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# « GILBERT YADROLI BIR O‘LCHOVLI INTEGRAL TENGLAMANI SONLAR NAZARIYASI USULLARI YORDAMIDA TAQRIBIY YECHISH » mavzusida

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## Приближенное решение одномерного интегрального уравнения с ядром Гильберта с помощью методов теории чисел


### Аннатоция

В этой диссертации, сочетая метод итерации с теоретикочисловым методом дается приближенное решение интегральных уравнений

$$y(x) = f(x) + \lambda \int_0^1 ctg\pi\,(t - x) K(x,t) y(t) dt$$

указана оценка погрешности

$$R_N = 0\left[N^{-\frac{\alpha-1}{2}} log^{0,5(\alpha-1)\beta+M-1} N\right].$$



**Основные слова:** *оптимальный коэффициент, параллелепипедная сетка, узловая точка, кубическая формула, вычет*


## Approximate Solution of one-dimensional integral equation with Hilbert kernel using methods of number theory


### Annotation

In this dissertation, combining the iteration method with the number-theoretic method, an approximate solution of the integral equations is given

$$y(x) = f(x) + \lambda \int_0^1 ctg\pi\,(t - x) K(x,t) y(t) dt$$

error estimate

$$R_N = 0\left[N^{-\frac{\alpha-1}{2}} log^{0,5(\alpha-1)\beta+M-1} N\right].$$



**Key words:** *optimal coefficient, box mesh, node point, cubic formula, residue*

**MUNDARIJA**

# KIRISH

***Matematika hamma aniq fanlarga asos. Bu fanni yaxshi bilgan bola aqlli, keng tafakkurli bo'lib o'sadi, istalgan sohada muvaffaqiyatli ishlab ketadi.***

**Sh. Mirziyoyev**

Hozirgi kunda ilm–fanga Prezidentimiz tomonidan alohida e'tibor berilmoqda. Ayniqsa, 2023 yilning Prezidentimiz tomonidan "Insonga e'tibor va sifatli ta'lim yili" deb e'lon qilinishi hamda bu yilda matematika, kimyo, biologiya va geologiya fanlarini rivojlantirishga alohida e'tibor berilishi biz yosh matematiklar uchun katta imkoniyatlar yaratdi.

Jamiyat ijtimoiy sohasining eng muhim tarkibiy qismlaridan biri ta'lim tarbiya sohasi bo'lib, uning rivojisi yosiy –huquqiy, iqtisodiy va ma'naviy sohalarga bevosita ta'sir etadi hamda ijtimoiy sohalar me'yoriy mohiyatini, kamolot darajasini belgilab beradi.

O'zbekistonda ta'lim tizimining isloh qilishning dasturiy hujjatlarida ta'kidlanganidek, mamlakatimiz ta'lim tizimi xodimlari oldida raqobat bardosh kadrlar tayyorlash, ta'lim tarbiya jarayonini jahon andozalar darajasiga yetkazishni ta'minlash asosiy vazifa qilib qo'yilgan.

Shu ma'noda olib qaraganda, yoshlarning yangi avlodi istiqbol masalalarini kun tartibiga dadil qo'yadigan va uni yecha oladigan, fikr yuritishning yuksak madaniyatini egallagan, siyosiy hamda ijtimoiy iqtisodiy hayotda o'ziga mustaqil yo'l topa oladigan qobiliyatga ega bo'lishi kerak.

Ushbu magistrlik dissertatsiyasi mavzusi ana shu talab va vazifalardan kelib chiqib tanlandi.

Fizika, biologiya, mexanika, ekonomika, sotsiologiya va hakozalarning murakkab xodisalarining matematik modellari, xususan qarshiliklar nazariyasida signallarni tiklash, mikro ob'ektlarni kuzatish reduksiyasi, spektroskopmasalalari, uchlamchi yulduzlar konfiguratsiyasining haqiqiy bo‘linish funksiyasini aniqlash, tutilgan yorug‘lik sistemasining egri chiziqlarini interpritatsiya qilish va turli xil chegaraviy masalalar to‘g‘ridan to‘g‘ri yoki maxsus usullar bilan Fredgolm yoki Volterra integral tenglamalariga va ularning sistemasiga keltiriladi ([7] qarang).

Fredgolm yoki Volterra integral tenglamalar va ularning sistemasini yechishning ko‘pgina analitik va taqribiy usullari mavjud. Analitik usulda yechish Laplas, Fure, Mellin va boshqalarning almashtirishlariga asoslangan bo‘lib, qo‘llanilishida tabiiy tusiqlar mavjud, ular ma’lum doiradagi masalarga qo‘llaniladi va EHM da hisoblash qiyin. Integral tenglamalarini yechishda kvadratur va kubatur formulalar, iteratsiya usuli, proeksion usullar (momentlar usuli, Galerkin-Petrov, kollakatsiya va boshqalar) ([1], [3], [14], [7] qarang).

Ikkinchi tarafdan aerodinamika, gidrodenimika, egiluvchanlik nazariyasi, elektrodinamika, matematik fizikaning chegaraviy masalasi, analitik funksiyalar chegaraviy masalasini yechish singulyar integrallarga keltiriladi ([6], [9], [20], [21] qarang). Bunday tenglamalar nazariyasi yaxshi rivojlangan.

Fredgolm va Volterr integral tenglamalarini taqribiy yechish usullari L.V.Kantorovich va V.I.Krilov[14], N.S.Baxvalova [1] , I.V. Berezina i I.P.Jidkova [3], A. F. Verlenya i A. S. Sizikova [ 7] va hokazolarning monografiyalarida bayon etilgan.

Koshi va Gil’berta yadrali singulyar integral tenglamalarni taqribiy yechish V. V .Ivanova [11] , B. G. Gabdulxaeva [8 ], I. V. Boykova [ 4 ] , S. M. Belotserkovskogo i I. K. Lifanova [2 ], Ye. Ye. Tirtishnikova [19] , V. A. Zolotorevskogo [10] va boshqalarning monografiya va maqolalarida yaxshi yoritilgan.

Shuning uchun, Fredgolm va Volterr yadroli bir o‘lchovli va ko‘p o‘lchovli integral tenglamalarni sonli yechishning effektiv algoritmlarini qurish hisoblash matematikasining aktual masalalaridan hisoblanadi.

Ushbu dissertatsiyada biz integral tenglamalarni yechishda sonlar nazariyasig usullariga asoslangan kubatur formulalar quramiz.

Oxirgi yillarda regulyar va singulyar integrallar uchun kvadratur va kubatur formulalar qurish nazariyasiga qiziqish ortganligi sababli sonlar nazariyasi usullarini qullash sezilarli darajada oshdi.

Hozirgi vaqtda sonlar nazariyasi usullari hisoblash matematikasining turli masalarini yechishda keng qo‘llanilayapdi. Ayniqsa regulyar integrallarga kubatur formulalar qurish, ko‘p o‘zgaruvchili funksiyalarni interpolyatsiyalash, ko‘p o‘lchovli Fredgolm 2-tur integral tenglamalarini yechish masalalarida qo‘llaniladi [ 15-19 ] , [22] , [23-24 ] , [25 ] , [26 ] .

Sonlar-nazariyasi usullari xuddi shunday Gil’bert yadroli singulyar integrallar uchun kubatur formula qurish masalasiga keng qo‘llanildi.

Shuni aytish kerakki sonlar nazariyasining turli bo‘limlari S. L. Sobolevaning ( sm. [12-13] ) invariant kubatur formulalar nazariyasiga ham qo‘llanilgan.

Sonlar nazariyasi usullaridan foydalanib, N. M. Korobov va Ye. Hlawkalar birinchi bo‘lib, karrali integrallarni taqribiy hisoblashda integrallash to‘rlarining maxsus sinflarini yaratdi. N. M. Korobov ularni “parallelepipedal to‘r” deb atadi, g‘arb adabiyotlarida esa ular “ Good lattice points ” degan nomni oldi.

K. K. Frolova [23] ning ishlarida parallelepipedal to‘r yordamida integrallashni umumlashtirishga qadam qo‘yilgan va keyin Sloan va Kachoyan[29] lar tamonidan rivojlantirilgan. Keyinchalik taqribiy integrallashning bu yo‘nalishiga V. A. Выkovskim [5] , Lyness [28] , Niederreiter [27] , Sloan [27] va boshqalar katta xissa qo‘shdilar.

Sonlar nazariyasi usullaridan foydalanib qurilgan kubatur formulalar , integralning karraligiga bog‘liq emas (Monte – Karlo usuliga o‘xshash), shuning uchun bunday formulalar integral tenglamalarni iteratsiya usuli bilan taqribiy yechishga qulay bo‘ladi.

Kubatur formulalar va sonlar nazariyasi orasidagi bog‘liqlikka qisqacha to‘xtalamiz. $G_n = \{ 0 \leq x_i \leq 1$ , i=1 , 2 ,… , $n\}$ birlik giperkubda

$f(x) = f(x_1, x_2, \dots, x_n)$ funksiya aniqlangan, uzluksiz bo‘lsin va har bir $x_i$

o‘zgaruvchi bo‘yicha davriy bo‘lsin.

Faraz qilaylik, $f(x)$ni $G_n$ da Fur’e qatoriga yoyish mumkin bo‘lsin

$$f(x) = \sum_{m_1, \dots, m_n = -\infty}^{\infty} C(m) e^{2\pi i (m,x)} ,$$

bunda

$$S(m) = C(m_1, \dots, m_n),\ m_1 x_1 + \dots + m_n x_n = (m, x).$$

Ma’lumki

$$S(0) = \int_{G_n} f(x) dx ,$$

u holda,

$$\int_{G_n} f(x) dx \cong \frac{1}{N} \sum_{k=1}^{N} f(x^{(k)}) \tag{1}$$

kubatur formulaning xatoligini

$$R_N(f) = \frac{1}{N} \sum_{m_1, \dots, m_n = -\infty}^{\infty}{}' C(m) S(m)$$

ko‘rinishda yozish mumkin. Bunda summadagi shtrix $(m_1, \dots, m_n) \neq (0, \dots, 0)$ ekanligini bildiradi. Aynan mana shu tenglik sonlar nazariyasi bilan kubatur formullar orasida bog‘liklik o‘rnatadi.

$$S(m) = \sum_{k=1}^{N} e^{2\pi i(m\,,\,x^{(k)})}$$

ichki summa trigonometrik summa bo‘ladi. Xususiy xolda $x^{(k)}$ sifatida teng taqsimlanmagan $x^{(k)} = \left(\left\{\frac{k}{N}\right\},\ \dots\ ,\left\{\frac{k^n}{N}\right\}\right)$ to‘rni olsak, u holda $S(m)$ sonlar nazariyasida ma’lum bo‘lgan ratsional trigonometrik yig‘indiga aylanadi, bunda N – natural son, $\{y\}$ - $y$ sonning kasr qismi.

$$S(m) = \sum_{k=1}^{N} e^{2\pi i\frac{m_1 k+\dots+m_n k^n}{N}} \ .$$

N . M. Korbov [ 39 ] har bir $x_i$ o‘zgaruvchisi bo‘yicha davriy bo‘lgan va Fur’e koeffitsienti

$$|S(m)| \le C(K(\overline{m}))^{-\alpha}\ , \qquad K(\overline{m}) = \prod_{j=1}^{n} max\left(1\,, |m_j|\right).$$

tengsizlikni qanoatlantiruvchi $f(x) = f(x_1, x_2, \dots, x_n)$ funksiyalarning $E_n^{\alpha}$(C) sinfini kiritdi. Bunda $\alpha > 1$ va S o‘zgarmas $m_1, \dots, m_n$ larga bog‘liq emas. Keyin, analitik sonlar nazariyasi usullarini qo‘llab, u parallelepipedal to‘r deb ataluvchi

$$x^{(k)} = M_{kn} = \left(\left\{\frac{k}{N}\right\},\ \dots\ ,\left\{\frac{k^n}{N}\right\}\right) \qquad (2)$$

to‘rni qurdi, bunda $a_1, \dots, a_n$ maxsus tanlangan natural sonlar (N modul bo‘yicha $\beta$ indeksli optimal koeffitsientlar ( sm. [ 39 ] , c. 98 )) va (2) to‘r uchun (1) kubatur formula

$$R_N(f) = 0\ \left(\frac{log^{\alpha n} N}{N^{\alpha}}\right)$$

diyarli optimal yaqinlashishiga ega bo‘ladi.

