpha-\varepsilon,2-\varepsilon;\frac{\xi-s}{\eta-s}\right)ds+O\left((\eta-\xi)^{\alpha+\varepsilon}\right)$$

tenglikka ega bo‘lamiz. Shunday qilib, (1.28) tenglikda ifodalarni hisobga olgan holda $\eta-\xi\to 0$ da limitga o‘tib, ushbu tenglikka kelamiz:

$$\lim_{\eta-\xi\to 0}\left(\frac{m+2}{4}\right)^{\alpha+\beta}(\eta-\xi)^{\alpha+\beta}\left(\frac{\partial u}{\partial \xi}-\frac{\partial u}{\partial \eta}\right)=$$

$$=\nu(-1)+\int_{-1}^{\xi}\psi(s)(\xi-s)^{\alpha-1+\varepsilon}ds=\nu(\xi) \tag{1.29}$$

1.6 Lemma isbot bo‘ldi.

V . $P(\alpha_0,\beta_0)\in A_0C_0$ bo‘lsin. Bu holda $\alpha=0$, $0<\beta<1$ va Riman funksiyasi quyidagi ko‘rinishda bo‘ladi:

$$R(\xi,\eta;\xi_0,\eta_0)=\left(\frac{\eta-\xi}{\eta_0-\xi}\right)^{\beta} \tag{1.30}$$

(1.30) ga asosan, (1.16) formula ushbu ko‘rinishda bo‘ladi:

$$u(\xi_0,\eta_0)=u(\eta_0-\varepsilon,\eta_0)+\frac{\varepsilon^{\beta}}{2(\eta_0-\xi_0)^{\beta}}\left(u(\xi_0,\xi_0+\varepsilon)-u(\eta_0-\varepsilon,\eta_0)\right)+$$

$$+\frac{\beta\varepsilon^{\beta}}{2}\int_{\xi_0}^{\eta_0-\varepsilon}\frac{u(\xi,\xi+\varepsilon)-u(\eta_0-\varepsilon,\eta_0)}{(\eta_0-\xi)^{\beta+1}}d\xi-\frac{1}{2}\left(\frac{4}{m+2}\right)^{\beta}\int_{\xi_0}^{\eta_0-\varepsilon}\frac{\nu(\xi)d\xi}{(\eta_0-\xi)^{\beta}} \tag{1.31}$$

Endi (1.31) da $\varepsilon\to 0$ da limitga o‘tib, keyin dastlabki $x\,,y$ o‘zgaruvchilarga qaytib, ushbu:

$$u(x,y)=\tau\left(x+\frac{2}{m+2}(-y)^{\frac{m+2}{2}}\right)-\frac{2^{\beta}(-y)^{1-\beta 0}}{m+2}\int_{-1}^{1}\nu\left(x+\frac{2t}{m+2}(-y)^{\frac{m+2}{2}}\right)(1-t)^{-\beta}dt \tag{1.32}$$

yechimga ega bo‘lamiz.

S. $P(\alpha_{0,}\beta_0)\in B_0C_0$ bo‘lsin. Bu holda $\beta=0$, $0<\alpha<1$ va Riman funksiyasi ushbu:

$$R(\xi,\eta;\xi_0,\eta_0)=\left(\frac{\eta-\xi}{\eta-\xi_0}\right)^{\alpha} \tag{1.33}$$

ko‘rinishda bo‘ladi.

(1.33) tenglikka asosan, (1.16) formulani quyidagi ko‘rinishda yozib olamiz:

$$u(\xi_0,\eta_0)=\frac{u(\xi_0,\xi_0+\varepsilon)+u(\xi_0+\varepsilon,\xi_0)}{2}+$$

$$+\frac{\varepsilon^{\alpha}}{2(\eta_0-\xi_0)^{\alpha}}\left(u(\eta_0-\varepsilon,\eta_0)-u(\xi_0+\varepsilon,\xi_0)\right)+$$

$$+\frac{\varepsilon^{\alpha}}{2}\int_{\xi_0}^{\eta_0-\varepsilon}\frac{u(\xi,\xi+\varepsilon)-u(\xi_0+\varepsilon,\xi_0)}{(\xi+\varepsilon-\xi_0)^{1+\alpha}}d\xi-\frac{1}{2}\left(\frac{4}{m+2}\right)^{\alpha}\int_{\xi_0}^{\eta_0-\varepsilon}\frac{v(\xi)d\xi}{(\xi+\varepsilon-\xi_0)^{\alpha}}$$

(1.34)

Endi (1.34) da $\varepsilon\to 0$ da limitga o‘tib va eski x, y o‘zgaruvchilarga qaytib, ushbu:

$$u(x,y)=\tau\left(x-\frac{2}{m+2}(-y)^{\frac{m+2}{2}}\right)-\frac{2^{\alpha}(-y)^{1-\beta_0}}{m+2}\int_{-1}^{1}v\left(x-\frac{2t}{m+2}(-y)^{\frac{m+2}{2}}\right)(1-t)^{-\alpha}dt \quad (1.35)$$

yechimga ega bo‘lamiz.

D. $P(\alpha_0,\beta_0)=C_0(0,-m/2)$ bo‘lsin. Bu holda $\alpha=\beta=0$ va $R(\xi,\eta;\xi_0,\eta_0)\equiv 1$.

Bu yerdan, (1.16) Riman formulasiga ko‘ra, ushbu:

$$u(\xi_0,\eta_0)=\frac{u(\xi_0,\xi_0+\varepsilon)+u(\eta_0-\varepsilon,\eta_0)}{2}+\frac{1}{2}\int_{\xi_0}^{\eta_0}\left(\frac{\partial u}{\partial\eta}-\frac{\partial u}{\partial\xi}\right)d\xi \qquad (1.36)$$

tenglikka kelamiz. (1.36) da $\varepsilon\to 0$ da limitga o‘tib va eski x, y o‘zgaruvchilarga qaytib, ushbu:

$$u(x,y)=\frac{\tau\left(x-\frac{2}{m+2}(-y)^{\frac{m+2}{2}}\right)+\tau\left(x+\frac{2}{m+2}(-y)^{\frac{m+2}{2}}\right)}{2}-$$
$$-\frac{(-y)^{\frac{m+2}{2}}}{m+2}\int_{-1}^{1}v\left(x+\frac{2t}{m+2}(-y)^{\frac{m+2}{2}}\right)dt \qquad (1.37)$$

Dalamber formulasini hosil qilamiz.

1.3-§ Giperbolik tipdagi tenglama uchun Koshi- Gursa masalasini yechish. Dalamber formulasi. Torning erkin tebranishi

$$u_{tt} = a^2 u_{xx} \qquad (1.38)$$

tenglama bilan tasvirlanadi[29].

(1.38) tenglamaning xarakteristikalari tenglamasi

$$dx^2 - a^2 dt^2 = 0$$

tenglikdan iborat.

Bundan esa (1.1) tenglamaning xarakteristikalarini topamiz:

$$x - at = c_1, \quad x + at = c_2, \quad c_1, c_2 = const.$$

Ushbu $\xi = x - at, \quad \eta = x + at$ formula bo‘yicha yangi o‘zgaruvchilarni tanlab, (1.1) tenglamada qatnashayotgan hosilalarni hisoblaymiz:

$$u_x = u_\xi \xi_x + u_\eta \eta_x = u_\xi + u_\eta, \quad u_{xx} = u_{\xi\xi} + 2u_{\xi\eta} + u_{\eta\eta},$$

$$u_t = u_\xi \xi_t + u_\eta \eta_t = a(-u_\xi + u_\eta), \quad u_{tt} = a^2(u_{\xi\xi} - 2u_{\xi\eta} + u_{\eta\eta}).$$

Bularni (1.1) tenglamaga qo‘yib,

$$\frac{\partial^2 u}{\partial\xi\partial\eta} = 0 \qquad (1.39)$$

kanonik tenglamani olamiz. (1.2) tenglamadan

$$\frac{\partial}{\partial\xi}\left(\frac{\partial u}{\partial\eta}\right) = 0 \quad \Rightarrow \quad \frac{\partial u}{\partial\eta} = f(\eta),$$

hosil qilamiz, bu yerda $f(\eta)$ $-$ixtiyoriy funksiya. Hosil bo‘lgan tenglamanda, ξ ni parametr deb qarab, uni η bo‘yicha integrallaymiz va

$$u(\xi,\eta) = \int f(\eta) d\eta + f_1(\xi) = f_2(\eta) + f_1(\xi) \qquad (1.40)$$

formulani hosil qilamiz. Bunda $f_1(\xi), \; f_2(\eta) -$ ixtiyoriy funksiyalar.

Eski x va t o‘zgaruvchilarga qaytsak, (1.3) formula

$$u(x,t) = f_1(x - at) + f_2(x + at) \qquad (1.41)$$

ko‘rinishda yoziladi.

Agar f_1 va f_2 funksiyalar ikki marta uzluksiz differensiallanuvchi bo‘lsa, (1.4) formula bilan aniqlangan $u(x,t)$ funksiya (1.1) tenglamaning umumiy yechimi bo‘ladi. (1.1) tenglamaning (1.4) yechimi **Dalamber yechimi** deyiladi.

x va t o‘zgaruvchilar tekisligida (1.1) tenglamaning

$$AC:\ x-at=0, \qquad BC:\ x+at=1 \qquad (1.42)$$

xarakteristikalari va $AB=\{(x,t):\ 0<x<1,\ t=0\}$ kesma bilan chegaralangan D sohani belgilaymiz, bunda $x>0,\ t>0$. Bu soha odatda (1.38) tenglama uchun **xarakteristik uchburchak** deyiladi.

D sohada (1.1) tenglama uchun quyidagi Koshi masalasini o‘rganamiz:

Koshi masalasi. (1.38) tenglamaning $u(x,t)\in C(\bar{D})\cap C^2(D)$, $u_t\in C(AB)$ sinfga tegishli va D da (1.1) tenglamani, $t=0$ da esa

$$u\big|_{t=0}=\tau(x), \quad 0\le x\le 1, \quad u_t\big|_{t=0}=\nu(x), \quad 0<x<1 \qquad (1.43)$$

boshlang‘ich shartlarni qanoatlantiruvchi $u(x,t)$ yechimi topilsin, bu yerda $\tau(x),\ \nu(x)$ − berilgan funksiyalar bo‘lib, quyidagi

$$\tau(x)\in C[0,1]\cap C^2(0,1), \quad \nu(x)\in C^1(0,1), \quad |\nu(x)|\le const[x(1-x)]^{-\varepsilon} \quad (1.44)$$

sinfga tegishlidir, bu yerda $0<\varepsilon<1$.