(2) parallelepipedal to‘r qurilgan $a_1, \dots, a_s$ optimal koeffitsientlar diofant yaqinlashishlar nazariyasi bilan yaqin bog‘liq, xususan kasr ulushlarning teng taqsimlanishi masalalari bilan.

$\gamma_1, \dots, \gamma_s$ lar [0,1] intervaldan olingan ixtiyoriy xaqiqiy sonlar bo‘lsin.

$$0 \leq x_1 \leq \gamma_1 \,, \dots \,, 0 \leq x_s \leq \gamma_s \qquad (3)$$

shartni qanoatlantiruvchi sohani qaraylik va (2) to‘rning shu sohaga yotuvchi nuqtalar sonini $T_N\,(\gamma_1, \dots, \gamma_s)$ orqali belgilaylik.

[ 39 ] da ko‘rsatilganidek, $a_1, \dots, a_s$ butun sonlarning optimal koeffitsient bo‘lishligining zaruriy va yetarlilik sharti

$$T_N\,(\gamma_1, \dots, \gamma_s) = \gamma_1 \dots \gamma_s \; N + R_O \,, \qquad (4)$$

bajarilishi bo‘ladi, bu yerda $R_O = 0(log^{\beta} N)$ va $\beta$ lar N ga bog‘liq emas.

Shunday qilib, parallelepipedal to‘r tugunlari shunday joylashganki, ularning (3) ko‘rinishdagi ixtiyoriy soha bilan ustma-ust tushishlar soni soha hajmini to‘rning barcha nuqtalari soniga ko‘paytirilganiga asimtotik ravishda teng bo‘ladi.

$M_{ks}$ to‘r nuqtalarining birlik kubda teng taqsimlanishi (4) dagi qoldiq hadning yaxshi baholanishiga bog‘liq. To‘r qanchalik teng taqsimlangan bo‘lsa, bu to‘r bilan tuzilgan kubatur formula shunchalik aniqroq bo‘ladi.

$a_1, \dots, a_s$ butun sonlar xar qanday tanlanganda ham (4) tenglikni qoldiq qismini $\log N$ ning biror darajasidan yaxshiroq baholab bo‘lmaydi.

Bundan ko‘rinadiki, optimal koeffitsientlarda qoldiq xadi eng yaxshi bahoga erishadi va shu bilan parallelepipedal to‘rda qurilgan kubatur formullarning optimalligini ta’minlaydi.

Parallelepipedal to‘rda qurilgan kubatur formullarning o‘ziga xos xususiyati ularning chekli trigonometrik polinomlar

$$P(x_1, \dots, x_s) = \sum_{\bar{m}_1 \dots \bar{m}_s \le C_0 N^{1-\varepsilon}} C(m_1, \dots, m_s) e^{2\pi i(m_1 x_1 + \dots + m_s x_s)}, \qquad (5)$$

ni aniq hisoblashida, ya’ni

$$\int_0^1 \dots \int_0^1 P(x_1, \dots, x_s) dx_1 \dots dx_s = \frac{1}{N} \sum_{k=1}^{N} P\left(\left\{\frac{k}{N}\right\}, \quad \dots, \left\{\frac{k^n}{N}\right\}\right)$$

tenglik har qanday (5) ko‘rinishdagi palinom uchun bajariladi.

V. S. Ryaben’kiy [ 53 ] sonlar nazariyasi usuli bilan tuzilgan to‘rlardan ko‘p o‘zgaruvchili funksiyalarni interpolyatsiyalash masalasida foydalanish mumkinligini ko‘rsatdi. $f(x_1, \dots, x_s) \in E_s^\alpha$ funksiyaning Fur’e koeffitsientiga parallelepipedal to‘rda qurilgan kubatur formulani qo‘llab,

$$f(x_1, \dots, x_s) = \frac{1}{N} \sum_{k=1}^{N} f\left(\left\{\frac{a_1 k}{N}\right\}, \dots, \left\{\frac{a_s k}{N}\right\}\right) \phi_k(x_1, \dots, x_s) +$$

$$+ 0\left(\frac{log^{\gamma_1} N}{N^{0.5(\alpha-1)}}\right), \qquad (6)$$

interpolyatsion formulani olamiz, bu yerda $\phi_k(x_1, \dots, x_s)$ - biror aniq funksiya va $\gamma_1$ esa N ga bog‘liq emas.

$E_s^\alpha$ sinfda s>2 da (6) formula boshqa formulalardan sezilarli darajada aniqroq bo‘lib, xatolik shu sinfda 1 / $N^{(\alpha-1)/s}$ tartibga ega.

Matematik fizika masalalarida ko‘pincha $\frac{\partial^{n_1 + \dots + n_s} f}{\partial x_1^{n_1} \dots \partial x_s^{n_s}}$ $(0 \le n_1 + \dots + n_s \le \alpha s, \ \ 0 \le n_\nu \le \alpha)$ tartibli hosilaga ega bo‘lgan funksiyalar bilan ishlashga to‘g‘ri keladi, bunday funksiyalarning sinfi $N_s^\alpha$ deb belgilanadi.

Haqiqatdan ham, masalan,

$$\varphi(x) = f(x) + \lambda \int_0^1 K(x, y) \varphi(y) dy$$

Fredgol’mning 2-tur integral tenglamasini qaraylik.

Faraz qilaylik, $f'(x)$, $\frac{\partial K}{\partial x}$, $\frac{\partial K}{\partial y}$ $va$ $\frac{\partial^2 K}{\partial x \partial y}$ hosilalar mavjud va uzluksiz bo‘lsin. Bu tenglamani iteratsiya usuli bilan yechganda

$$\int_0^1 \dots \int_0^1 K(x, x_1) \dots K(x_{s-1}, x_s)\, f(x_s) dx_1 \dots dx_s \ , \qquad (7)$$

ko‘rinishdagi integrallarni yechishga to‘g‘ri kelar edi, bunda integral ostidagi

$$\mathrm{F}\,(x_1, \dots, x_s) = K\,(x, x_1) \dots K\,(x_{s-1}, x_s) f\,(x_s)$$

funksiyani $H_s^\alpha$ sinfga tegishli deb qarash mumkin.

F funksiyani $H_s^\alpha$ sinfga tegishli deb qarab, biz (7) integral to‘g‘risida to‘liq ma’lumotga ega bo‘lamiz.

Agar (7) integralni hisoblash uchun tugunlari koordinata o‘qlariga parallel bo‘lgan to‘r (reshyotka) hosil qiladigan kubatur formuladan foydalansak, u holda xatolik uchun

$$R = 0\left(\frac{1}{N^{\frac{1}{s}}}\right) \qquad (8)$$

dan yaxshi baho olalmaymiz.

Bu usul bilan olingan natija s ning oshishi bilan yanada yomonlashadi. Shuning uchun (7) integralni hisoblashda katta s lar uchun klassik kubatur formulalar yaroqsiz bo‘lib qoladi.

Sodir bo‘lgan qiyinchiliklarni integral ostidagi funksiya $H_s^1$ ga tegishli bo‘lgan holdagi kubatur formulalar yordamida yengish mumkin.

Bunday kubatur formulalarning to‘ri sonlar nazariyasida ta’riflangan xarakterga ega bo‘lish kerak.

Sonlar nazariyasi xarakteridagi to‘rini qo‘llash ixtiyoriy $\varepsilon > 0$ da va $H_s^1$ sinfda xatoligi

$$R = 0\left(\frac{1}{N^{1-\varepsilon}}\right) . \tag{9}$$

dan oshmaydigan kubatur formulani hosil qiladi.

Bu baho eng yaxshi baho hisoblanadi, chunki $H_s^1$ sinfda to‘rni boshqacha har qanday tanlaganda ham olinadigan baho

$$R = 0\left(\frac{1}{N}\right)$$

dan yaxshi bo‘lmaydi.

Ko‘rinib turibdiki (9) baho $H_s^1$ sinfda klassik usulda olingan (8) va xuddi shunday Monte – Karlo usulida olingan

$$R = 0\left(1/\sqrt{N}\right) .$$

bahodan ancha yaxshi.

**Dissertatsiya ishi 3 ta bob va adabiyotlar ro‘yxatidan iborat.**

1-bob 1.1-§ da sonlar nazariyasining keyinchalik foydalanadigan ta’riflar va ba’zi oldindan ma’lum tasdiqlar keltirilgan.

1.2-§ da sonlar nazariyasining usllaridan foydalanib optimal koeffitsientlar hisoblangan.

2-bob 2.1-§ da Volterrning integral tenglamasi sonlar nazariyasining usllaridan foydalanmay taqribiy hisoblangan va xatoligi ko‘rsatilgan. Xatolik sonlar nazariyasining usullaridan foydalangan xoldagidan ancha kattaligi ko‘rsatilgan. Misol ishlab ko‘rsatilgan.

2.2-§ da $E_s^\alpha$ sinfiga kiruvchi funksiyalar uchun olingan har qanday kvadratur formula $H_s^\alpha$ sinfidagi davriy funksiyalar uchun ham o‘rinli bo‘ladi , chunki 7 - lemmaning birinchi xulosasiga ko‘ra, bunday funksiyalar $E_s^\alpha$ sinfiga tegishlidir . Bunda $H_s^\alpha$ sinfidan ixtiyoriy funksiya uchun kvadratur formulalarini qurish masalasini $E_s^\alpha$ sinfidagi kvadratur formulalari masalasiga keltirish mumkinligini ko‘rsatgan.

3-bob 3.1-§ da parallelopepidal to‘r qurilgan.

3.2-§ esa ko‘p o‘lchovli Volterr integral tenglamalarni sonlar nazariyasi usullari yordamida taqribiy yechish masalasi ko‘rib chiqilgan.

## ASOSIY QISM

## I BOB. FUNKSIYALARNI INTERPOLYASIYALASH

### 1.1-§. Interpolyatsion ko‘phadlarning mavjudligi va yagonaligi. Lagranj interpolyatsion formulasi

Biz asosan algebraik interpolyatsiyalash bilan shugullanamiz. Masalaning quyilishi quyidagichadir. Darajasi p dan yukori bo‘lmagan shunday kupxad kurilsinki, u berilgan (p+ 1) ta $x_0, x_1, \ldots, x_p$ nuqtalarda berilgan

$f(x_0), f(x_1), \ldots, f(x_n)$

qiymatlarni qabo‘l qilsin. Bu masalani geometrik ta’riflash ham mumkin: darajasi p dan ortmaydigan shunday R(x) ko‘phad qurilsinki, uning grafigi berilgan (n+1) ta $M_k(x_k\ f(x))$ $(k = \overline{0,n})$ nuqtalardan o‘tsin.

Demak, $c_t$ koeffitsientlarni shunday aniqlash kerakki,

$$P(x) = c_0 + c_1 x + \cdots + c_n x^n \qquad (1.6)$$

ko‘phad uchun ushbu

$$P(x_k) = f(x_k) \qquad k = 0,1,2 \ldots, n \qquad (1.7)$$

tengliklar bajarilsin. B u tengliklarni ochib yozsak, $c_t(t = \overline{0,n})$ larga nisbatan (n+1) noma’lumli (n+1) ta tenglamalar sistemasi hosil bo‘ladi:

$$\begin{cases} \boldsymbol{c_0 + c_1 x_0 + c_2 x_0^2 + \cdots + c_n x_0^n = f(x_0)} \\ \boldsymbol{c_0 + c_1 x_1 + c_2 x_1^2 + \cdots + c_n x_1^n = f(x_1)} \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ \boldsymbol{c_n + c_1 x_n + c_2 x_n^2 + \cdots + c_n x_n^n = f(x_n)} \end{cases} \qquad \textbf{(1.8)}$$

Bu sistemaning determinanti Vandermond determinantidir: $W(x_0, x_1, x_2, \ldots, x_p)$. Masala mazmunidan ravshanki, $x_k$ nuqtalar bir-biridan farqli, demak bu determinant noldan farqlidir. Shuning uchun ham (1.8) sistema va shu bilan birga kuyilgan interpolyatsiya masalasi yagona yechimga ega. Bu sistemani yechib, $C_t$ larni topib (1.6) ga qo`ysak, R(x) kuphad aniqlanadi. Biz R(x) ning oshkor kurinishini topish uchun boshqacha yo`l tutamiz, avvalo fundamental ko‘phadlar deb ataluvchi $Q_{nj}(x)$ larni, ya’ni

$$Q_{nj}(x_i) = \delta_i^j = \begin{cases} 0, & i \neq j \quad bo`lganda \\ 1, & i = j \quad bo`lganda \end{cases}$$

shartlarni sanoatlantiradigan ya - darajali kupdadlarni ko`ramiz. U holda

$$L_n^x = \sum_{j=0}^{n} f(x_j) Q_{nj}(x) \qquad (1.9)$$

izlanayotgan interpolyatsion ko‘phad bo‘ladi. Haqiqatan ham, barcha i = 0, 1, 2, . . . n uchun

$$L_n^x = \sum_{j=0}^{n} f(x_j) Q_{nj}(x_i) = \sum_{j=0}^{n} f(x_j)\, \delta_i^j = f(x_i)$$

va ikkinchi tomondan $L_n(x)$ n-darajali ko‘phaddir.

Endi $Q_{n,j}(x)$ ning oshkor kurinishini topamiz, $i \neq j$ bo‘lganda $Q_{n,j}(x) = 0$, shuning uchun ham $Q_{nj}(x\ )$ ko‘phad $i \neq j$ bo‘lganda x —$x_i$ ga bo‘linadi. Shunday qilib, n- darajali ko‘phadning p ta bo‘luvchilari bizga ma’lum, bundan zsa

$$Q_{n,j}(x) = C \prod_{i \neq j} (x - x_i)$$

kelib chiqadi. Noma’lum ko‘paytuvchi C ni esa

$$Q_{n,j}(x) = C \prod_{i \neq j} \left(x_j - x_i\right) = 1$$

shartdan topamiz; natijada:

$$Q_{n,j}(x) = \prod_{i \neq j} \left(\frac{x - x_i}{x_j - x_i}\right) = 1$$

Bu ifodani (1.9) ga quyib, kerakli ko‘phadni aniqlaymiz:

$$L_n(x) = \sum_{i=0}^{n} f(x)_i \prod_{i \neq j} \left( \frac{x - x_i}{x_j - x_i} \right) \tag{1.10}$$

Bu kuhad Lagranj interpolyatsion, ko‘phadi deyiladi.

Bu formulaning xususiy hollarini ko`raylik: n = 1 bo‘lganda, Lagranj ko‘phadi ikki nuqtadan o‘tuvchi to‘g‘ri chiziq formulasini beradi:

$$L_n = \frac{x - x_1}{x_1 - x_0} f(x_0) + \frac{x - x_0}{x_0 - x_1} f(x_1)$$

Agar n = 2 bo‘lsa, u vaqtda kvadratik interpolyatsion ko‘phadga ega bo‘lamiz, bu kuhad uchta nuqtadan o‘tuvchi va vertikal uqqa. ega bo‘lgan parabolani aniklaydi;

$$L_2(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} f(x_0) + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} f(x_1) + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} f(x_2)$$

Misol . 0, 1, 2 nuqtalarda mos ravish da 1, 2, 5 qiymatlarni qabo‘l qiluvchi kvadratik ko‘phad qurilsin.

Bu qiymatlarni oxirgi formulaga qo‘yamiz:

$$L_2(x) = \frac{(x - 1)(x - 2)}{(0 - 1)(0 - 2)} 1 + \frac{(x - 0)(x - 2)}{(1 - 0)(1 - 2)} 2 + \frac{(x - 0)(x - 1)}{(2 - 0)(2 - 1)} 5 = x^2 + 1$$

Endi Lagranj interpolyatsion formulasining boshqa ko‘rinishini keltiramiz. Buning uchun $\omega_{n+1}(x) = \prod_{i=0}^{n} (x - x_i)$

ko‘phadni kiritamiz. Bundan hosila olsak

$$\omega`_{n+1}(x) = \sum_{k=0}^{n} [\prod_{i \neq j} (x - x_i)]$$

Kvadrat qavs ichidagi ifoda $x = x_j$ va $k \neq j$ bo‘lganda nolga aylanadi, chunki {$x_i$ — $x_j$}) ko‘paytuvchi qatnashadi. Demak,

$$\omega`_{n+1}(x) = \prod_{i \neq j} (x - x_i)$$

Shuning uchun ham $\prod_{i \neq j} \left(\frac{x - x_i}{x_j - x_i}\right)$ Lagranj koeffitsientini

$$\frac{\omega_{n+1}(x)}{\omega`_{n+1}(x_j)(x - x_i)}$$

ko‘rinishda yozish mumkin. Bundan esa Lagranj ko‘phadi quyidagi ko‘rinishga ega bo‘ladi:

$$L_n(x) = \sum_{j=0}^{n} \frac{f(x_j)\omega_{n+1}(x)}{\omega`_{n+1}(x_j)(x - x_i)} \qquad (1.11)$$

Endi tugunlar bir xil uzok lik da joylashgan: $x_1 - x_0 = x_2 - x_1 = \ldots = x_n - x_{n-1} = h$ xususiy holni ko‘ramiz. Bu holda soddalik uchun $x = x_1 + th$ almashtirish bajaramiz, u holda

$$\boldsymbol{x - x_j = h(t - j)\ ,\ \omega_{n+1}(x) = h^{n+l}\omega^*_{n+1}(t)}$$

bu yerda

$$\boldsymbol{\omega^*_{n+1}(t) = t(t-1)\ \ldots\ (t-n),\ \ \omega^*_{n+1}(xj) = (-1)n - 1j!\,(n-1)!\,hn}$$

bo‘lib, (1.6) Lagranj interpolyatsion ko‘phadi quyidagi ko‘rinishni oladi:

$$L_n(x_0 + th) = \omega^*_{n+1}(x) \sum_{i=0}^{n} \frac{(-1)^{n-j} f(x_j)}{(t-j)j!\,(n-j)!} \qquad (1.12)$$

## 1.2-§ Interpolyatsiyalash xatoligi

Biz *f ( x )* fiinksiyani interpolyatsion $L_n(x)$ ko‘phadga almashtirganimizda

$$\boldsymbol{\tau_n(x) = f(x) - L_n(x)}$$

xatolikka yo‘l qo‘yamiz. Bu *interpolyatsiyalash xatoligi* deyiladi. Tugun nuqtalarda xatolik nolga teng. $[a\ ,b\ ]$ ga tegishli ixtiyoriy *x* nuqtadagi ifodasini topamiz va baholaymiz. Buning uchun quyidagi funksiyani qaraymiz:

$$\varphi(z) = f(z) - L_n(x) - K\omega_{n+1}(z) \qquad (1.13)$$

bu yerda $z \in [a; b]$ , *K* - o‘zgarmas va

$$\omega_{n+1}(z) = (z - x_0)(z - x_1)(z - x_2) \dots (z - x_n) \qquad (1.14)$$

(6) dagi o`zgarmas K ni $\varphi(x) = 0$ shartdan topamiz

$$K = \frac{f(x) - L_n(x)}{\omega_{n+1}(x)} \qquad (1.15)$$

*f( z )* funksiya [a,b] da *n* + 1 marta uzluksiz differensiallanuvchi bo`lsin deymiz. $\varphi(z)$ funksiya $[a$ ,b] da *n + 2* ta nuqtada nolga teng, ular *x*, $x_0$,$x_1$, $x_2$,…$x_n$ Roll teoremasiga asosan, $\varphi`(z)$ $[a; b]$ga tegishli *n* + 1 ta, $\varphi``(z)$ $[a; b]$ *n* ta nolga ega bo`ladi va hokazo. $\varphi^{n+1}(z)$ $[a; b]$ da kamida bitta nolga ega bo‘ladi, ya'ni $\varphi^{n+1}(\xi) = 0,\ \xi \in [a; b]$ (1) dan n+1 marta hosila olib, $z = \xi$ desak , quyidagiga ega bo`lamiz:

$$f^{n+1}(\xi) = K(n + 1)! \qquad (1.16)$$

(1.15)va (1.16) dan

$$f(x) - L_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}\omega_{n+1}(x) \qquad (1.17)$$

kelib chiqadi. Bundan

$$|f(x) - L_n(x)| = \frac{M_{n+1}}{(n+1)!}|\omega_{n+1}(x)| \qquad (1.18)$$

Bahoga ega bo`lamiz, buyeda $M_{n+1} = \underbrace{sup}_{[a;b]} \left|f^{(n+1)}(x)\right|$ ga teng.

**Ta'rif** . Quyidagi to'rt shartni qanoatlantiruvchi ushbu S(f,x) = $S_3$( f,x, $\Delta_n$) funksiya interpolyatsion kubik splayn deyiladi:

1) Har bir [$x_i$ , $x_{i+1}$] (1 = $\overline{0,n}$) oraliqda S($f$,$x$) ∈ $N_3$(R)

2) $S(x) \in C^2\ [a;b]$;

3. To'rning $x_k$(k=0,n) tugunlarida S ($f$, $x_k$) = $f_k$ tenglik o'rinli;

4. S '' ( f,x ) uchun

$$S''(f,a) = S''(f,b)=0 \tag{1.19}$$

chsgaraviy shartlar bajariladi. Bu turt shartni kanoatlantiruvchi yagona S($f$, $x$) splayn mavjudligini kursatamiz. Buning uchun avval kuyidagi yordamchi faktlarni keltiramiz.

***Lemma***. Faraz kilaylik, A = [$a_{ij}$] n-tartibli kvadrat matritsaning elementlari

$$\min_{1\le i\le n}\{|a_{ii}| - \sum_{i\ne 1}|a_{ii}|\} = q > 0 \tag{1.20}$$

shartni kanoatlantirsin. U holda $A\bar{x} = \bar{b}$ sistema yagona yechimga ega bulib, uning yechimi

$$\max_{1\le k\le n}|x_k| \le q^{-1} \max_{1\le k\le n}|b_k| \tag{1.21}$$

tengsizlikni kanoatlantiradi.