Koshi masalasining yechimi mavjud deb faraz qilamiz. U holda, bu yechim (1.41) formula bilan aniqlanadi. (1.41) formuladagi f_1 va f_2 funksiyalarni shunday topamizki, natijada (1.43) boshlang‘ich shartlar qanoatlantirilsin, ya’ni

$$u(x,0)=f_1(x)+f_2(x)=\tau(x), \qquad (1.45)$$

$$\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}=af_1'(x)+af_2'(x)=\nu(x). \qquad (1.46)$$

(1.46) tenglikni integrallab, quyidagini

$$-f_1(x)+f_2(x)=\frac{1}{a}\int_0^x \nu(s)ds+C \qquad (1.47)$$

hosil qilamiz. (1.45) va (1.47) ifodalardan f_1 va f_2 funksiyalarni topamiz:

$$f_1(x)=\frac{1}{2}\tau(x)-\frac{1}{2a}\int_0^x \nu(s)ds-\frac{C}{2}, \qquad f_2(x)=\frac{1}{2}\tau(x)+\frac{1}{2a}\int_0^x \nu(s)ds+\frac{C}{2}.$$

Bu qiymatlarni (1.41) formulaga qo‘yib, ushbu

$$u(x,t)=\frac{1}{2}\left[\tau(x-at)+\tau(x+at)\right]+\frac{1}{2a}\cdot\int_{x-at}^{x+at}\nu(s)ds \qquad (1.48)$$

formulaga ega bo‘lamiz. (1.48) formulaga **Dalamber formulasi** deyiladi.

Koshi masalasining yechimi mavjud deb, (1.48) formulani keltirib chiqardik. Masala yechimining yagonaligi (1.48) formulani keltirib chiqarish usulidan darhol kelib chiqadi. Agar berilgan funksiyalar (1.44) sinfga tegishli bo‘lsa, u holda Koshi masalasining yechimi $u(x,t)\in C(\bar{D})\cap C^2(D)$ sinfga tegishli bo‘ladi.

Tor tebranish tenglamasi uchun Koshi-Gursa(Darbu) masalasini yechish.

Bu bandda D sohada (1.38) tenglama uchun qo‘yilgan Koshi - Gursa-1 va Koshi-Gursa-2 masalalarini yechamiz.

Koshi-Gursa-1 masalasi. (1.38) tenglamaning $u(x,t)\in C(\bar{D})\cap C^2(D)$, sinfga tegishli va D da (1.1) tenglamani va

$$u\big|_{t=0}=\tau(x), \quad 0\le x\le 1, \qquad (1.49)$$

$$u\big|_{AC}=\psi(x), \quad 0\le x\le 1/2 \qquad \left(u\big|_{BC}=\psi_1(x), \quad 1/2\le x\le 1\right) \qquad (1.50)$$

shartlarni qanoatlantiruvchi $u(x,t)$ yechimi topilsin, bu yerda $\tau(x),\ \psi(x),(\psi_1(x))$ $-$berilgan funksiyalar bo‘lib, quyidagi

$$\tau(x)\in C[0,1]\cap C^2(0,1), \quad \psi(x)\in C\left[0;\frac{1}{2}\right]\cap C^2\left(0;\frac{1}{2}\right), \qquad (1.51)$$

$\left(ёки\ \ \psi_1(x)\in C\left[\frac{1}{2};1\right]\cap C^2\left(\frac{1}{2};1\right), \tau(1)=\psi_1(1)\right)$ sinfga tegishli bo‘lib, $\tau(0)=\psi(0)$ shartni qanoatlantirsin.

Koshi-Gursa-1 masalasini yechishda (1.38) tenglamaning (1.41) umumiy yechimidan foydalanib, noma’lum f_1 va f_2 funksiyalarni ushbu

$$\begin{cases} f_1(x)+f_2(x)=\tau(x), \\ f_1(0)+f_2(2x)=\psi(x) \end{cases}$$

sistemadan topamiz. Buni yechib,

$$f_1(x)=\tau(x)-\psi\left(\frac{x}{2}\right)+f_1(0), \qquad f_2(x)=\psi\left(\frac{x}{2}\right)-f_1(0),$$

ifodalarga ega bo‘lamiz. Bularni (1.41)ga qo‘yib, quyidagini

$$u(x,t)=\tau(x-at)-\psi\left(\frac{x-at}{2}\right)+\psi\left(\frac{x+at}{2}\right) \tag{1.52}$$

hosil qilamiz.

(1.52) formula (1.38), (1.49), (1.50) Koshi-Gursa-1 masalasining yechimidir.

Agar berilgan funksiyalar (1.51) sinfga tegishli bo‘lsa, u holda Koshi-Gursa-1 masalasining yechimi $u(x,t)\in C(\bar{D})\cap C^2(D)$ sinfga tegishli bo‘ladi.

Ikkinchi hol xam xuddi shunday yechiladi va uning yechimi

$$u(x,t)=\tau(x+at)-\psi_1\left(\frac{x+at+1}{2}\right)+\psi_1\left(\frac{x-at+1}{2}\right) \tag{1.53}$$

ko‘rinishda bo‘ladi.

Koshi-Gursa-2 masalasi. (1.38) tenglamaning $u_t\in C(AB)$, $u(x,t)\in C(\bar{D})\cap C^2(D)$ sinfga tegishli va D sohada (1.1) tenglamani va (1.50),

$$u_t\big|_{t=0}=\nu(x), \quad 0<x<1 \tag{1.54}$$

shartlarni qanoatlantiruvchi $u(x,t)$ yechimi topilsin, bu yerda

$\nu(x)$, $\psi(x)$ $-$berilgan funksiyalar bo‘lib, quyidagi $\nu(x)\in C^1(0,1)$,

$$|\nu(x)| \le const\left[x(1-x)\right]^{-\varepsilon},\ 0<\varepsilon<1, \quad \psi(x)\in C\left[0;\frac{1}{2}\right]\cap C^2\left(0;\frac{1}{2}\right), \quad (1.55)$$

$\left(yoki\ \ \psi(x)\in C\left[\frac{1}{2};1\right]\cap C^2\left(\frac{1}{2};1\right)\right)$ sinfga tegishlidir.

Masalani yechimini (1.48) ko‘rinishda qidiramiz. (1.48) formulani (1.50) shartni birinchisiga qo‘yib,

$$u\big|_{at=x} = \psi(x) = \frac{1}{2}\left[\tau(0)+\tau(2x)\right] + \frac{1}{2a}\cdot\int_0^{2x}\nu(s)ds \qquad (1.56)$$

quyidagini hosil qilamiz. (1.56) tenglikda $2x=z$ almashtirish bajarib, so‘ngra z ni $x$ga almashtirib, $\tau(0)=\psi(0)$ tenglikni e’tiborga olib, ushbu

$$\tau(x) = 2\psi\left(\frac{x}{2}\right) - \psi(0) - \frac{1}{a}\int_0^x \nu(s)ds \qquad (1.57)$$

ifodaga ega bo‘lamiz.

(1.57) ni (1.48) formulaga qo‘yib, Koshi-Gursa-2 masalasining yechimini topamiz:

$$u(x,t) = \psi\left(\frac{x-at}{2}\right) + \psi\left(\frac{x+at}{2}\right) - \psi(0) - \frac{1}{a}\cdot\int_0^{x-at}\nu(s)ds\,. \quad (1.58)$$

Xuddi shu usulda (1.54) va $u\big|_{BC} = \psi_1(x),\ \ \frac{1}{2}\le x\le 1$ shartlarni qanoatlantiruvchi Koshi-Gursa-2 masalasining yechimini ushbu ko‘rinishda topiladi:

$$u(x,t) = \psi_1\left(\frac{1+x-at}{2}\right) + \psi_1\left(\frac{1+x+at}{2}\right) - \psi_1(1) - \frac{1}{a}\int_{x+at}^{1}\nu(s)ds\,. \quad (1.59)$$

Koshi-Gursa-2 masalasi yechimining yagonaligi Koshi masalasi yechimining yagonaligidan kelib chiqadi. Agar berilgan funksiyalar (1.55) sinfga tegishli bo‘lsa, u holda Koshi-Gursa-2 masalasi yechimi $u(x,t)\in C(\bar{D})\cap C^2(D),\ u_t\in C(AB)$ sinfga tegishli bo‘ladi.

Tor tebranish tenglamasi uchun Gursa masalasini yechish.

D sohada (1.1) tenglama uchun quyidagi Gursa masalasini yechamiz:

Gursa masalasi. (1.1) tenglamaning $u(x,t)\in C(\bar{D})\cap C^2(D)$, sinfga tegishli va D da (1.1) tenglamani va

$$u\big|_{AC}=\psi_1(x),\quad 0\le x\le 1/2,\qquad u\big|_{BC}=\psi_2(x),\quad 1/2\le x\le 1 \qquad (1.60)$$

shartlarni qanoatlantiruvchi $u(x,t)$ yechimi topilsin, bu yerda $\psi_1(x)$, $\psi_2(x)$ −berilgan funksiyalar bo‘lib, quyidagi

$$\psi_1(x)\in C[0;1/2]\cap C^2(0;1/2),\quad \psi_2(x)\in C[1/2;1]\cap C^2(1/2\,;1) \qquad (1.61)$$

sinfga tegishli bo‘lib, $\psi_1\left(\frac{1}{2}\right)=\psi_2\left(\frac{1}{2}\right)$ shartni qanoatlantirsin.

Gursa masalasini yechishda (1.38) tenglamaning (1.41) umumiy yechimi foydalanib, noma’lum f_1 va f_2 funksiyalarni ushbu

$$\begin{cases} f_1(0)+f_2(2x)=\psi_1(x), \\ f_1(2x-1)+f_2(1)=\psi_2(x) \end{cases}$$

sistemadan topamiz. Bu sistemani yechib,

$$f_1(x)=\psi_2\left(\frac{x+1}{2}\right)-f_2(1),\qquad f_2(x)=\psi_1\left(\frac{x}{2}\right)-f_1(0),$$

ifodalarga ega bo‘lamiz.