**Splayn interpolyatsiyasining yaqinlashuvi bo'yicha natijalar isbotsiz beriladi.**

Interpolyatsiyasining segmenti [a;b] berilgan, to`r $x_0<\ldots< x_n$ bu yerda $x_0$=a, $x_n=b$ berilgan segmentda kondensatsiyalanadi, *ya'ni n* → ∞ (uchastkalar soni ortib bormadi) va $n \rightarrow \infty$ $\bar{h} =\cdot \max_{i=1,\dots,n} h_i \rightarrow 0$ (segmentlar sonining ko‘payishi bilan ma'lum bo‘lim uchun maksimal segment uzunligini tavsiflovchi h parametri nolga intiladi)

Quyidagi natijalar shuni ko‘rsatadiki, to‘r [a, b] segmentida aniqlanganda, interpolyatsiya kubik splayin $S_{\bar{h}}(x)$ [a;b] oraliqda berilgan f (x) funksiyaga yaqinlashadi, $S_{\bar{h}}(x)$ splaynning hosilalari uning (mos keladigan) hosilalariga yaqinlashadi.

Konvergentsiya natijasi tuzilgan funktsiya fazosi, yaqinlashuv tartibi $\bar{h}\rightarrow 0$ va yaqinlashuv faktining o‘zi $\Omega\bar{h}$, $\bar{h}\rightarrow 0$ panjara ketma-ketligini tanlashga bog‘liqligi (mustaqilligi) quyidagilarga bog‘liq:

1) f (x) funksiyaning silliqligi;

2) uning $L_2$[a, b] funksiya fazosida differensiallanuvchi funksiyalar sinfiga mansubligi;

3) interpolyatsiya qiluvchi kubik splaynni qurishda qo‘shimcha (chegaraviy) shartlarni tanlash.

Masalan, [a, b] segmentida yuqoridagi shartlarga qarab 1), 2), 3) interpolyatsiya kub splaynning funksiyaga yaqinlashuvi 4, 3, 2, 3/2 tartibli bo‘lishi mumkin.

Agar yaqinlashish sodir bo‘lsa, u holda splaynning birinchi hosilasining funktsiyaning birinchi hosilasiga yaqinlashishi ro‘yxatdagi birinchi to‘rtta holat uchun kafolatlanadi, ya'ni (4, 3, 2, 3/2) va quyidagi tartibga ega bo‘ladi. 1 kam, ya'ni mos ravishda (3, 2, 1 , 1/2).

Splaynning ikkinchi hosilasining funktsiyaning ikkinchi hosilasiga yaqinlashishi ro‘yxatdagi birinchi ikkita holat uchun kafolatlanadi, ya'ni (4, 3) va mos ravishda (2, 1) tartibli.

**Natijani yuqori konvergentsiya bilan shakllantiramiz:**

**2-TEOREMA.** [a, b] segmentida bir xil to‘r $x_0 < ...< x_n$. ko‘rsatilsin, bunda $x_0$=a $x_n$=b, n ta qadamlar soni bilan $h = \frac{b-a}{n}$ aniqlanadi

f (x) funksiya [a, b] oraliqda to‘rt marta uzluksiz differentsiallanuvchi bo‘lsin.

$P_{3,h}$ (x) nuqtalar ustidagi h qadam ($x_i$ , $f_i$ ), $i$=0,1, 2, 3 bo‘lgan to‘rda qurilgan ko‘pi bilan 3 darajali interpolyatsiya polinomi bo‘lsin.

$P''_{3,h}(x_0)$, $P''_{3,h}$ $(x_1)$ uning ikkinchi hosilasining to‘rning birinchi ikkita tugunidagi qiymatlarini bildiradi: $x_0$ = a, $x_1$=a + h .

Q $_{3,h}$ (x) ($x_i$, $f_i$), $i$= n, n−1, n−2, n−3 nuqtalar ustidagi qadam h bo‘lgan to‘rda qurilgan eng ko‘pi 3-darajali interpolyatsiya polinomi bo‘lsin.

$Q''_{3,h}(x_n)$ , $Q''_{3,h}(x_n-1)$ uning ikkinchi hosilasining oxirgi ikkita panjara tugunidagi qiymatlarini bildirsin: $x_n-1$ = b − h, $x_n$ = b .

Keyin $a_i$ $b_i$ $c_i$ $d_i$ $i$=1, ...n koeffitsientli interpolyatsiya kubik splayn $\hat{S}_h$(x), SLAE asosida n ta bo‘limli to‘rda (1.1) formula bo‘yicha tuzilgan.

$$\begin{cases} \frac{h_1}{3} c_0 + \frac{h_1}{6} c_1 = \frac{h_1}{3} P''_{3,h}(x_0) + \frac{h_1}{6} P''_{3,h}(x_1) \\ c_{i-1}\, h_i + 2\,(h_i + h_{i+1})\, c_i + c_{i+1}\, h_{i+1} = 6 \cdot \left( \frac{f_{i+1} - f_i}{h_{i+1}} - \frac{f_i - f_{i-1}}{h_i} \right), \; i = 1,\ldots n-1 \\ \frac{h_n}{6} c_{n-1} + \frac{h_n}{3} c_n = \frac{h_n}{6} Q''_{3,h}(x_{n-1}) + \frac{h_n}{3} Q''_{3,h}(x_n) \end{cases}$$

bir xil kondansativ to‘rlar ketma-ketligi bo‘yicha: $\bar{\Omega}$h, h→0 4-tartibli [a, b] segmentidagi f (x) funksiyaga bir xilda yaqinlashadi:

$$\max_{x\in[a,b]} \left| f(x) - \hat{S}_h(x) \right| \le \hat{M}h^4$$

$S_h(x)$ ning birinchi va ikkinchi hosilalari f(x) ning 3 va 2-tartibli mos keladigan (birinchi va ikkinchi) hosilalariga bir xilda yaqinlashadi:

$$\max_{x\in[a,b]} \left| f'(x) - \hat{S}'_h(x) \right| \le \hat{M}h^3$$

$$\max_{x\in[a,b]} \left| f''(x) - \hat{S}''_h(x) \right| \le \hat{M}h^2$$

где $\hat{M} = const \cdot \max_{x\in[a,b]} \left| f^{IV}(x) \right|$

**Izox:**

1) 2-teoremaning ma'nosi quyidagicha: agar ma'lum to‘rt marta uzluksiz differensiallanuvchi f (x) funksiya uchun $\widehat{M}$ doimiysining yuqori chegarasi ma'lum bo‘lsa, bir xil n to‘rning shunday sonli segmentlarini tanlash mumkinki, $\hat{S}_h$ (x) splayn va f (x ) funktsiyasi o‘rtasidagi farq (ularning hosilalaridagi farqlarni o‘z ichiga olgan holda) oldindan belgilangan qiymat e > 0 dan kichik edi, masalan, $10^{-4}$ dan kichik.

$$\max_{x\in[a,b]} \left| f(x) - \hat{S}_h(x) \right| \le 10^{-4}$$

$$\max_{x\in[a,b]} \left| f'(x) - \hat{S}'_h(x) \right| \le 10^{-4}$$

$$\max_{x\in[a,b]} \left| f''(x) - \hat{S}''_h(x) \right| \le 10^{-4}$$

2) Yuqoridagi konstantaning yuqori chegarasi ma'lum to‘rt marta uzluksiz differentsiallanuvchi funktsiya uchun ma'lum yoki ma'lum bo‘lishidan qat'i nazar, interpolyatsiya qiluvchi kub splaynning funktsiyaga (shu jumladan ularning hosilalariga) yaqinlashishi va formulada ko‘rsatilgan tartibda sodir bo‘ladi teorema. Bu shuni anglatadiki, etarlicha katta miqdordagi to‘r bo‘limlari

uchun splayn va funktsiya o‘rtasidagi farqlar (shu jumladan ularning hosilalaridagi farqlar) o‘zboshimchalik bilan kichik bo‘ladi.

3) 2-teoremani shakllantirishda ko‘phadlar o‘rniga $P_{3,h}(x)$ va $Q_{3,h}(x)$ (ularning darajasi 3 dan yuqori emas) yuqori darajali polinomlardan foydalanish mumkin:

$P_{4,h}$ (x) 4 dan yuqori bo‘lmagan interpolyatsiya polinomi bo‘lib, birinchi beshta to‘r nuqtasi ustida, ya’ni nuqtadan ($x_i$, $f_i$ ), i = 0,1,2,3,4 ustidagi qadam h bo‘lgan panjara ustiga qurilgan va uning ikkinchi hosilasining qiymatlari $P''_{4,h}$ ($x_0$) , $P''_{4,h}$ ($x_1$) birinchi ikkita panjara tugunlarida: $x_0$ = a, $x_1$=a + h .

## II- bob Kub splayn orqali funksiyalarni interpalyatsiyalash

Silliqligi yuqori bo`lmagan funksiyalar uchun ko‘phadlar yaqinlashish apparati sifatida qator noqulayliklarga ega. Bulardan eng asosiysi shundan iboratki, bunday funksiyalarning biror nuqta atrofidagi holati, ularning to`la holati bilan uzviy bog`liqdir. Bundan tashqari interpolyatsion ko‘phadlarning nuqsoni sifatida ularning har doim ham interpolyatsiyalanuvchi funksiyaga yaqinlashavermasligidir. Eng yaxshi tekis yaqinlashuvchi ko‘phadlarning kamchiligi sifatida shuni kursatish mumkinki, ularni qurish juda qiyin va odatda bunday ko‘phadning darajasi ortishi bilan koeffitsientlari ham tez usib boradi. Oxirgi vaqtlarda shu nuqsondan holi bo‘lgan boshqa yaqinlashish apparatlari ishlab chiqilmokda. Nazariy tadqiqot va tatbiqlarda yaxshi natija beradigan apparat — splayn-funksiyalar apparatidir. Splaynning ta’rifi bilan tanishaylik.

Haqiqiy o`qdagi [a,b] oraliqda ushbu

$$\Delta_n: a = x_0 < x_1 < \cdots < x_n = b$$

tur berilgan bulsin. Faraz qilaylik, $N_m$(R ) darajasi m dan ortmaydigan ko‘phadlar to‘plami, $S^{\{k\}} = S^{\{k\}}$ [a,b] o`zi va k tartibgacha hosilalari-[a,b] oraliqda uzluksiz bo‘lgan funksiyalar to‘plami bo‘lsin.

**Ta'rif** . Quyidagi ikkita shartni qanoatlantiruvchi ushbu $S_m(x) = S_m(x, \Delta_n)$ funksiya defekti 1 ga teng bo'lgan m- darajali polinominal splayn deyiladi:

1) Har bir $[x_i, x_{i+1}]$ $(1 = \overline{0,n})$ oraliqda $S_m(x) \in N_m$ (R)

2) $S_m(x) \in C^{(m-1)}$ $[a;b]$

Bu yerdagi $\{x_i\}$ nuqtalar splayn tugunlari deyiladi. $S_m(x)$ splaynning m-hosilasi [a, b] oraliqda uzilishga ega bulishi ham mumkin.

Agar k=0,1,…m, lar uchun $S_m^{(k)}(a + 0) = S_m^k(b - a)$ tengliklar bajarilsa, $S^m(x)$ splayn b — a davrli davriy splayn deyiladi. Ta'rifni kanoatlantiruvchi splaynlar bilan bir qatorda shunday splaynlar ham karaladiki, ularning silliqligi $\Delta_n$to'rning turli kismlarida turlichadir. Bunday splaynlar [a, b] oraliqning turli qismlarida turli silliklikka ega bo'lgan funksiyalarni ikinlashtirishda foydalaniladi.

Odatda, splayn yagona ravishda ashщlanishi uchun [a, b] oraliqning chetki a va b nuqtalarida chegaraviy shartlar deb ataluvchi kushimcha shartlar qo`yiladi. Amalda uchinchi Darajali, ya'ni kubik splaynlar keng qo`llaniladi.

Splaynlarning hisoblash matematikasida keng qullanilayotganligi sabablaridan yana biri ularning qiymatlarini EXMlarda hisoblashning kulayligi va ular yordamida interpolyatsiyalash kabi jarayonlarning keng sinfdagi turlar uchun yaxshi yaqinlashishligidadir (yukorida aytilgandek kuphad bilan interpolyatsiyalash bunday emas). Bundan buyon biz interpolyatsion kubik va $S_3^{``}(x) = S_3^{``}(b) = 0$ chegaraviy shartlarni kanoatlantiruvchi splaynlar bilan shug'ullanamiz.

## 2.1-§ **Kub splayn, uning kanonik shakli va xossalari, tabiiy chegara shartlari**

**Ta'rif** 2. [a, b] segmentining $x_i$ , i = 0,...n to‘rida aniqlangan kubik splayn tabiiy chegara shartlariga ega splayn (***Kub splayn, uning kanonik shakli va xossalari, tabiiy chegara shartlari***) deyiladi, agar:

$$S''(a) = 0;\ S''(b) = 0 \qquad (2.1)$$

(segmentning [a, b] chegaralaridagi splaynning ikkinchi hosilalari yo‘qoladi).

Izoh: To‘rda $x_i$ , $i = 0,...n$, ***Kub splayn, uning kanonik shakli va xossalari, tabiiy chegara shartlari*** bilan kubik splaynning [a, b] segmentini ko‘rsatish uchun 3n-1 shartlarga mos keladigan 4n koeffitsient kerak bo‘ladi.

Misol: Segment [−1; 1], panjara $x_0$ = -1; $x_1$ = 0; $x_2$ = 1, segmentlar soni n = 2, panjara qadam h = 1. Nolning chap tomonidagi segment [−1;0] va nolning o‘ng tomonidagi maydon [0;1].

1) $$S(x) = \begin{cases} x^3 + 3x^2, & x \in [-1; 0] \\ -x^3 + 3x^2, & x \in [0; 1] \end{cases}$$

***Kub splayn, uning kanonik shakli va xossalari, tabiiy chegara shartlari*** bilan kub splayn hisoblanadi, chunki:

nolning chap tomonidagi maydonda: $S''(x) = 6x + 6$, $S''(-1) = 6\cdot(-1) + 6 = 0$,

nolning o‘ng tomonidagi maydonda: $S''(x) = -6x + 6$, $S''(1) = -6\cdot 1 + 6 = 0$.

[−1;1] oraliqda $S(x)=x^3$ EHC (***Kub splayn, uning kanonik shakli va xossalari, tabiiy chegara shartlari***) bilan kubik splayn emas, chunki

$S''(x)=6x$, $S''(-1)=-6$, $S''(1)=6$ .

**Kub splayin orqali interpolyatsiya** [a, b] oraliqda $f\,(x)$ funksiya va n ta segment va $x_i$, $i$=1,2,3,…n tugunli to‘rni ko‘rib chiqaylik.

**Ta'rif 3**. [a, b] segmentining $x_i$, $i = 0,...n$ to‘rida aniqlangan kubik splayn S(x), agar splayn va funktsiya qiymatlari bo‘lsa, f (x) funktsiyasini interpolatsiya qiladi. panjara tugunlari bir xil:

$$S(x_i) = f(x_i), i = 0,...n \tag{2.2}$$

Interpolyatsiya qiluvchi kubik splaynni qurish uchun 4n-2 shartga mos keladigan 4n koeffitsientni aniqlash kerak: ular orasida 3n-3 konjugatsiya shartlari va n+1 interpolyatsiya shartlari, qarang (2.5)-(2.7) va (2.9).

Shartlarga qaraganda ko‘proq talab qilinadigan koeffitsientlar mavjud bo‘lganligi sababli, interpolyatsiya kubik splaynni qurish muammosini yagona hal qilish uchun yana ikkita shart kerak.

Ko‘pincha ishlatiladi:

- tabiiy chegara shartlari (1.5): $S''(a) = 0$; $S''(b) = 0$

- funksiyaning ikkinchi hosilalari va splaynning mos kelishi sharti

$$S''(a) = f''(a) = \mu 1;\ S''(b) = f''(b) = \mu_2 \tag{2.3}$$

– funksiya va splaynning birinchi hosilalarining mos kelishi sharti

$S'(a) = f'(a) = \nu 1$; $S'(b) = f'(b) = \nu_2$

shuningdek, boshqa turdagi (shu jumladan birlashtirilgan) chegara shartlari, shu jumladan splayn davriyligi uchun shartlar.

Splaynning mavjudligi, yagonaligi va usuli haqidagi teoremalardan birini tuzamiz va isbotlaymiz.

1-TEOREMA. Qiymatlari [a, b] segmentining $x_i$ n$i$, = 0,... to‘rida berilgan har qanday f(x) funksiyasi uchun interpolyatsiya kubik splayn S(x) ko‘rinishning chegara shartlariga ega.

$$S''(a) = \mu_1;\ S''(b) = \mu_2 \tag{2.4}$$

mavjud va noyobdir. Uning $c_i$, $i=1,\dots n$ koeffitsientlarini topish uchun (SLAE) ni yechish kerak.

$$\begin{cases} c_0 = \mu_1 \\ c_{i-1}\, h_i + 2\,(h_i + h_{i+1})\, c_i + c_{i+1}\, h_{i+1} = 6\cdot\left(\dfrac{f_{i+1}-f_i}{h_{i+1}} - \dfrac{f_i - f_{i-1}}{h_i}\right), \\ \qquad i = 1,\dots n-1 \\ c_n = \mu_2 \end{cases}$$

(2.5)

va keyin formulalar yordamida uning qolgan koeffitsientlarini $ai$, $bi$, $di$, $i=1,\dots n$ hisoblaymiz.

$$ai = fi\ ,\ i=1,\dots n \tag{2.6}$$

$$d_i = \frac{c_i - c_{i-1}}{h_i}\ ,\ i=1,\dots n \tag{2.7}$$

$$b_i = \frac{f_i - f_{i-1}}{h_i} + c_i\,\frac{h_i}{3} + c_{i-1}\,\frac{h_i}{6},\quad i=1,\dots n \tag{2.8}$$

SLAE (2.10) usuli bilan echilishi mumkin.

Izox: Teorema bayonida interpolyatsiya qiluvchi kub splaynni qurish imkoniyati ko‘rib chiqiladi va splaynning ikkinchi hosilalari segment chegaralaridagi interpolyatsiya qilingan funktsiyaning ikkinchi hosilalari bilan mos kelishi shart emas, solishtiring (2.4) va. (2.3).

Kub splaynning *4n* koeffitsientini topish uchun 4n tenglamadan foydalaniladi: $3n-3$ konjugatsiya shartlari, 2 chegara sharti (2.3) $n+1$ interpolyatsiya sharti (1.6), shuningdek, uydirma (qo‘shimcha) koeffitsient. $c_0 = \mu_1$

Tenglamalardan koeffitsientlarni chiqarib tashlash $a_i$, $b_i$, $d_i$, $i$ =1,...$n$ $c_i$ , $i$ = 1, ...$n$ ni topish uchun SLAE (2.4) ni oling.

SLAE Sweep Application teoremasining shartlariga mos keladi, bu esa har qanday o‘ng tomon uchun uning yechimining mavjudligi va o‘ziga xosligini bildiradi.

Qolgan koeffitsientlar uchun (2.6) - (2.8) formulalar olinadi. Har qanday (2.4) uchun splaynning barcha koeffitsientlari yagona topiladi, bu esa (2.2) ko‘rinishdagi chegaraviy shartlarga ega interpolyatsiya qiluvchi kubik splaynning mavjudligi va yagonaligini bildiradi.

Isbot:

1) Splayn koeffitsientlarini topish uchun barcha shartlarni yozamiz.

(1.1) ga muvofiq [$x_{i-1}$, $x_i$] segmentidagi splaynning qiymati formula bilan aniqlanadi.

$$S_i(x) = a_i + b_i(x - x_i) + \frac{c_i}{2}(x - x_i)^2 + \frac{d_i}{6}(x - x_i)^3$$

Biz $S_i(x)$ ni ajratamiz va [$x_{i-1}$, $x_i$] segmentida splaynning birinchi va ikkinchi hosilalarining qiymatlarini olamiz:

$$S'_i(x) = b_i + c_i(x - x_i) + \frac{d_i}{2}(x - x_i)^2$$

$$S''_i(x) = c_i + d_i(x - x_i)$$

Agar xi tugunini [$x_{i-1}$, $x_i$] kesmaning o‘ng chegarasi deb hisoblasak, splayn va uning hosilalarini hisoblash uchun $S_i(x)$ dan foydalanish kerak. Keyin

$$S_i(x_i) = a_i$$

$$S'_i(x_i) = b_i$$

$$S''_i(x_i) = c_i$$

Agar $x_i$ tugunini $[x_i, x_{i+1}]$ kesmaning chap chegarasi deb hisoblasak, splayn va uning hosilalarini hisoblash uchun $S_{i+1}(x)$ dan foydalanish kerak. Keyin

$$S_{i+1}(x_i) = a_{i+1} + b_{i+1}(x_i - x_{i+1}) + \frac{c_{i+1}}{2}(x_i - x_{i+1})^2 + \frac{d_{i+1}}{6}(x_i - x_{i+1})^3$$

$$S'_{i+1}(x_i) = b_{i+1} + c_{i+1}(x_i - x_{i+1}) + \frac{d_{i+1}}{2}(x_i - x_{i+1})^2$$

$$S''_{i+1}(x_i) = c_{i+1} + d_{i+1}(x_i - x_{i+1})$$

Splaynning uzluksizligi, uning birinchi va ikkinchi hosilalari uzluksizligi uchun shartlar (qarang: konjugatsiya shartlari):

$$S_i(x_i) = S_{i+1}(x_i), \quad i = 1, \ldots n-1$$

$$S'_i(x_i) = S'_{i+1}(x_i), \quad i = 1 \ldots n-1$$

$$S''_i(x_i) = S''_{i+1}(x_i), \quad i = 1, \ldots n-1$$

Biz ularni quyidagi shaklda yozamiz

$$a_i = a_{i+1} - b_{i+1}h_{i+1} + \frac{c_{i+1}}{2}(h_{i+1})^2 - \frac{d_{i+1}}{6}(h_{i+1})^3, \; i = 1, \ldots n-1 \tag{2.9}$$

$$b_i = b_{i+1} - c_{i+1}(h_{i+1}) + \frac{d_{i+1}}{2}(h_{i+1})^2, \; i = 1, \ldots n-1 \tag{2.10}$$

$$c_i = c_{i+1} - d_{i+1} \cdot h_{i+1}, \; i = 1, \ldots n-1 \tag{2.11}$$

Qani $h_i+1 = x_i+1-x_i$, $i = 1, \ldots n$ panjara qadamlarining ilgari kiritilgan belgilari mavjud.