Bularni (1.4)ga qo‘yib, $u\left(\frac{1}{2},\frac{1}{2a}\right)=f_1(0)+f_2(1)=\psi_1\left(\frac{1}{2}\right)=\psi_2\left(\frac{1}{2}\right)$ ni e’tiborga olib, quyidagini

$$u(x,t)=\psi_1\left(\frac{x+at}{2}\right)+\psi_2\left(\frac{x-at+1}{2}\right)-\psi_1\left(\frac{1}{2}\right) \qquad (1.62)$$

hosil qilamiz. Bu esa (1.38), (1.60) Gursa masalasining yechimidir.

Agar berilgan funksiyalar (1.61) sinfga tegishli bo‘lsa, u holda Gursa masalasining yechimi $u(x,t)\in C(\bar{D})\cap C^2(D)$ sinfga tegishli bo‘ladi. Gursa masalasining yagonaligi Asgeyrsson prinsipidan [29] yoki (1.62) formulani hosil qilish usulidan ham kelib chiqadi.

I bob bo‘yicha xulosalar

I bobda

$$-(-y)^m u_{xx} + u_{yy} + \left(\alpha_0/(-y)^{1-m/2}\right)u_x + (\beta_0/y)u_y = 0 \quad (1.1)$$

Tenglama uchun shakli o‘zgargan Koshi masalasining yechimini beruvchi Dalamber va Darbu formulalari keltirib chiqarilgan.

Giperbolik tenglamalar nazariyasida $R(\xi,\eta;\xi_0,\eta_0)$-Riman funksiyasi fundamental ahamiyatga ega, shu sababli (1.1) tenglamadagi α_0 va β_0 parametrlarining turli qiymatlarida Riman funksiyasi qurilgan va shakli o‘zgargan Koshi masalasi Riman metodi yordamida yechilgan.

(1.1) tenglamaning R_1 sinfga tegishli umumlashgan yechimlari haqida so‘z yuritilgan bo‘lib, giperbolik tipdagi tenglama uchun Koshi- Gursa va Koshi-Gursa(Darbu) masalalari yechimini beruvchi Dalamber va Darbu formulalari keltirilgan.

II BOB. SINGULYAR KOEFFITSIENTLI BUZILUVCHAN GIPERBOLIK TIPDAGI TENGLAMA UCHUN BITSADZE-SAMARSKIY MASALASINING KORREKTLIGINI O‘RGANISHNING METODIKASI VA BAYONI

2.1-§ Γ masalasining qo‘yilishi

Ushbu

$$-(-y)^m u_{xx} + u_{yy} + (\beta_0 / y) u_y = 0\ ,\ \ m > 0\ ,\ -m/2 \le \beta_0 < 1 \qquad (2.1)$$

tenglamani

$z = x + iy$, $\operatorname{Im} z < 0$ yarim tekislikda $AC: x - \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = -1$, $BC: x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 1$ xarakteristikalar va $y = 0$ o‘qning AB $A = A(-1,0)$, $B = B(1,0)$ kesmasi bilan chegaralangan kompleks tekislikning D sohasida o‘rganamiz.

AB kesmada $E_1 = E_1(c_1, 0)$ i $E_2 = E_2(c_2, 0)$ bu yerda $-1 < c_1 < c_2 < 1$. Nuqtalarni qaraymiz. $\bar{I}$ kesmada esa quyidagi xossalarga ega bo‘lgan $p_k(x) \in C^2(\bar{I})$, $k = 1,2$ funksiyani o‘rganamiz:

1^0. $\bar{I} = [-1,1]$ kesmaning nuqtalaridan iborat bo‘lgan $p_k(x)-$diffeomorfizmni olamiz, $[c_k, 1]$. $k = 1,2$;

2^0. Ushbu $p_2(x) > p_1(x) > x$, $\forall x \in \bar{I} \setminus \{1\}$, $p'_k(x) > 0$ $p_k(-1) = c_k$, $p_k(1) = 1$ .xossalarga ega bo‘lgan $p_k(x) = b_k x + a_k$, $k = 1,2$, bunda $a_k + b_k = 1$, $a_k - b_k = c_k$ chiziqli funksiyani qaraymiz

Quyidagi belgilashlarni kiritamiz

$$\theta(x_0) = \frac{x_0 - 1}{2} - i\left[\frac{m+2}{4}(1 + x_0)\right]^{\frac{2}{m+2}}, \qquad (2.2)$$

$$\theta^*{}_k(p_k(x_0) = \frac{p_k(x_0) + c_k}{2} - i\left[\frac{m+2}{4}(p_k(x_0) - c_k)\right]^{\frac{2}{m+2}}\ , \qquad (2.3)$$

Bunda $\theta(x_0)$ va $\theta^*{}_k(p_k(x_0))$ (2.1) tenglamaning $(x_0,0)$ va $(p_k(x_0),0)$ nuqtalardan chiquvchi xarakteristikalari bilan AC xarakteristikaning affikslari

$$E_kB_k: x-\frac{2}{m+2}(-y)^{\frac{m+2}{2}}=c_k\,,\ B_k\in BC\ ,\quad k=1{,}2\,.$$

Γ ***masalasi.*** D sohada (2.1) tenglamaning quyidagi shartlarni qanoatlantiruvchi $u(x,y)\in C(\overline{D})$ regulyar yechimi topilsin

$$u(x,0)=\tau(x),\ x\in\bar{I}\,; \tag{2.4}$$

$$u[\theta_0(x)]=\mu_1u[\theta^*{}_1(p_1(x))]+\mu_2u[\theta^*{}_2(p_2(x))]+\rho(x),\ x\in\bar{I}\,,\quad \mu_1>0\,,\mu_2>0 \tag{2.5}$$

$\tau(x)\,,\rho(x)\in C^3(\bar{I})$ - ma'lum funksiya bo'lib, $\overline{D}$ sohada $u(x,y)$ funksiya uchun (2.5) shartda $x=-1$ hol ushbu $\tau(-1)=\mu_1\tau(c_1)+\mu_2\tau(c_2)+\rho(-1)$ muvofiqlashtiruvchi shart o'rinli

(2.5) izlanayotgan $u(x,y)$ funksiyaning qiymatlarini AC, E_1B_1 va E_2B_2 xarakteristikalari bilan bog'laydigan Bitsadze-Samarskiy [14,26,30,31] sharti hisoblanadi.

2.2-§ $-m/2<\beta_0<1$ hol uchun Γ masalani o'rganish

Ushbu $p_k(x)=b_kx+a_k\,,b_k=\dfrac{1-c_k}{2},0\quad a_k=\dfrac{1+c_k}{2}$ chiziqli funksiyani qaraymiz. Ushbu bobning asosiy natijalaridan biri bo'lgan quyidagi teoremani keltiramiz:

2.1. Teorema. *Γ masala ushbu*

$$\rho(-1)=\rho'(-1)=\rho''(-1)=0;\quad \tau(-1)=\tau'(-1)=\tau''(-1)=0$$
$$\tau(c_1)=\tau'(c_1)=\tau''(c_1)=0;\quad \tau(c_2)=\tau'(c_2)=\tau''(c_2)=0$$

$$\delta_1+\delta_2<1,\qquad \delta_k=\mu_kb_k^{1-2\beta}\ ,\quad k=1{,}2 \tag{2.6}$$

shartlar bajarilsa yagona yechimga ega bo'ladi.

Isbot. 2.1 Teoremaning isboti $-m/2<\beta_0<1$ va $\beta_0=-m/2$ hollar uchun alohida keltiriladi. Bunda Darbu va Dalamber formulalari ushbu

$$u(x,0)=\tau(x),\ x\in\bar{I}\ ;\quad \lim_{y\to-0}(-y)^{\beta_0}\frac{\partial u}{\partial y}=\nu(x),\ \ x\in I\,, \tag{2.7}$$

Boshlang‘ich shartlarni qanoatlantiradigan hollarni qaraymiz.

Quyida keyingi ishlarda qo‘llaniladigan operatorlarni qaraymiz

$$D_{a,x}^{\ell} f(x) = \begin{cases} \dfrac{1}{\Gamma(-\ell)} \displaystyle\int_a^x \dfrac{f(t)dt}{(x-t)^{1+\ell}}, & \ell < 0, \\ \dfrac{d^{n+1}}{dx^{n+1}} D_{a,x}^{\ell-(n+1)} f(x), & \ell > 0, \end{cases}$$

(2.8)

$$D_{x,b}^{\ell} f(x) = \begin{cases} \dfrac{1}{\Gamma(-\ell)} \displaystyle\int_x^b \dfrac{f(t)}{(t-x)^{1+\ell}} dt, & \ell < 0, \\ (-1)^{n+1} \dfrac{d^{n+1}}{dx^{n+1}} D_{x,b}^{\ell-(n+1)} f(x), & \ell > 0, \end{cases}$$

Ta’rifga asosan

$$D_{a,x}^{0} f(x) = D_{x,b}^{0} f(x) = f(x) \qquad (2.9\ )$$

tenglik o‘rinli.

Ta’kidlaymizki ushbu

$$D_{c,p(x)}^{l} f(x) = \begin{cases} \dfrac{1}{\Gamma(-l)} \displaystyle\int_c^{p(x)} \dfrac{f(t)dt}{(p(x)-t)^{1+l}}, & l < 0, \\ \dfrac{d}{dp(x)} D_{c,p(x)}^{l-1} f(x), & 0 < l < 1. \end{cases} \qquad (2.10)$$

Operatordan ham foydalanamiz

D sohada (1.7) boshlang‘ich shartlarni qanoatlantiruvchi shakli o‘zgargan Koshi masalasining yechimi Darbu formulasi quyidagicha ifodalanadi vid [32,33,34]

$$u(x,y) = \gamma_1 \int_{-1}^{1} \tau\left[x + \frac{2t}{m+2}(-y)^{\frac{m+2}{2}}\right](1-t)^{\beta-1}(1+t)^{\beta-1} dt +$$

$$+\gamma_2 (-y)^{1-\beta_0} \int_{-1}^{1} \nu\left[x + \frac{2t}{m+2}(-y)^{\frac{m+2}{2}}\right](1-t)^{-\beta}(1+t)^{-\beta} dt, \qquad (2.11)$$

bu yerda

$$\gamma_1 = \frac{\Gamma(2\beta)}{\Gamma(\beta)\Gamma(\beta)} 2^{1-2\beta}, \quad \gamma_2 = -\frac{\Gamma(2-2\beta)2^{2\beta-1}}{(1-\beta_0)\Gamma^2(1-\beta)}. \qquad (2.12)$$

(2.11) formula (2.2) va (2.3) shartlarga asosan quyidagi munosabat o‘rinli

$$u[\theta_0(x)] = \gamma_1 \Gamma(\beta)[(1+x)/2]^{1-2\beta} D_{-1,x}^{-\beta} (1+x)^{\beta-1} \tau(x) +$$

$$+\gamma_2\Gamma(1-\beta)[(m+2)/2]^{1-2\beta}D_{-1,x}^{\beta-1}(1+x)^{-\beta}\nu(x) \ , \qquad (2.13)$$

$$u[\theta^*(p(x))]=\gamma_1\Gamma(\beta)\left(\frac{p(x)-c}{2}\right)^{1-2\beta}D_{c,p(x)}^{-\beta}(x-c)^{\beta-1}\tau(x)+$$

$$+\gamma_2\Gamma(1-\beta)[(m+2)/2]^{1-2\beta}D_{c,p(x)}^{\beta-1}(x-c)^{-\beta}\nu(x) \ .$$

Shu bilan birga quyidagi ayniyatlar o‘rinli ekanligini ta’kidlaymiz

$$D_{-1,x}^{1-\beta}D_{-1,x}^{\beta-1}(1+x)^{-\beta}\nu(x)=(1+x)^{-\beta}\nu(x) \ , \qquad (2.14)$$

$$D_{-1,x}^{1-\beta}(1+x)^{1-2\beta}D_{-1,x}^{-\beta}(1+x)^{\beta-1}\tau(x)=(1+x)^{-\beta}D_{-1,x}^{1-2\beta}\tau(x) \ , \qquad (2.15)$$

$$D_{-1,x}^{1-\beta}D_{c,p(x)}^{\beta-1}(x-c)^{-\beta}\nu(x)=b^{1-2\beta}(1+x)^{-\beta}\nu(p(x)), \qquad (2.16)$$

$$D_{-1,x}^{1-\beta}(p(x)-c)^{1-2\beta}D_{c,p(x)}^{-\beta}(x-c)^{\beta-1}\tau(x)=b^{1-2\beta}(1+x)^{-\beta}D_{c,p(x)}^{1-2\beta}\tau(x) \ . \qquad (2.17)$$

(2.17) ayniyatni isbotlaymiz

$$I(x)=D_{-1,x}^{1-\beta}(p(x)-c)^{1-2\beta}D_{c,p(x)}^{-\beta}(x-c)^{\beta-1}\tau(x)=$$

.

$$=\frac{1}{\Gamma^2(\beta)}\frac{d}{dx}\int_{-1}^{x}\frac{(p(t)-c)^{1-2\beta}}{(x-t)^{1-\beta}}dt\int_{c}^{p(t)}\frac{(s-c)^{\beta-1}\tau(s)ds}{(p(t)-s)^{1-\beta}}$$

Bu yerda integrallash tartibini o‘zgartirish natijasida ushbuga

$$I(x)=\frac{1}{\Gamma^2(\beta)}\frac{d}{dx}\int_{c}^{bx+a}(s-c)^{\beta-1}\tau(s)ds\int_{(s-a)/b}^{x}\frac{(bt+a-c)^{1-2\beta}dt}{(x-t)^{1-\beta}(bt+a-s)^{1-\beta}}$$

ega bo‘lamiz.

Ichki integralda $t=\frac{s-a}{b}+\left(x-\frac{s-a}{b}\right)\sigma$ almashtirishni bajaramiz

$$I(x)=\frac{b^{-\beta}}{\Gamma^2(\beta)}\frac{d}{dx}\int_{c}^{bx+a}\frac{(s-c)^{-\beta}\tau(s)ds}{(bx+a-s)^{1-2\beta}}\int_{0}^{1}\sigma^{\beta-1}(1-\sigma)^{\beta-1}\left(1-\frac{bx+a-s}{c-s}\sigma\right)^{1-2\beta}d\sigma \ .$$

Keyingi hisoblashalar uchun Gaussning

$$\int_{0}^{1}\sigma^{a-1}(1-\sigma)^{c-a-1}(1-x\sigma)^{-b}d\sigma=\frac{\Gamma(a)\Gamma(c-a)}{\Gamma(c)}F(a,b,c;x), \quad c-a-b>0 \qquad (2.18)$$

Gipergeometrik funksiyasni tadbiq etib ushbu

$$I(x)=\frac{b^{-\beta}}{\Gamma(2\beta)}\frac{d}{dx}\int_{c}^{bx+a}\frac{(s-c)^{\beta}}{(bx+a-s)^{1-2\beta}}F\left(\beta,2\beta-1,2\beta;\frac{bx+a-s}{c-s}\right)\tau(s)ds \qquad (2.19)$$

munosabatni olamiz yoki

$$I(x)=\frac{b^{-\beta}}{\Gamma(2\beta)}\lim_{\varepsilon\to 0}\frac{d}{dx}\int\limits_{c}^{bx+a-\varepsilon}\frac{(s-c)^{\beta}}{(-1)^{1-2\beta}}\left(\frac{bx+a-s}{c-s}\right)^{2\beta-1}\times$$
$$\times F\left(\beta,2\beta-1,2\beta;\frac{bx+a-s}{c-s}\right)\tau(s)ds\,.$$

Bunda

$$\frac{d}{dx}x^{b}F(a,b,c;x)=bx^{b-1}F(a,b+1,c;x)\,, \qquad (2.20)$$

Formulani qo‘llab, x o‘zgaruvchi bo‘yicha differensiallash natijasida quyidagi munosabatni olamiz

$$I(x)=\frac{b^{1-\beta}}{\Gamma(2\beta)}\lim_{\varepsilon\to 0}\left[\frac{\varepsilon^{2\beta-1}\tau(bx+a-\varepsilon)}{(bx+a-c-\varepsilon)^{\beta}}F\left(\beta,2\beta-1,2\beta;\frac{\varepsilon}{c-bx-a+\varepsilon}\right)+\right.$$
$$\left.+\frac{b(2\beta-1)}{(bx+a-c)^{\beta}}\int\limits_{c}^{bx+a-\varepsilon}\frac{\tau(s)ds}{(bx+a-s)^{2-2\beta}}\right], \qquad (2.21)$$

ushbu

$$b(2\beta-1)\int\limits_{c}^{bx+a-\varepsilon}\frac{\tau(s)ds}{(bx+a-s)^{2-2\beta}}=\frac{d}{dx}\int\limits_{c}^{bx+a-\varepsilon}\frac{\tau(s)ds}{(bx+a-s)^{1-2\beta}}-\frac{\tau(bx+a-\varepsilon)}{\varepsilon^{1-2\beta}}b$$

tenglik orqali (1.21) ni quyidagicha ifodalaymiz

$$I(x)=\lim_{\varepsilon\to 0}\left\{\frac{b^{1-\beta}}{\Gamma(2\beta)}\frac{\varepsilon^{2\beta-1}\tau(bx+a-\varepsilon)}{(bx+a-c-\varepsilon)^{\beta}}\left[F\left(\beta,2\beta-1,2\beta;\frac{\varepsilon}{c-bx-a+\varepsilon}\right)-b\left(\frac{bx+a-c-\varepsilon}{bx+a-c}\right)^{\beta}\right]+\right.$$

$$\left.+\frac{1}{(bx+a-c)^{\beta}}\frac{b^{1-\beta}}{\Gamma(2\beta)}\frac{d}{dx}\int\limits_{c}^{bx+a-\varepsilon}\frac{\tau(s)ds}{(bx+a-s)^{1-2\beta}}\right\} \qquad (2.22)$$

(2.22) da $\varepsilon\to 0$ limitga o‘tib, ushbu

$$I(x)=\lim_{\varepsilon\to 0}\varepsilon^{2\beta}\left[\frac{F\left(\beta,2\beta-1,2\beta;\frac{\varepsilon}{c-bx-a+\varepsilon}\right)-1}{\varepsilon}+\frac{1-\left(1-\frac{\varepsilon}{bx+a-c}\right)^{\beta}}{\varepsilon}\right]=0$$

tenglikni e’tiborga olib quyidagiga ega bo‘lamiz

$$I(x)=\frac{b^{1-\beta}}{(bx+a-c)^{\beta}}\frac{1}{\Gamma(2\beta)}\frac{d}{dx}\int_{c}^{bx+a}\frac{\tau(s)ds}{(bx+a-s)^{1-2\beta}}=\frac{b^{1-\beta}}{b^{\beta}(1+x)^{\beta}}\frac{d}{dx}D_{c,p(x)}^{-2\beta}\tau(x)=$$

$$=b^{1-2\beta}(1+x)^{-\beta}D_{c,p(x)}^{1-2\beta}\tau(x) \qquad (2.23)$$

Bu yerda $a-c=b$.

(2.17) ayniyat isbotlandi.

(2.14), (2.15), (2.16) formulalar ham (2.17) kabi isbotlanadi.

Γ masalani o‘rganishni davom ettiramiz.

(2.13) dan $u[\theta(x)]$, $u[\theta_k(p_k(x))]$ ning qiymatlarini (2.5) ga qo‘yamiz

$$2^{2\beta-1}\gamma_1\Gamma(\beta)D_{-1,x}^{1-\beta}(1+x)^{1-2\beta}D_{-1,x}^{-\beta}(1+x)^{\beta-1}\tau(x)+\gamma_2\left(\frac{m+2}{2}\right)^{1-2\beta}\Gamma(1-\beta)D_{-1,x}^{1-\beta}D_{-1,x}^{\beta-1}(1+x)^{-\beta}\nu(x)=$$

$$=\sum_{k=1}^{2}\Big[\mu_k\gamma_1 2^{2\beta-1}\Gamma(\beta)D_{-1,x}^{1-\beta}(p_k(x)-c_k)^{1-2\beta}D_{c_k,p_k(x)}^{-\beta}(x-c_k)^{-\beta}\tau(x)+$$

$$+\mu_k\gamma_2\left(\frac{m+2}{2}\right)^{1-2\beta}\Gamma(1-\beta)D_{-1,x}^{1-\beta}D_{c_k,p_k(x)}^{\beta-1}(x-c_k)^{-\beta}\nu(x)\Big]+D_{-1,x}^{1-\beta}\rho(x). \qquad (2.24)$$

(2.14)-(2.17) ayniyatlarni e’tiborga olib (1.24) ni quyidagicha yozamiz

$$\nu(x)=\mu_1 b_1^{1-2\beta}\nu(p_1(x))+\mu_2 b_2^{1-2\beta}\nu(p_2(x))+F(x), \qquad (2.25)$$

bu yerda

$$F(x)=\frac{1}{\gamma_2\Gamma(1-\beta)}\left(\frac{2}{m+2}\right)^{1-2\beta}\Big[(1+x)^{\beta}D_{-1,x}^{1-\beta}\rho(x)-2^{2\beta-1}\gamma_1\Gamma(\beta)D_{-1,x}^{1-2\beta}\tau(x)+$$

$$+2^{2\beta-1}\gamma_1\Gamma(\beta)\sum_{k=1}^{2}\mu_k b_k^{1-2\beta}D_{c_k,p_k(x)}^{1-2\beta}\tau(x)\Big]\in C^2[-1,1]. \qquad (2.26)$$

(2.25) tenglamani quyidagi ko‘rinishda yozamiz

$$\nu(x)=\delta_1\nu(p_1(x))+\delta_2\nu(p_2(x))+F(x), \qquad (2.27)$$

bunda $\delta_1=\mu_1 b_1^{1-2\beta}$, $\delta_2=\mu_2 b_2^{1-2\beta}$.