Interpolatsiya shartlarini koʻrib chiqamiz (2.11).

$S(x_i) = f(x_i)$, $i = 0, \ldots n$

(endi $n+1$ tarmoq tugunlarida $S(x)$ splayn va $f(x)$ funksiyasi qiymatlarining mos kelishi kerak). $i=1,...n$ indekslar uchun $x_i$ tugunidagi splayn $S_i(x)$ formula bilan hisoblanishi mumkinligi sababli interpolyatsiya shartlarini shaklda yozamiz.

$S_i(x_i) = f(x_i),\ i=1,...n$

$$a_i = f_i,\ i = 1,...n \tag{2.12}$$

*(kanonik shaklda yozilgan kubik splaynning "kichik" koeffitsientlari (2.1) tarmoq tugunlaridagi funktsiya qiymatlari bilan mos kelishi kerak).*

$i=0$ indeksi uchun, ya'ni $x_0 = a$ tugunida splaynni $S_1(x)$ sifatida hisoblash mumkin:

$S(a) = S_1(x_0)$

$x_0$ tugunidagi interpolyatsiya sharti shaklni oladi $S_1(x_0) = f(x_0)$,

$$S_1(x_0) = a_1 + b_1(x_0 - x_1) + \frac{c_1}{2}(x_0 - x_1)^2 + \frac{d_1}{6}(x_0 - x_1)^3 = f(x_0)$$

To‘r oralig‘i yozuvidan foydalanib, biz $h_1 = x_1 - x_0$ ni olamiz

$$f_0 = a_1 - b_1(h_1) + \frac{c_1}{2}(h_1)^2 - \frac{d_1}{6}(h_1)^3 \tag{2.13}$$

Splaynning ikkinchi hosilasini segmentning chap chegarasiga, ya'ni $x_0=a$ tuguniga o‘rnatish uchun chegara sharti (2.10) va $S_1(x)$ formulasidan foydalanamiz:

$$S''(a) = \mu_1$$
$$S''(a) = S''_1(x_0),$$
$$S''_1(x_0) = c_1 + d_1(x_0 - x_1) = \mu_1$$

$$c_1 - d_1 h_1 = \mu \tag{2.14}$$

Ikkinchi hosilani segmentning o‘ng chegarasiga, ya'ni tugunga o‘rnatish uchun $x_n=b$ biz (2.14) va $S_n(x)$ formulasidan ham foydalanamiz.

$$S''(b) = \mu_2$$
$$S''(b) = S''_n(x_n)$$
$$S''(b) = S''_n(x_n) = c_n + d_n(x_n - x_n) = \mu_2$$

$$c_n = \mu_2 \qquad (2.15)$$

Splayn koeffitsientlarini tanlash shartlari yoziladi. Keling, sxemalarga o‘tamiz.

(2.11) va (2.14) dan, ya'ni

$$c_i = c_{i+1} - d_{i+1} \cdot h_{i+1}, \qquad i = 1,\ldots n-1$$
$$\mu_1 = c_1 - d_1 h_1 \qquad (2.16)$$

shundan kelib chiqadiki, $\mu_1$ qiymati berilgan $c_0$ qo‘g‘irchoq o‘zgaruvchisi yuqoridagi formulalarni bir xilda yozishga imkon beradi.

$$c_0 = \mu_1$$
$$c_0 = c_1 - d_1 h_1$$
$$c_1 = c_2 - d_2 \cdot h_2$$
$$\ldots\ldots$$
$$c_{n-1} = c_n - d_n \cdot h_n$$

va keyin $d_i$ koeffitsientlarini hisoblash uchun yagona formulalarni (2.16)

$$d_i = \frac{c_i - c_{i-1}}{h_i}, \quad i = 1,\ldots n$$

oling:

Yuqoridagi formulalar (2.10) dan keyin (2.9) va (2.13) dan di koeffitsientlarini yo‘q qilish imkonini beradi. (2.10) dan biz olamiz.

$$b_i = b_{i+1} - c_{i+1}(h_{i+1}) + \frac{(c_{i+1} - c_i)}{2 \cdot h_{i+1}}(h_{i+1})^2 =$$

$$= b_{i+1} + c_{i+1}(h_{i+1})(-\frac{1}{2}) + c_i(h_{i+1})(-\frac{1}{2}) =$$

$$= b_{i+1} + \frac{c_{i+1} + c_i}{2}(h_{i+1})(-1), \qquad i = 1,\dots n-1$$

Keling, barcha formulalarda (2.15) va (2.19) splaynning "pastki" koeffitsientlarini, ya'ni $a_i$, $i=1,\dots$ $n$ ni to‘r tugunlaridagi funktsiya qiymatlari bilan almashtiramiz, ya'ni $f_i$ , $i=1, \dots n$ , shuningdek (2.13) dan foydalangan holda biz $d_i$ , $i = 1, \dots n$ yozuvidan chiqaramiz:

$$f_i = f_{i+1} + b_{i+1}h_{i+1}(-1) + \frac{c_{i+1}}{2}(h_{i+1})^2 + \frac{c_{i+1} - c_i}{6 \cdot h_{i+1}}(h_{i+1})^3(-1), \; i = 1,\dots n -$$

$$f_0 = f_1 + b_1(h_1)(-1) + \frac{c_1}{2}(h_1)^2 + \frac{c_1 - c_0}{6 \cdot h_1}(h_1)^3(-1)$$

Bu tengliklarni xuddi shunday yozish mumkin:

$$f_i = f_{i+1} + b_{i+1}h_{i+1}(-1) + \frac{c_{i+1}}{3}(h_{i+1})^2 + \frac{c_i}{6}(h_{i+1})^2, \qquad i = 0,\dots n-1$$

Ulardan ifodalash mumkin

$$b_{i+1}h_{i+1} = f_{i+1} - f_i + \frac{c_{i+1}}{3}(h_{i+1})^2 + \frac{c_i}{6}(h_{i+1})^2, \qquad i = 0,\dots n-1$$

va formulaning analogini yozing (12.13)

$$b_{i+1} = \frac{f_{i+1} - f_i}{h_{i+1}} + \frac{c_{i+1}}{3}h_{i+1} + \frac{c_i}{6}h_{i+1}, \qquad i = 0,\dots n-1$$

Oldindan olingan iboralarni ko‘rib chiqing

$$b_{i+1} - b_i = \frac{c_{i+1} + c_i}{2}h_{i+1} \qquad i = 1,\dots n-1$$

(2.14) dan foydalanib, biz ulardan $b_i$ koeffitsientlarini chiqarib tashlaymiz:

$$\left(\frac{f_{i+1}-f_i}{h_{i+1}}+\frac{c_{i+1}}{3}h_{i+1}+\frac{c_i}{6}h_{i+1}\right)-\left(\frac{f_i-f_{i-1}}{h_i}+\frac{c_i}{3}h_i+\frac{c_{i-1}}{6}h_i\right)=$$
$$=\frac{c_{i+1}+c_i}{2}h_{i+1} \qquad i=1,...n-1$$

Har bir tenglamaning chap tomoniga $c_i$ koeffitsientlarini, o‘ng tomoniga esa tarmoq tugunlaridagi funktsiya qiymatlarini yozamiz:

$$\frac{c_{i+1}+c_i}{2}h_{i+1}-\left(\frac{c_{i+1}}{3}h_{i+1}-\frac{c_i}{6}h_{i+1}\right)+\left(\frac{c_i}{3}h_i+\frac{c_{i-1}}{6}h_i\right)=$$
$$=\frac{f_{i+1}-f_i}{h_{i+1}}-\frac{f_i-f_{i-1}}{h_i}, \quad i=1,...n-1$$

Atamalarni guruhlaymiz:

$$\frac{c_{i+1}}{6}\cdot h_{i+1}+\left(\frac{c_i}{3}h_{i+1}+\frac{c_i}{3}h_i\right)+\left(\frac{c_{i-1}}{6}h_i\right)=\frac{f_{i+1}-f_i}{h_{i+1}}-\frac{f_i-f_{i-1}}{h_i}$$
$$i=1,...n-1$$

Biz tenglamalarni (2.14) va (2.15) shartlar bilan to‘ldiramiz, splayn koeffitsientlarini topish uchun SLAE ni yozamiz.

$$\begin{cases} c_0=\mu_1 \\ c_{i+1}\cdot h_{i+1}+2\cdot c_i\cdot(h_{i+1}+h_i)+c_{i-1}\cdot h_i=6\cdot\left(\frac{f_{i+1}-f_i}{h_{i+1}}-\frac{f_i-f_{i-1}}{h_i}\right), \\ \qquad\qquad i=1,...n-1 \\ c_n=\mu_2 \end{cases}$$

3) Keling, SLAE (2.11) yechimining mavjudligi va o‘ziga xosligini tekshiramiz. Matritsa uch diagonaldir. Matritsaning birinchi va oxirgi qatorlari qattiq diagonal ustunlikka ega: $1>k_1=0, 1>k_2=0.$

Barcha panjara qadamlari ijobiy: $h_i > 0,\ n=1,...n$ . Shuning uchun, matritsaning qatorlarida (ikkinchi qatordan oxirgi qatorgacha) asosiy diagonalning chap va o‘ng tomonida joylashgan koeffitsientlar nolga teng emas.

$h_i+1 > 0;\ h_i > 0,\ i = 1,...n-1$ va qattiq diagonal ustunlik kuzatiladi:

$2 \cdot (h_i+1 + h_i) > h_{i+1} + h_i,\ i = 1,...n-1$

Sweep Application teoremasiga ko‘ra, SLAE (2.11) yechimi uning har qanday o‘ng tomoni uchun mavjud, noyob va uni supurish orqali topish mumkin. Shunday qilib, to‘rda berilgan har qanday funktsiya uchun interpolyatsiya qiluvchi kubik splaynning mavjudligi va o‘ziga xosligi isbotlangan. Bundan tashqari, uni qurish usuli ko‘rsatilgan: formulalar (2.5) - (2.8) va tozalash usuli. Segmentlar soni $n=4$ va tugunlar bilan $[a,b]$ segmentining $\Omega_h$ to‘rida interpolyatsiya kubik splayn koeffitsientlarini topish uchun SLAE ning umumiy ko‘rinishini keltiramiz.