(2.25) funksional tenglamaning yechimini $x=1$ da chegaralangan funksiyalar sinfidan izlaymiz. Agar bu shartdan voz kechsak (2.27) tenglamaga mos keladigan bir jinsli funksiona

$$\nu(x)=\delta_1\nu(p_1(x))+\delta_2\nu(p_2(x)) \qquad (2.28)$$

notrivial yechimlarga ega bo‘lishi mumkin [21,29,35]

Misol. Faraz qilamiz $p_1(x) = bx + a$, $p_2(x) = p_1(p_1(x)) = b^2x + ba + a$, $a - b = c_1$, $c_1 b + a = c_2$, bo‘lsin, u holda quyidagi funksiya

$$v(x) = (1-x)^{\lambda}, \quad \text{bunda} \quad \lambda = \ell og_b \frac{\sqrt{\delta_1^2 + 4\delta_2} - \delta_1}{2\delta_2}$$

(2.27) bir jinsli tenglamaning notrivial yechimidan iborat bo‘ladi

$$v(p_1(x)) = (1 - p_1(x))^{\lambda} = b^{\lambda}(1-x)^{\lambda},$$

$$v(p_2(x)) = (1 - p_2(x))^{\lambda} = b^{2\lambda}(1-x)^{\lambda},$$

bu qiymatlarni (2.28) ga qo‘yish natijasida ushbu kvadrat

$$\delta_2 b^{2\lambda} + \delta_1 b^{\lambda} - 1 = 0.$$

Tenglamaga ega bo‘lamiz.Bu yerdan

$$\lambda = \ell og_b \frac{\sqrt{\delta_1^2 + 4\delta_2} - \delta_1}{2\delta_2}$$

(2.6) shartga asosan $\lambda < 0$, yekanligi o‘rinli, bu esa (2.28) funksional tenglamaning $x = 1$dagi yechimining chegaralanmaganligini isbotlaydi.

Bu holda Γ masalaga mos bir jinsli masalaning yechimi quyidagi ko‘rinishdan iborat bo‘ladi [32] :

$$u(x,y) = (-y)^{1-\beta_0} \int_{-1}^{1} \left(1 - x - \frac{2t}{m+2}(-y)^{(m+2)/2}\right)^{\lambda} \left(1-t^2\right)^{-\beta} dt =$$

$$= 2^{1-2\beta} B(1-\beta, 1-\beta) \left(1 - x + \frac{2}{m+2}(-y)^{(m+2)/2}\right)^{\lambda} (-y)^{1-\beta_0} \times$$

$$\times F\left(1-\beta, -\lambda, 2-2\beta; \frac{4}{m+2}(-y)^{(m+2)/2} / \left(\frac{2}{m+2}(-y)^{(m+2)/2} - x + 1\right)\right). \qquad (2.29)$$

Shu sababli (2.25) tenglamaning yechimini $x = 1$ nuqtada chegaralangan funksiyalar sinfidan izlaymiz.

(1.25) tenglamani quyidagicha yozib olamiz

$$v(x) = \delta_1 v(\lambda(x)) + \delta_2 v(r(x)) + F(x), \qquad (2.30)$$

bu yerda $\lambda(x) = p_1(x)$, $r(x) = p_2(x)$. (1.30) tenglamani yechish uchun kombinatsiyalashgan ketma-ket yaqinlashish va iteratsiya metodidan foydalanamiz [45].

Ketma-ketlikning $v_0(x), v_1(x), \ldots, v_n(x), \ldots$ hadlarini hosil qilish uchun quyidagi rekurrent formuladan foydalanamiz

$$v_n(x) = \delta_1 v_n(\lambda(x)) + \delta_2 v_{n-1}(r(x)) + F(x). \tag{2.31}$$

$v_0(x)$ uchun quyidagi funksional tenglamaning yechimini qaraymiz

$$v_0(x) = \delta_1 v_0(\lambda(x)) + F(x). \tag{2.32}$$

(2.32) funksional tenglamani yechish uchun iteratsiya metodidan foydalanamiz. Bu tenglamada x ni $\lambda(x)$ ga almashtirib quyidagiga ega bo‘lamiz

$$v_0(\lambda(x)) = \delta_1 v_0(\lambda(\lambda(x))) + F(\lambda(x)). \tag{2.33}$$

Hozir (2.33) ni (2.32) ifodaga qo‘yib

$$v_0(x) = \delta_1^2 v_0(\lambda_2(x)) + \delta_1 F(\lambda_1(x)) + F(x), \tag{2.34}$$

ni hosil qilamiz, bu yerda $\lambda_1(x) = \lambda(x)$, $\lambda_2(x) = \lambda_1(\lambda_1(x))$. (2.34) munosabat (2.32) tenglamaning birinchi iteratsiyasidan iborat. Bu jarayonni n – had uchun qo‘llaymiz

$$v_0(x) = \delta_1^{n+1} v_0(\lambda_{n+1}(x)) + \sum_{k=0}^{n} \delta_1^k F(\lambda_k(x)) \tag{2.35}$$

bu yerda $\lambda_0(x) = x$, $\lambda_{n+1}(x) = \lambda_n(\lambda_1(x)) = \lambda_1(\lambda_n(x))$,

yeslatib o‘tamiz (2.6) ga asosan

$$\lim_{n\to\infty} \delta_1^n = 0. \tag{2.36}$$

Demak $\lambda(x) > x, \lambda(1) = 1,$ u holda $\lambda_n(x) = \lambda_{n-1}(\lambda_1(x)) = \lambda_1(\lambda_{n-1}(x)) > \lambda_{n-1}(x)$ munosabat o‘rinli, ya’ni $\{\lambda_n(x)\}$ funksional ketma-ketlik monoton o‘suvchi va yuqoridan bir bilan chegaralangan, ya’ni $\lambda_n(x) \le 1$, $\forall x \in \bar{I}$. Monoton o‘suvchi va chegaralangan funksiyaning limiti haqidagi teoremaga asosan, quyidagi ifodaga ega bo‘lamiz

$$\lim_{n\to\infty} \lambda_n(x) = \lambda^0(x) , \quad x \in \bar{I} . \tag{2.37}$$

Ushbu $\lambda_n(x)=\lambda_1(\lambda_{n-1}(x))$ tenglikdan $n\to\infty$ da limitga o‘tib $\lambda^0(x)=\lambda_1(\lambda^0(x))$ ga ega bo‘lamiz. Bizga $\lambda^0(x)\equiv 1$, $\forall x\in\bar{I}$ ma’lum $\lambda(x)$ uchun yagona $x=1$ qo‘zg‘almas nuqtaga ega bo‘lamiz.

$$\lim_{n\to\infty}\lambda_n(x)=1 \quad \forall x\in\bar{I}\,. \tag{2.38}$$

(2.35) tenglikda $n\to\infty$ limitga o‘tib (2.36) munosabatga asosan $v_0(x)$ uchun

$$v_0(x)=\sum_{k=0}^{\infty}\delta_1^k F(q_k(x)) \tag{2.39}$$

- (2.32) funksional tenglamaning yechimini olamiz.
Quyidagi tenglik har doim o‘rinli

$$|v_0(x)|\le\sum_{k=0}^{\infty}\delta_1^{\,k}\,|F(\lambda_k(x))|\le\sum_{k=0}^{\infty}M\,\delta_1^{\,k}=\frac{M}{1-\delta_1}\,, \tag{2.40}$$

bunda $\max\limits_{x\in\bar{I}}|F(x)|=M$. Sledovatelno, funksionalnыy ryad pravoy chasti (2.39) tenglikning chap qismidagi funksional qator yaqinlashuvchi. $F(x)$ va $\bar{I}$ uzluksizligidan $v_0(x)\in C(\bar{I})$ munosabat o‘rinli. Bundan esa $v_0(x)\in C^2(\bar{I})$ shartning bajarilishi kelib chiqadi.

(2.31) rekurrent munosabatdan $n=1$ uchun $v_1(x)$ ning ifodasini topamiz:

$$v_1(x)=\delta_1 v_1(\lambda(x))+\delta_2 v_0(r(x))+F(x) \tag{2.41}$$

$v_0(x)$ (2.39) tenglik orqali ifodalanadigan ma’lum funksiya.

Funksional tenglamaga iteratsiya metodini qo‘llab, (2.41) tenglamaning yechimini olamiz

$$v_1(x)=\sum_{k=0}^{\infty}\delta_1^k\delta_2 v_0(r(\lambda_k(x)))+\sum_{k=0}^{\infty}\delta_1^k F(\lambda_k(x))\,. \tag{2.42}$$

Ishonch hosil qilish mumkinki (2.42) tenglikning o‘ng qismidagi funksional qator tekis yaqinlashuvchi va $v_1(x)\in C^2(\bar{I})$ munosabat o‘rinli.

Bu jarayonni davom ettirish natijasida quyidagi ifodani aniqlaymiz:

$$v_n(x)=\sum_{k=0}^{\infty}\delta_1^k\delta_2 v_{n-1}(r(\lambda_k(x)))+\sum_{k=0}^{\infty}\delta_1^k F(\lambda_k(x)) \quad v_n(x)\in C^2(\bar{I})\,. \qquad (2.43)$$

Shunday qilib, quyidagi funksional ketma-ketlikka ega bo‘lamiz

$$v_0(x),v_1(x),\ldots,v_n(x)\ldots\,. \qquad (2.44)$$

Endi esa (2.44) funksional ketma-ketlikning yaqinlashishini isbotlaymiz, buning uchun quyidagi funksional qatorni qaraymiz

$$v_0(x)+\left[v_1(x)-v_0(x)\right]+\left[v_2(x)-v_1(x)\right]+\ldots+\left[v_n(x)-v_{n-1}(x)\right]+\ldots\,. \qquad (2.45)$$

(2.45) funksional qator uchun mojorant qatorni aniqlab olamiz.

(2.42) dan (2.35) ifodani ayirib (2.40) tenglikka asosan quyidagiga ega bo‘lamiz

$$\left|v_1(x)-v_0(x)\right|\le\sum_{k=0}^{\infty}\delta_1^k\delta_2 v_0(r(\lambda_k(x)))\le\frac{\delta_2}{1-\delta_1}\frac{M}{1-\delta_1}=\frac{M}{\delta_2}\left(\frac{\delta_2}{1-\delta_1}\right)^2\,. \qquad (2.46)$$

(2.46) ifodaga asosan quyidagi tengsizlik o‘rinli

$$\left|v_n(x)-v_{n-1}(x)\right|\le\sum_{k=0}^{\infty}\delta_1^k\delta_2\left|v_{n-1}(r(\lambda_k(x)))-v_{n-2}(r(\lambda_k(x)))\right|\le$$

$$\le\frac{\delta_2}{1-\delta_1}\frac{M}{\delta_2}\left(\frac{\delta_2}{1-\delta_1}\right)^n=\frac{M}{\delta_2}\left(\frac{\delta_2}{1-\delta_1}\right)^{n+1}\,. \qquad (2.47)$$

(2.45) funksional qator $\sum_{n=0}^{\infty}\frac{M}{\delta_2}\left(\frac{\delta_2}{1-\delta_1}\right)^{n+1}$ sonli qator orqali mojorantlanadi va u $\bar{I}$ da tekis yaqinlashadi.

Shunday qilib, $\{v_n(x)\}$ funksional ketma-ketlik tekis yaqinlashuvchi va quyidagi limit o‘rinli

$$\lim_{n\to\infty} v_n(x)=v(x),\ \forall\alpha\in\bar{I} \qquad v(x)\in C^2(\bar{I})$$

2.1 teorema isbotlandi.

2.3-§ Γ masalani $\beta_0 = -m/2$ hol uchun o‘rganish

Bu paragrafda Γ masalani (2.1) da tenglama β_0 parametrning $\beta_0 = -m/2$ qiymati uchun, ya’ni ushbu

$$-(-y)^m u_{xx} + u_{yy} - \frac{m}{2y} u_y = 0 \quad . \qquad (2.48)$$

tenglama uchun o‘rganamiz.

$\beta_0 = -m/2$ hol uchun $p_k(x)$, $k = 1,2$ funksiya 1^0 va 2^0 shartlarni qanoatlantiradi

2.2. Teorema Γ *masala quyidagi shartlar bajarilganda*

$$\delta_1 + \delta_2 < 1, \qquad \delta_1 = \max_{x \in \bar{I}} \left| \mu_1 p_1'(x) \right|, \; \delta_2 = \max_{x \in \bar{I}} \left| \mu_2 p_2'(x) \right| \qquad (2.49)$$

bir qiymatli yechiladi.

Isbot. D^- sohada (2.48) tenglamani va ushbu $u(x,0) = \tau(x), \quad x \in \bar{I}$,

$$\lim_{y \to -0} (-y)^{-\frac{m}{2}} \frac{\partial u}{\partial y} = \nu(x), \; x \in I \qquad (2.50)$$

boshlang`ich shartlarni qanoatlantiruvchi shakli o‘zgargan Koshi masalasining yechimi Dalamber formulasi quyidagicha ifodalanadi

$$u(x,y) = \frac{\tau\left(x - \frac{2}{m+2}(-y)^{\frac{m+2}{2}} \right) + \tau\left(x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}} \right)}{2} - \\ - \frac{(-y)^{\frac{m+2}{2}}}{m+2} \int_{-1}^{1} \nu\left[x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}} \right] \nu(t)\,dt \quad . \qquad (2.51)$$

(2.51) formuladan va (2.2) , (2.3) shartlarga asosan quyidagi munosabatlarga ega bo‘lamiz

$$u[\theta(x)] = \frac{\tau(-1)+\tau(x)}{2} - \frac{1}{2}\int_1^x v(z)dz \qquad (2.52)$$

$$u[\theta_k(p_k(x))] = \frac{\tau(c_k)+\tau(p_k(x))}{2} - \frac{1}{2}\int_{c_k}^{p_k(x)} v(z)dz. \qquad (2.53)$$

Hozir esa (2.52) va (2.53) munosabatlarni nolokal (2.5) shartga qo‘yib, x bo‘yicha differensiallab, noma`lum $v(x)$ ga nisbatan quyidagi funksional munosabatga ega bo‘lamiz:

$$v(x) = \omega_1(x)v(p_1(x)) + \omega_2(x)v(p_2(x)) + F(x),\ x \in I. \qquad (2.54)$$

Bunda

$$\omega_1(x) = \mu_1 p'_1(x)\ ,\ \omega_2(x) = \mu_2 p'_2(x), \qquad (2.55)$$

$$\begin{aligned} F(x) &= \tau'(x) - \mu_1 p'_1(x)\tau'(p_1(x)) - \\ &- \mu_2 p'_2(x)\tau'(p_2(x)) - 2\rho'(x)\ ,\ F(x) \in C^1(\overline{I}). \end{aligned} \qquad (2.56)$$

(2.54) funksional tenglamaning yechimini $x = 1$ nuqtada chegaralangan funksiyalar sinfidan izlaymiz.

(2.54) Funksional tenglamani quyidagi ko‘rinishda yozib olamiz

$$v(x) = \omega_1(x)v(\lambda(x)) + \omega_2(x)v(r(x)) + F(x) \qquad (2.57)$$

bu yerda $\lambda(x) = p_1(x),\ r(x) = p_2(x)$. (2.57) tenglamani yechish uchun kombinatsiyalashgan ketma-ket yaqinlashish va iteratsiya metodidan foydalanamiz [45].

Ketma-ketlikning $v_0(x), v_1(x), \ldots, v_n(x), \ldots$ hadlarini hosil qilish uchun quyidagi rekurrent formuladan foydalanamiz

$$v_n(x) = \omega_1(x)v_n(\lambda(x)) + \omega_2(x)v_{n-1}(r(x)) + F(x) \qquad (2.58)$$

(2.58) tenglamaning tadqiq etish $\beta_0 \in \left(-(m/2), 1\right)$ hol kabi amalga oshiriladi. 2.2 teorema isbotlandi.

II bob bo‘yicha xulosalar

II bobda singulyar koeffisientli giperbolik tipdagi tenglama uchun izlanayotgan funksiya qiymatini uchta parallel xarakteristika bilan bog`laydigan Bitsadze-Samarskiy masalasi o‘rganildi.

Ushbu masalani tadqiq etishda kombinatsiyalashgan ketma-ket yaqinlashish va iteratsiya metodidan foydalanildi [36-38].

Γ masala β_0 parametrning $-(m/2) < \beta_0 < 1$ va $\beta_0 = -m/2$ hollarida o‘rganildi.

III BOB. SINGULYAR KOEFFITSIENTLI BUZILUVCHAN GIPERBOLIK TIPDAGI TENGLAMA UCHUN NOLOKAL MASALALARNING ASOSIY NATIJALARI BAYONI

3.1-§ Singular koeffitsientli giperbolik tipdagi tenglama uchun Bitsadze–Samarskiy sharti qatnashgan 1-nolokal masala

Ω orqali $y<0$ da (2.1) tenglamaning

$$AC: \quad x-2(-y)^{(m+2)/2}\big/(m+2)=-1, \qquad BC: x+2(-y)^{(m+2)/2}\big/(m+2)=1$$

xarakteristikalari va $J\equiv AB=\{(x,y):\ -1<x<1,\quad y=0\}$ kesma bilan chegaralangan sohani belgilaymiz.

$$-(-y)^m u_{xx}+u_{yy}+\frac{\beta_0}{y}u_y=0, \quad m>0, \quad -\frac{m}{2}<\beta_0<1 \qquad (3.1)$$

Ω sohada (3.1) tenglama uchun quyidagi Bitsadze – Samarskiy sharti qatnashgan nolokal masalani yechamiz.

$Б_1C-$ **masala.** (3.1) tenglamaning $u(x,y)\in C(\bar{\Omega})\cap C^2(\Omega)$, sinfga tegishli va Ω sohada (3.1) tenglamani va

$$\lim_{y\to 0} u(x,y)=\tau(x), \quad (x,0)\in \bar{J}, \qquad (3.2)$$

$$D_{-1x}^{1-\beta}u\left[\theta_1(x)\right]=a_1(x)\lim_{y\to -0}(-y)^{\beta_0}u_y(x,y)+b_1(x), \quad (x,0)\in J \qquad (3.3)$$

shartlarni qanoatlantiruvchi $u(x,y)$ yechimi topilsin, bu yerda $a_1(x),\ b_1(x)$ $-$berilgan funksiyalar bo‘lib, quyidagi shartlarni qanoatlantiradi:

$$a_1(x)\in C(\bar{J})\cap C^2(J), \qquad (3.4)$$

$$\tilde{a}_1(x)\equiv(1+x)^{\beta}a_1(x)-\gamma_2\left[(m+2)/2\right]^{1-2\beta}\Gamma(1-\beta)\neq 0, \quad \forall\,(x,0)\in \bar{J}, \qquad (3.5)$$

$$b_1(x)\in C^2(J), \qquad (3.6)$$

$b_1(x)$ funksiya mos ravishda $x \to -1$ va $x \to 1$ da $1-\beta$ va $1-2\beta$ dan kichik tartibda cheksizlikka intilishi mumkin, bu yerda $\gamma_2 = -\dfrac{\Gamma(2-2\beta)2^{2\beta-1}}{(1-\beta_0)\Gamma^2(1-\beta)}$.