$x_0 < x_1 < x_2 < x_3 < x_4$., где $x_0 = a,\ x_4 = b$:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ h_1 & 2\cdot(h_1+h_2) & h_2 & 0 & 0 \\ 0 & h_2 & 2\cdot(h_2+h_3) & h_3 & 0 \\ 0 & 0 & h_3 & 2\cdot(h_3+h_4) & h_4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} =$$

$$= 6 \times \begin{bmatrix} \mu_1 \\ \dfrac{f_2 - f_1}{h_2} - \dfrac{f_1 - f_0}{h_1} \\ \dfrac{f_3 - f_2}{h_3} - \dfrac{f_2 - f_1}{h_2} \\ \dfrac{f_4 - f_3}{h_4} - \dfrac{f_3 - f_2}{h_3} \\ \mu_2 \end{bmatrix}$$

Здесь $h_1 = x_1 - x_0;\ h_2 = x_2 - x_1;\ h_3 = x_3 - x_2;\ h_4 = x_4 - x_3,$

$f_0 = f(x_0);\ f_1 = f(x_1);\ f_2 = f(x_2);\ f_3 = f(x_3);\ f_4 = f(x_4).$

# III BOB. SONLAR NAZARIYASI USULLARI YORDAMIDA GILBERT YADROLI SINGULYAR INTEGRAL TENGLAMALARNI TAQRIBIY YECHISH

## 3.1-§. Singulyar integral tenglamalarni algebraik tenglamalar sistemasiga keltirish.

Ushbu ishda ko‘rilayotgan singulyar integral tenglamalarni taqribiy yechish masalasi, singulyar integral tenglamani sonlar nazariyasi to‘ri bilan qurilgan kvadratur formulalarini qo‘llash orqali algebraik tenglamalar sistemasiga keltirishga asoslangan

Quyidagi

$$\varphi(P) = \int\limits_{G_s} \prod_{j=1}^{s} ctg\pi\left(x_j - y_j\right) K(P,Q)\varphi(Q)dQ + f(P) \qquad 3.1)$$

Gilbert yadroli ko‘p o‘lchovli sinulyar integral tenglamani ko‘rib chiqaylik, bunda

$$\mathrm{P} = (x_1, \dots, x_s), \quad \mathrm{Q} = (y_1, \dots, y_s), \quad \mathrm{dQ} = (dy_1, \dots, dy_s).$$

(7.1) tenglamada ozod hadi va yadro

$$f(P) \in E_s^a(C_1), \quad K(P,Q) \in E_{2s}^a(C_2) \qquad (3.2)$$

shartlarni qanoatlantirsin.

Quyidagicha belgilash kiritamiz:

$$\mathrm{D}(\lambda) = 1 + \sum_{\nu=1}^{\infty} \frac{(-1)^{\nu}}{\nu!} \lambda^{\nu} \int\limits_{G_s} K\begin{pmatrix} P_1, \dots, P_\nu \\ P_1, \dots, P_\nu \end{pmatrix} dP_1 \dots dP_\nu\,, \qquad (3.3)$$

bu yerda

$$K\begin{pmatrix} P_1, \dots, P_v \\ P_1, \dots, P_v \end{pmatrix} = \begin{vmatrix} K(P_1,Q_1)\Phi_1(P_1), \dots, K(P_1,Q_V)\Phi_v(P_1) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. \\ K(P_v,Q_1)\Phi_1(P_v), \dots, K(P_v,Q_V)\Phi_v(P_v) \end{vmatrix}, \qquad (3.4)$$

va

$$\phi_k(P) = i\sum_{m_1\dots m_s<N_1} sign(m_1\dots m_s)e^{2\pi i\left[m_1\left(y_1-\left\{\frac{a_1k}{N}\right\}\right)+\dots+m_s\left(y_s-\left\{\frac{a_sk}{N}\right\}\right)\right]}, \qquad (3.5)$$

$k = 1,2,\dots,\quad N_1 = (Nlog^{\beta}N)^{0.5}$.

Quyidagi teorema o‘rinli

**Теорема.** (3.1) tenglamaning yadrosi va ozod hadi (3.2) shartlarni qanoatlantirsin. Agar D(l) ≠ 0 bo‘lsa, (3.1) tenglamaning yechimi uchun

$$\Phi(\mathrm{P}) = f(\mathrm{P}) + \frac{\lambda}{N}\sum_{k=1}^{N} K(P, M_{ks})\varphi(M_{ks})\phi_k(P) + O\left(N^{-a}log^{\alpha\beta}N\right)$$

$$(j = 1,2,\dots,N)$$

tenglikni qanoatlantiradi, bunda $\varphi(M_{ks})$ miqdor

$$\Phi(M_{js}) = f(M_{js}) + \frac{\lambda}{N}\sum_{k=1}^{N} K(M_{js}),(M_{ks})\phi(M_{kS}) \qquad (j = 1,2,\dots,N)$$

algebraik tenglamalar sistemasini qanoatlantiradi va $\phi_k(P)$ funksiya (3,5) formula bilan aniqlanadi

## 3.2-§. Bir o‘lchovli Gilbert yadroli integral tenglamalarni sonlar nazariyasi usullari yordamida taqribiy yechish

Bu paragrafda sonlar nazariyasi usullarini qo‘llab, quyidagi

$$y(x) = f(x) + \lambda\int_0^1 ctg\pi\,(t-x)K(x,t)y(t)dt \qquad (3.6)$$

singulyar integral tenglamani taqribiy yechamiz.

Faraz qilaylik, (3.6) tenglamaning $f(x)$ va $K(x,\mathrm{t})$ yadrosi mos ravishda $E_1^\alpha(C_1)$ va $E_2^\alpha(C_2)$, $\alpha > 1$ sinflarga tegishli bo‘lsin.

$$\mathrm{A} = 2^{\alpha+1}\left(3 + \frac{2}{\alpha+1}\right) \qquad (3.7)$$

va λ parametr $|\lambda| < \frac{(\alpha-1)}{3\alpha s A C_2}$ shartni bajarsin, $C_2$ esa $E_2^\alpha(C_2)$ sinfning konstantasi bo‘lsin.

$N_1$ va M sonlar quyidagi tengliklar bilan aniqlangan bo‘lsin:

$$N_1=\left[N^{\frac{1}{2}}\log^{\frac{\beta}{2}} N\right], \quad \mathrm{M}=\left[\frac{(\alpha-1\log N-(\alpha\beta-2)\log\log N}{2(\log\log N-\log((3\alpha-1)|\lambda| s A C_2}\right]. \qquad (3.8)$$

bunda $\beta$ optimal koeffisient indeksi. (8.1) tenglamaning yechimini

$$y\,(x) = \sum_{n=1}^{\infty}\lambda^n\,\varphi_n(x) \qquad (3.9)$$

Neyman qatori ko‘rinishda izlaymiz. Bu yerda $\varphi_{in}(x)$ aniqlanishi kerak.

(3.4) ni (3.1) ga qo‘yib va integrallash amali bilan qo‘shish amallarining o‘rinlarini almashtirib,

$$\sum_{n=0}^{\infty}\lambda^n\,\varphi_n\,(x) = f(x) +$$

$$+\sum_{n=1}^{\infty}\ \lambda^n\int_0^1 ctg\pi\,(\mathrm{t}-\ x)K_j(x,t)\varphi_{j\,,n-1}(x)dt,$$

ni hosil qilamiz.

$\lambda$ ning bir xil darajalari oldidagi koeffitsientlarni tenglashtirib, ba’zi almashtirishlarni bajargandan so‘ngquyidagiga ega bo‘lamiz

$$\varphi_o(x) = f(x),$$

$$\varphi_n(x) = \int_{G_n} \sum_{j,\dots,j_n=1}^{q} K_{j_1}(x,t_1)\mathrm{ctg}\pi(t_1 - x) \times$$

$$\times \prod_{l=2}^{n} K_{j_l}(t_{l-1,}t_l)\mathrm{ctg}\pi(t_l - t_{l-1}) f_{j_n}(t_n)\mathrm{d}t_1 \dots \mathrm{d}t_n \qquad (3.10)$$

Ixchamlashtirish maqsadida

$$\left.\begin{aligned} \prod\nolimits_n (x,\bar{t}) = ctg\pi(t_1 - x)\prod_{l=1}^{n} ctg(t_1 - t_{l-1}), \\ F_n(x,\bar{t}\,) = \sum_{j_1,\dots,j_n}^{q} K_{j_1}(x,t_1)\prod_{l=1}^{n} K_{j_l}(t_{l-1},t_l) f_{j_n}(t_n). \end{aligned}\right\}$$

belgilash kiritamiz.

U holda (1.9) dan

$$\varphi_n(x) = \int_{G_n} \prod\nolimits_n (x,\bar{t})\, F_n(x,\bar{t}) d\bar{t} \qquad (3.11)$$

ga ega bo‘lamiz.

8-lemmadan foydalanib, $F_{in}(x,\bar{t}\,)$ funksiyani $t_1,\dots,t_n$ larning funksiyasi sifatida $E_n^{\alpha}(s^n A^n C_2^n C_1)$ sinfga, $x,t_1,\dots,t_n$ larning funksiyasi sifatida esa $E_{n+1}^{\alpha}(s^n A^{n+1} C_2^{n+1} C_1)$ sinfga tegishli ekanligini ko‘rsatish mumkin.

Quyidagi o‘rinli

**Teorema.** N$\geq 2^{\alpha}$ , $\alpha > 1$ bo‘lsin, $f_i(x)$ va $K_{ij}(x$,t) funksiyalar mos ravishda $E_1^{\alpha}(C_1)$ va $E_2^{\alpha}(C_2)$ sinflarga tegishli bo‘lsin, hamda $N_1$ va M kattaliklar aniqlangan bo‘lsin. Agar $\lambda$ parametr $|\lambda\,| < \frac{\alpha-1}{3\alpha s A C_2}$ shartni qanoatlantirsa, u holda ixtiyoriy $i = 1,2, \dots , s$ larda (3.6) tenglama yechimlari uchun quyidagi tenglik o‘rinli

$$y(x) = f(x) + \frac{1}{\mathrm{N}} \sum_{n=1}^{M} \lambda^n \sum_{k=1}^{M} F_n(x\, M_{kn}) \times$$

$$\times \Psi_{knN_1}(x) + 0 \left[ N^{-\frac{\alpha-1}{2}} log^{0,5(\alpha-1)\beta+M-1} N \right] \tag{3.12}$$

Bu yerda

$$\Psi_{knN_1}(x) = \sum_{k(\overline{m})<N_1} i^n sign \left[ \prod_{j=1}^{n} \sum_{s=1}^{j-1} m_{n-s} \right] e^{2\pi i \left[ (\sum_{s=1}^{n} m_s) x - \frac{K \sum_{s=1}^{k-1} (a_s m_s)}{N} \right]} \tag{3.13}$$

Isbot. $P_{inN_1}(x, \bar{t}\,)$ trigonometrik polinomni

$$\tilde{\hat{C}}(x, \overline{m}\,) = \frac{1}{\mathrm{N}} \sum_{k=1}^{N} F_n\ (x\, M_{kn})\ \mathrm{e}^{-2\pi i \frac{(\overline{m}\,, \bar{a})k}{N}}\ , \tag{3.14}$$

$$P_{nN_1}(x, \bar{t}) = \sum_{k(\overline{m})<N_1} \tilde{\hat{C}}\ (x, \overline{m}) \mathrm{e}^{2\pi \mathrm{i}(\overline{\mathrm{m}}, \bar{\mathrm{t}}\,)} \tag{3.15}$$

tengliklar yordamida quramiz.

(3.5) ifodani (3.6) ga qo‘yib,

$$P_{nN_1}(x, \bar{t}\,) = \frac{1}{\mathrm{N}} \sum_{k=1}^{N} F_n\ (x, M_{kn}) \sum_{k(\overline{m})<N_1} \mathrm{e}^{2\pi \mathrm{i} \left[ \overline{\mathrm{m}}, \bar{\mathrm{t}} - \frac{\bar{\alpha}\mathrm{k}}{\mathrm{N}} \right]} \tag{3.16}$$

polinomni hosil qilamiz.

Endi $F_{in}(x, \bar{t}\,)$ ni

$$F_n(x,\bar{t}\,) = \quad P_{nN_1}(x,\bar{t}\,) + r_{nN_1}(x,\bar{t}\,) \tag{3.17}$$

ko‘rinishda yozamiz.