$\theta_1(x)$ funksiya

$$\theta_1(x) = \left(\frac{x-1}{2};\ -\left(\frac{m+2}{4}(1+x) \right)^{2/(m+2)} \right), \tag{3.7}$$

formula orqali aniqlanadi.[39]

Izoh. Agar $a_1(x)=0$ va $u\left[\theta_1(-1)\right]=0 \Leftrightarrow u(-1,0)=\tau(-1)=0$ bo‘lsa, u holda $Б_1C$ masala (3.1) tenglama uchun qo‘yilgan Koshi-Gursaning-1 masalasiga keltiriladi.

$$u(x,y) = \gamma_1 \int_{-1}^{1} \tau\left[x + \frac{2t}{m+2}(-y)^{\frac{m+2}{2}} \right] \left(1-t^2\right)^{\beta-1} dt +$$

$$+\gamma_2 (-y)^{1-\beta_0} \int_{-1}^{1} \nu \left[x + \frac{2t}{m+2}(-y)^{\frac{m+2}{2}} \right] \left(1-t^2\right)^{-\beta} dt \tag{3.7$_1$}$$

ko‘rinishdagi shakli o‘zgargan Koshi masalasining yechimi va (3.7) ifodadan foydalanib, $u\left[\theta_1(x)\right]$ funksiyani hisoblaymiz:

$$u\left[\theta_1(x)\right] = u\left(\frac{x-1}{2}; -\left(\frac{m+2}{4}(1+x) \right)^{2/(2+m)} \right) = \gamma_1 \int_{-1}^{1} \tau\left[\frac{x-1}{2} + \frac{x+1}{2}t \right] \left(1-t^2\right)^{\beta-1} dt +$$

$$+\gamma_2 \left(\frac{m+2}{4}(1+x) \right)^{\frac{2(1-\beta_0)}{2+m}} \int_{-1}^{1} \nu \left[\frac{x-1}{2} + \frac{x+1}{2}t \right] \left(1-t^2\right)^{-\beta} dt .$$

Bunda, ushbu

$$z = \frac{x-1}{2} + \frac{x+1}{2}t, \quad t = \frac{2z+1-x}{1+x}, \quad dt = \frac{2dz}{1+x}, \quad 1+t = \frac{2(z+1)}{1+x},$$

$$1-t = \frac{2(x-z)}{1+x}, \quad t=-1, \quad z=-1; \quad t=1, \quad z=x; \quad 1-2\beta = \frac{2(1-\beta_0)}{2+m}$$

almashtirishlarni bajarib, uni ushbu ko‘rinishda yozib olamiz: [40]

$$u\left[\theta_1(x)\right] = \gamma_1 \left((x+1)/2\right)^{1-2\beta} \int_{-1}^{x} \tau(z)(x-z)^{\beta-1} \left(1+z\right)^{\beta-1} dz +$$

$$+\gamma_2 \left((m+2)/2\right)^{1-2\beta} \int_{-1}^{x} \nu(z)(x-z)^{-\beta} \left(1+z\right)^{-\beta} dz,$$

yoki bunga

$$D_{ax}^{-\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x \frac{f(t)\,dt}{(x-t)^{1-\alpha}}, \qquad x > a \;, \qquad (3.8)$$

(3.8) formulani qo‘llab, $u\left[\theta_1(x)\right]$ funksiyani quyidagicha topamiz:

$$u\left[\theta_1(x)\right] = \gamma_1 2^{2\beta-1}\Gamma(\beta)(1+x)^{1-2\beta} D_{-1x}^{-\beta}(1+x)^{\beta-1}\tau(x) +$$

$$+\gamma_2\left[(m+2)/2\right]^{1-2\beta}\Gamma(1-\beta)D_{-1x}^{\beta-1}(1+x)^{-\beta}\nu(x). \qquad (3.9)$$

(3.9) ni (3.3) shartga qo‘yib, so‘ngra

$$D_{ax}^{\alpha} D_{ax}^{-\alpha} f(x) = f(x), \qquad D_{xb}^{\alpha} D_{xb}^{-\alpha} f(x) = f(x) \qquad (3.10)$$

$$D_{ax}^{1-\beta}(x-a)^{1-2\beta} D_{0x}^{-\beta}(x-a)^{\beta-1} f(x) = (x-a)^{-\beta} D_{0x}^{1-2\beta} f(x), \qquad (3.11)$$

ayniyatlar va (3.5) dan foydalanib, noma’lum $\nu(x)$ funksiyani

$$\nu(x) = \left[\gamma_1 2^{2\beta-1}\Gamma(\beta) D_{-1x}^{1-2\beta}\tau(x) - (1+x)^{\beta} b_1(x)\right] \Big/ \tilde{a}_1(x) \qquad (3.12)$$

ko‘rinishda topamiz.

(3.4), (3.5), (3.6) shartlar va $\left|R_2(x)\right| \le c_2(1+x)^{2\beta-1}$. bahoga ko‘ra (3.8) ifodadan quyidagi xulosaga kelamiz: $\nu(x)$ funksiya $C^2(J)$ sinfga tegishli bo‘lib, u $x \to \pm 1$ intilganda $1-2\beta$ dan kichik tartibda cheksizlikka intiladi.

(3.6) formula orqali topilgan $\nu(x)$ funksiyani (3.7$_1$) formulaga qo‘yib, Ω sohada (3.1) tenglama uchun quyilgan $Б_1C$ masalasi yechimini shakli o‘zgargan Koshi masalasi yechimi (3.7$_1$) orqali qurib olinadi.

Shakli o‘zgargan Koshi masalasi yechimining mavjudligi va yagonaligidan (3.1) tenglama uchun quyilgan $Б_1C$ masalasi yechimining mavjudligi va yagonaligi kelib chiqadi.

$Б_1C$ masala to‘liq o‘rganildi.

3.2-§ Singular koeffitsientli giperbolik tipdagi tenglama uchun Bitsadze–Samarskiy sharti qatnashgan 2-nolokal masala

$Б_2C-$ **masala.** (3.1) tenglamaning $u(x,y)\in C(\bar{\Omega})\cap C^2(\Omega)$, sinfga tegishli va Ω sohada (3.1) tenglamani va

$$\lim_{y\to -0}(-y)^{\beta_0}u_y(x,y)=\nu(x),\quad (x,0)\in J \tag{3.13}$$

$$D^{\beta}_{-1x}(1+x)^{2\beta-1}u\left[\theta_1(x)\right]=a_2(x)u(x,0)+b_2(x),\quad (x,0)\in J \tag{3.14}$$

shartlarni qanoatlantiruvchi $u(x,y)$ yechimi topilsin, bu yerda

$\nu(x),\ a_2(x),\ b_2(x)$ –berilgan funksiyalar bo‘lib, quyidagi shartlarni qanoatlantiradi: [41,42]

$$a_2(x)\in C(\bar{J})\cap C^2(J), \tag{3.15}$$

$$\tilde{a}_2(x)\equiv(1+x)^{1-\beta}a_2(x)-\gamma_1\Gamma(\beta)2^{2\beta-1}\neq 0,\quad \forall\,(x,0)\in\bar{J}, \tag{3.16}$$

$$b_2(x)\in C(-1,1]\cap C^2(J), \tag{3.17}$$

$b_2(x)$ funksiya $x\to -1$ da $1-\beta$ dan kichik tartibda cheksizlikka intiladi, $x\to 1$ da esa chegaralagan,

$$\nu(x)\in C^2(J), \tag{3.18}$$

$\nu(x)$ funksiya $x\to\pm 1$ intilganda $1-2\beta$ dan kichik tartibda cheksizlikka intiladi, $\theta_1(x)$ funksiyaning ko‘rinishi (3.7) formulada berilgan.

Izoh. Agar $a_2(x)=0$ bo‘lsa, u holda $Б_2C$ masala (3.1) tenglama uchun quyilgan Koshi-Gursaning ikkinchi masalasiga to‘g’ridan - to‘g’ri keltirilmaydi, chunki $u(x,y)$ funksiyaning $(-1,0)$ nuqtadagi qiymati noma’lum. $Б_2C$ masalani Koshi-Gursaning ikkinchi masalasiga keltirish uchun $\lim_{x\to -1}(1+x)^{2\beta-1}u\left[\theta_1(x)\right]=0$ bo‘lishi kerak.

$Б_2C$ **masalasini yechamiz.** (3.9) ni (3.10) shartga qo‘yib, so‘ng (3.10),

$$D_{ax}^{\alpha}(x-a)^{2\alpha-1}D_{ax}^{\alpha-1}(x-a)^{-\alpha}\, f(x)=(x-a)^{\alpha-1}D_{ax}^{2\alpha-1}f(x), \qquad (3.19)$$

ayniyatlar va (3.16) dan foydalanib, noma'lum $\tau(x)$ funksiyani ushbu ko‘rinishda topamiz:

$$\tau(x)=\frac{1}{\tilde{a}_2(x)}\left[\gamma_2\,\Gamma(1-\beta)\left(\frac{m+2}{2}\right)^{1-2\beta}D_{-1x}^{2\beta-1}\nu(x)-(1+x)^{1-\beta}b_2(x)\right]. \qquad (3.20)$$

Shunday qilib, (3.12), (3.13), (3.14), (3.15) shartlar va $\left|R_2(x)\right|\le c_2(1+x)^{2\beta-1}$. bahoga ko‘ra (3.20) formuladan $\tau(x)$ funksiya $C(\bar{J})\bigcap C^2(J)$ sinfga tegishli bo‘ladi.

(3.17) formula orqali topilgan $\tau(x)$ funksiyani (3.7_1) formulaga qo‘yib, Ω sohada (3.1) tenglama uchun quyilgan $Б_2C$ masalasi yechimini shakli o‘zgargan Koshi masalasi yechimi (3.7_1) orqali qurib olamiz. $Б_2C$ masalasi yechimining bir qiymatli yechilishi shakli o‘zgargan Koshi masalasining korrektligidan kelib chiqadi.

III bob bo‘yicha xulosalar

Ushbu bobda

$$-(-y)^m u_{xx} + u_{yy} + \frac{\beta_0}{y} u_y = 0, \quad m > 0, \quad -\frac{m}{2} < \beta_0 < 1$$

singular koeffitsientli giperbolik tipdagi tenglama uchun Bitsadze-Samarskiy shartli masalalar yechimining bir qiymatli yechilishi o‘rganiladi. Qo‘yilgan masalalarning korrektligini aniqlashda singulyar integral tenglamalar metodi va shakli o‘zgargan Koshi masalasining yechimi formulasidan foydalanilgan.

XULOSA

Dissertatsiyaning dastlabki bobida soha ichida buziladigan singulyar koeffitsientli Gellerstedt tenglamasi shakli o‘zgargan Koshi masalasi yechimini beradigan formulalar keltirib chiqarilgan va giperbolik tipdagi tenglamalar uchun Koshi, Koshi-Gursa va Koshi-Gursa (Darbu) masalalari o‘rganilgan.

Singulyar koeffisientli giperbolik tipdagi tenglama uchun Bitsadze-Samarskiy tipidagi nolokal chegaraviy masala tadqiq qilingan bu masalaning korrektligi ketma-ket yaqinlashish va iteratsiya usullari yordamida kichik maxsuslikka ega bo‘lgan siljishli integral va funksional tenglamalarni yechish orqali ko‘rsatilgan.[43]

Tadqiqot ishida integral tenglamalar nazariyasining, matematik fizikaning chegaraviy masalalari nazariyasining metodlaridan, shuningdek iteratsiya va ketma-ket yaqinlashish usullaridan, hamda ekstremum prinsipidan foydalanilgan.[44]

Soha ichida buziladigan giperbolik tenglama uchun Bitsadze-Samarskiy shartli ya’ni chegaraviy xarakteristikada izlanayotgan yechimning qiymatlarini soha ichida yotuvchi uchta maxsus egri chiziq nuqtalari bilan bog‘lovchi chegaraviy masala yechilgan;[45]

FOYDALANILGAN ADABIYOTLAR RO‘YXATI

1.	O‘zbekiston Respublikasi Prezidentining “O‘zbekiston Respublikasining 2021– 2023 yillarga mo‘ljallangan Investitsiya dasturini amalga oshirish chora – tadbirlari to‘g’risida”gi (PQ-4937-son. 28.12.2020y.) Qarori. // www.lex.uz
2.	O‘zbekiston Respublikasi Prezidentining 2017 yil 7 fevraldagi PF-4947-sonli “2017-2021 yillarda O‘zbekiston Respublikasi rivojlantirishning beshta ustuvor yo‘nalishi bo‘yicha Harakatlar strategiyasi to‘g’risida”gi Farmoni. // www.lex.uz
3.	O‘zbekiston Respublikasi Prezidentining 2021-yil 14-iyuldagi PF-6260-sonli “Yoshlarni har tomonlama qo‘llab quvvatlash va ularning ijtimoiy faolligini yanada oshirishga oid qo‘shimcha chora – tadbirlar to‘g’risida” gi qarori.
4.	O‘zbekiston Respublikasi Prezidentining 2021-yil 19-yanvardagi 23-sonli “O‘zbekistonda yoshlarga oid davlat siyosatini 2025-yilgacha rivojlantirish konsepsiyasini tasdiqlash to‘g’risida” gi qarori.
5.	O‘zbekiston Respublikasi Prezidentining 2020-yil 7-maydagi PF-4708 sonli “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy tadqiqotlarni rivojlantirish chora - tadbirlari to‘g’risida”gi qarori.
6.	O‘zbekiston Respublikasi Prezidentining 2019-yil 9-iyuldagi “2020-2022-yillarda iqtisodiyot tarmoqlari va ijtimoiy soha uchun matematika bo‘yicha oliy malakali kadrlar tayyorlash chora – tadbirlari dasturini ishlab chiqish to‘g’risida” gi qarori.
7.	Трикоми Ф. О линейных уравнениях в частных производных второго порядка смешанного типа М.-Л. Гос.тех.издат 1947,-192 с.
8.	Holmgren E. Sur un porleme aux limites pour l’eqution $y^m z_{xx}+z_{yy}=0$// A Arkiv for matematik, astronomi och fysik.1927, vand 19B.No 14.
9.	Gellersted S. Sur un probleme aux limites pour ane equation lineaire aux derives partielles du second ordre de type mixte. Thesis, Uppsala, 1935.

10.	Михлин С.Г. Об интегральном уравнении F.TRICOMI / / ДАН СССР. 1948, Т.59,No 6. с. 1053-1056.
11.	Михлин С.Г. Лекции по линейным интегральным уравнениям. М. : Физ.мат.гиз, 1959.-232 с.
12.	Бицадзе А.В. Уравнения смешанного типа. М.: Изд-во АН СССР. 1959,-164 с.
13.	Бицадзе А.В. Некоторые классы уравнений в частных производных. М .: Наука, 1981,-448 с.
14.	Бицадзе А.В.,Самарский А.А. О. некоторых простейших обобщениях линей-ных эллиптических краевых задач / / ДАН СССР, 1969, т. 185, No4, с. 739-740.
15.	Кароль И.Л. Об одной краевой задаче для уравнения смешанного эллиптико-гиперболиеского типа / / ДАН СССР. 1953, т.88, No 2, с. 197-200.
16.	Бабенко К.И. К теории уравнений смешанного типа. Докторская диссертatsiя (библиотека математического института им. В. А. Стеклова РАН), 1952.
17.	Солдатов А.П. Одномерные сингулярные оператора и краевые задачи теории функции М.: Наука, 1991 .
18.	Моисеев Е.И. Уравнения смешанного типа со спектральным параметром. М.: Изд-во МГУ 1988.-149 с.
19.	Моисеев Е.И. Дифференц уравнения. 1993. Т. 29, №1. С. 104-105.
20.	Садыбеков М.А. Краевые задачи в областях с отходом от характеристики для уравнений гиперболического и смешанного типов второго порядка. Докторская диссертatsiя (библиотека института математики им. В. И. Рома-новского АН РУз).
21.	Кальменов Т.Ш. Краевые задачи для линейных уравнений в частных производных гиперболического типа. Шымкент. Гылым, 1993,-327 с.
22.	Салахитдинов М.С ., Уринов А.К. Аралаш турдаги тенгламалари.

23.	Джураев Т.Д. Краевые задаче для уравнений смешанного и смешанно-сос-тавного типов.Ташкент. Фан, 1979, -238 с.
24.	Джураев Т.Д. ,Абдиназаров С. Неклассическая краевая задача для уравнения высокого порядка с кратными характеристиками / / ДАН СССР.1991, т.317, No1,с. 19-22.
25.	Мирсабуров М. Нелокальная краевая задача для уравнения Геллерстедта / /Математические заметки.2000. т.67, вып. 5, с. 721-729.
26.	Мирсабуров М. Краевая задача для одного класса уравнений смешанного типа с условием Бицадзе-Самарского на параллельных характеристиках / / Дифференциальные уравнения. 2001, т.37, No 9. с. 1281-1284.
27.	Уринов А.К. Нелокальные краевые задачи для уравнений смешанного типа в односвязной и двусвязной областях. Докторская диссертatsiя (библиотека инсти-тута математики им. В.И.Романовкого.АН РУз). 1993.
28.	Березин И.С. О задаче Коши для линейных уравнений второго порядка с на-чальными данными на линии параболичности. / / Математический сборник 24 (66), N 2, с. 301-320.
29.	Нахушев А.М. Обратные задачи для вырождающихся уравнений и инте-гральные уравнения. Вольтерра третьего рода / / Дифференциальные уравнения. 1974, т 107, No 1. с. 100-101.
30.	Алимов Ш.А. Об одной спектральной задаче типа задачи Бицадзе-Самарского ДАН СССР.1986, т. 287, т. No 6.с. 1289-1290.
31.	Нахушев А.М. О некоторых краевых задачах для гиперболических уравнений и уравнений смешанного типа. / / Дифференциальные уравнения.1969, т 5, No1. с. 44-59.
32.	Салахитдинов М.С., Мирсабуров М. Нелокальные задачи для уравнений смешанного типа с сингулярными коэффициентами. Ташкент 2005. “Universitet”, “Yangiyo‘l poligraf servis” 224 с.

33.	Смирнов М.М. Уравнения смешанного типа. М.: Наука. 1970,-296 с.
34.	Смирнов М.М. Уравнения смешанного типа. М.: Высшая школа. 1985,-304 с.
35.	Бердышев А.С.,Тойжонова Г.Д. Краевая задача с наклонной производной для параболо-гиперболического уравнения в области с отходом от характеристики / /Известия НАН Республики Казахстан. Серия физ.-мат, наук, 1995, N 5, с. 13-20.
36.	Салахитдинов М.С. Уравнения смешанно-составного типа.Ташкент: Фан. 1974,-156с.
37.	Салахитдинов М.С., Мирсабуров М. О некоторых краевых задачах со смещением для уравнения гиперболического типа, вырождающегося на границе / / Известия АН Узбекистана. Серия физ-мат. Наука, 1980, No 1, с 16-21.
38.	Салахитдинов М.С.,Толипов А. О некоторых краевых задачах для уравнений смешанного типа с одной и двумя линиями вырождения / / Дифференциальные уравнения. 1972, т 8, No 1. с. 134-142.
39.	Мусхелишвили Н.И. Сингулярные интегральные уравнения. М.: Наука,1968,-512с.
40.	Салахитдинов М.С., Мирсабуров М. Задачи Бицадзе-Самарского для одного класса вырождающегося гиперболических уравнений / / Дифференциальные уравнения.2002, т. 38, No 2, с. 271-276.
41.	Салахитдинов М.С., Мирсабуров М. О некоторых краевых задачах для гиперболического уравнения вырождающегося внутри области / / Дифференциальные уравнения. 1981, т 17, No 1. с. 129-136.
42.	Салахитдинов М.С.,Уринов А.К. Краевые задачи для уравнений смешанного типа со спектральным параметром. Ташкент. Фан, 1997, –165 с.
43.	Т. А. Чоршанбиев, М. А. Мусурмонов, Б. Ж. Олтиев, “АРАЛАШ ТИПДАГИ ТЕНГЛАМА УЧУН СОҲАНИНГ ЭЛЛИПТИК ҚИСМИ

	ЧЕГАРАСИДА ЛОКАЛ ВА НОЛОКАЛ МАСАЛАЛАР ”/ TA’LIM TIZIMIDA FAN, INNOVATSIYA VA RAQAMLI TEXNOLOGIYALARNI RIVOJLANTIRISH ISTIQBOLLARI: MUAMMO VA YECHIMLAR XALQARO ILMIY-AMALIY ANJUMANI. Sirdaryo 2022-YIL 10-11-IYUN. 475-478 b.
44.	Мерганов Р. А., Олтиев Б.Ж., Шерматов Ш.Х. “ О единственности решения задачи с условием Бицадзе Самарского на параллельных характеристиках для одного класса уравнений смешанного типа” / MATHEMATICAL ANALYSIS AND ITS APPLICATIONS IN MODERN MATHEMATICAL PHYSICS. Samarkand September 23-24, 2022. p. 283-285.
45	Хасанова Д.У., Мерганов Р.А., Олтиев Б. Ж. “Об одном выделение Нефредгольмовых операторов ядра которых имеют особенности первого порядка в изолированной особой точке”. / Алгебра ва анализнинг долзарб масалалари мавзусидаги республика илмий-амалий анжумани. Термиз 2022 йил 18-10 ноябрь. 201-203 б.