(3.2) va (3.7)-(3.8) munasobatlardan foydalanib, tenglikni

$$y(x) = f + \frac{1}{\mathrm{N}}\sum_{k=1}^{N}\sum_{n=1}^{M}\lambda^n \Psi_{knN_1} F_{in}(x, M_{kn}) + \sum_{n=1}^{M}\lambda^n R_{nN_1}(x) +$$

$$+\sum_{n=M+1}^{\infty}\lambda^n\,\varphi_n(x), \tag{3.18}$$

ko‘rinishda yozib olamiz, bunda

$$R_{inN_1}(x)$$

$$= \int_{G_n}\prod\nolimits_n (x\,,\bar{t})d\bar{t}, \tag{3.19}$$

$$\Psi_{knN_1}(x) = \sum_{k(\overline{m})<N_1}\int_{G_n}\prod\nolimits_n (x,\bar{t}\,)\, e^{2\pi \mathrm{i}\left[\overline{\mathrm{m}},\bar{\mathrm{t}}\,)-\frac{\overline{\alpha}\mathrm{k}}{\mathrm{N}}\right]}\mathrm{d}\bar{t}.$$

Quyidagi

$$\int_0^1 ctg\pi(x-y)e^{-2\pi imx}dx = isign(n)e^{-2\pi imx}$$

tenglikni n marta qo‘llab,

$$\int_0^1 \dots .\int_0^1\prod\nolimits_n (x,\bar{t}\,)e^{2\pi \mathrm{i}\left[\overline{\mathrm{m}},\bar{\mathrm{t}}\,-\frac{\overline{\alpha}\mathrm{k}}{\mathrm{N}}\right]}d\bar{t} =$$

$$= i^n sign\left[\prod_{j=1}^{n}\sum_{s=0}^{j-1} m_{n-s}\right] e^{2\pi i\left[x\,\sum_{s=1}^{n} m_s - \frac{K\sum_{s=0}^{n} a_{sv+l} m_{sv+l}}{N}\right]}$$

ni hosil qilamiz.

U holda

$$\Psi_{knN_1}(x) = \sum_{K(\overline{m})<N_1} i^n \, sign\left[\prod_{j=1}^{n}\sum_{s=0}^{j-1} m_{n-s}\right]\times$$

$$\times\, e^{2\pi i\left[\left[x_l \sum_{s=1}^{n} m_s\right]-\left[k\sum_{s=1}^{n} a_v m_v\right]/N\right]}$$

Ko‘rinib turibdiki, bu yig‘indida

$$m_n = 0,\ m_n + m_{n-1} = 0, \dots,\ m_n + m_{n-1} + \cdots + m_1 = 0$$

tengliklardan hech bo‘lmaganda bittasi o‘rinli bo‘ladigan qo‘shiluvchilar nolga aylanadi.

Endi (3.9) tenglikdagi oxirgi ikki yig‘indini baholash bilan shug‘ullanamiz. $C_{ir}(x,\overline{m})\ va\ \ C_{iF}(P,\overline{m})$ lar $r_{inN_1}$ va $F_{inN_1}$ funksiyalarning mos ravishda Fure koeffitsientlari bo‘lsin.

U holda

$$r_{nN_1}(\mathrm{x},\bar{t}) = \sum\nolimits_{\overline{m}} C_r(x\ ,\overline{m}\ )e^{2\pi i(\overline{m},\bar{t})}, \qquad (3.20)$$

$$F_n(x,\bar{t}\ ) = \sum\nolimits_{\overline{m}} C_F(x,\overline{m}\ )e^{2\pi i(\overline{m},\bar{t})} \qquad (3.21)$$

bunda $\sum_{\overline{m}}$ $n - o‘lchovli$ fazoning barcha butun nuqtalari bo‘yicha yig‘indini bildiradi.

(3.6) va (3.8) tengliklardan

$$C_r(\mathrm{x},\overline{m}) = \begin{cases} C_F(x,\overline{m}) - \tilde{\hat{C}}(x,\overline{m}), \text{ yesli } \mathrm{K}\,(\overline{m}) < N_1, \\ C_F(x,\overline{m}) \quad , \text{yesli } \mathrm{K}\,(\overline{m}) < N_1, \end{cases}$$

tenglikka ega bo‘lamiz.

(3.11) – (3.12) ni (3.10) ga qo‘yib va oxirgi munasobatni hisobga olib,

$$R_{nN_1}(x) = \sum_{k(\overline{m})\leq N_1} \left| \, C_F(x,\overline{m}) - \tilde{\hat{C}}(x,\overline{m})\right|$$

$$+ \sum_{k(\overline{m})\leq N_1} | \, C_F(x,\overline{m})|. \qquad (3.22)$$

Birinchi summani baholash uchun 9-lemmadan foydalanamiz. Lekin hozirgi holda n- o‘zgaruvchi kattalik shuning uchun $C_0$, $C_1$ va C $=C_0^{\alpha}$+ns$B^n$ , bunda $(\mathrm{V} < 3 + 2/(\alpha - 1)\ )$ o‘zgarmaslarning n ga bog‘liqligini aniqlash kerak bo‘ladi.

Optimal koeffitsientlar ta’rifidan va 2-lemmadan $C_0 < 2\left[2 + \frac{3}{\log N}\right]^n$ .

$$F_n(x,\bar{t}\ )\in E_n^{\alpha}\ (s^n A^n C_2^n C_1)\ , \qquad (3.23)$$

bo‘lgani uchun

$$\mathrm{C}=C_1(sAC_2)^n.$$

bo‘ladi.

K$(\overline{m})< N_1$ bo‘lganda

$$e^{-2\pi i(\overline{m},\bar{t}\,)} \in E_n^{\alpha}\,(N_1^{\alpha})$$

ekanini ko‘rish qiyin emas.

Endi, $F_n(x,\bar{t}\ )\,ye^{-2\pi i(\overline{m}\bar{t}\,)}$ funksiyaga 8-lemmani qo‘llab,

$$\left| \, C_F(x,\overline{m}) - \tilde{\hat{C}}(x,\overline{m})\right|$$

$$= \left| \int_{G_n} F_{in}(x,\bar{t}\ )ye^{-2\pi i(\overline{m}\bar{t}\,)}\,\mathrm{d}\bar{t} - \frac{1}{N}\sum_{k=1}^{N} F_{in}\,(\mathrm{x}, M_{kn})\mathrm{e}^{-2\pi \mathrm{i}(\bar{\mathrm{a}}\,,\bar{\mathrm{m}})\mathrm{k/N}} \right|$$

$$\leq$$

$$\leq C_1(sAC_2)^n \mathrm{A} N_1^{\alpha} N^{-\alpha}(C_0^{\alpha}+\mathrm{n}B^n)log^{\alpha\beta} N_1$$

ni hosil qilamiz.

Ma’lumki, N$\geq 2^{\alpha}$ da

$$C_0^{\alpha}+\mathrm{n}B^n<A^n\left[\frac{n}{2^{(\alpha+1)n}}+\frac{(2+\frac{3}{\log N})^{a_n}}{2^{a_n}(\,3+\frac{2}{\alpha-1})^n}\right]<A^n$$

Shuning uchun

$$\left|\,C_F(x,\overline{m})-\tilde{\hat{C}}(x,\overline{m})\right|\leq S_1\mathrm{A}\ (\mathrm{s}A^2C_2)^n\,N_1^{\alpha}N^{-\alpha}log^{\alpha\beta}N_1.$$

Bu bahodan foydalanib va

$$\sum_{K(\overline{m})<N_1}1\leq 3^n\,N_1log^{n-1}N_1$$

ekanini hisobga olib,

$$\textstyle\sum_{K(\overline{m})<N_1}\left|\,C_F(x,\overline{m})-\tilde{\hat{C}}(x,\overline{m})\right|\ \leq S_1\mathrm{A}\ (3\mathrm{s}A^2C_2)^n\,N_1^{\alpha+1}N^{-\alpha}log^{\alpha\beta+n-1}\mathrm{N}.$$

ni topamiz.

(3.13) ifodadagi ikkinchi summani baholash uchun 3-lemmadan foydalanamiz.

$$\textstyle\sum_{k(\overline{m})>N_1}|\,C_F(x,\overline{m})|\leq C_1\ (3\mathrm{sA}C_2)^n a\left[\frac{a}{a-1}\right]^n\frac{log^{n-1}N_1}{N_1^{a-1}}.$$

Natijada,

$$\left|R_{nN_1(x)}\right|\leq(3\mathrm{s}A^2C_2)^n\ C_1\Big[A^{n+1}N_1^{a+1}N^{-a}log^{\alpha\beta+n-1}N_1+$$

$$+a(\tfrac{a}{a-1})^nN_1^{1-a}log^{n-1}N_1\Big].$$

Bundan ba’zi o‘zgartirishlar bajarib,

$$\left|\sum_{n=1}^{M}\lambda^n\left|R_{inN_1(x)}\right|\right|\leq 3|\lambda|\mathrm{sA}C_1C_2\left[\frac{A^2}{1-3s|\lambda\,|A^2C_2}+\frac{a^2}{a(1-3|\lambda\,|sAC_2}\right]\times$$

$$\times N^{-(a-1)/2}log^{0.5(a-1)\beta+M-1}\mathrm{N}. \qquad (3.24)$$

(3.14) ni hisobga olib, (3.9) dagi oxirgi qatorni baholaymiz

$$\left|\sum_{n=M+1}^{\infty}\lambda^{n}\varphi_{n}(x)\right| = \left|\sum_{n=M+1}^{\infty}\lambda^{n}\int_{G_n}F_{n}(x,\bar{t}\,)\,d\bar{t}\right| \leq$$

$$\leq \sum_{n=M+1}^{\infty}|\lambda|^{n}\sum_{\overline{m}}\frac{(sAC_2)^{n}C_1}{K(\overline{m})^{a}} = \sum_{n=M+1}^{\infty}(|\lambda|sAC_2)^{n}C_1\sum_{\overline{m}}K\,(\overline{m})^{-a} \leq$$

$$\leq C_1\sum_{n=M+1}^{\infty}|\lambda|\left(sAC_2\frac{3a-1}{a-1}\right)^{n} = \frac{[(3a-1)^{s}|\lambda|qAC_2]^{M+1}\,C_1}{a-|\lambda|(3a-1)sAC_2-1}. \qquad (3.25)$$

(3.15) va (3.16) baholarni birlashtirib

$$\sum_{n=1}^{M}\lambda^{n}R_{nN_1} + \sum_{n=M+1}^{\infty}\lambda^{n}\varphi_{n} \leq \mathrm{D}N^{-(a-1)/2}log^{\frac{\beta(a-1)}{2+M-1}}N$$

ni hosil qilamiz.

Bu yerda

$$\mathrm{D} = |\lambda|\ \mathrm{sA}C_1C_2\left[\frac{3A^2}{1-3s|\lambda|A^2C_2}+\frac{3a^2}{a-3a|\lambda|sAC_2-1}\right.$$
$$\left.+\frac{3a-1}{a-(3a-1)|\lambda|sAC_2}\right].$$

Bundan, M ning qiymatini hisobga olsak, quyidagiga ega bo‘lamiz

$$y(x) = y(x) + \frac{1}{N}\sum_{n=1}^{M}\lambda^n \sum_{k=1}^{N} F_n(x, M_{kn})\Psi_{knN_1}(x) +$$
$$+ 0\left[N^{-\frac{a-1}{2}} log^{0,5(a-1)\beta+M-1} N.\right]$$

Teorema isboti tugadi.

## III BOB YUZASIDAN XULOSALAR

Uchinchi bobda singulyar integral tenglamalarni taqribiy yechish masalasi, singulyar integral tenglamani sonlar nazariyasi to'ri bilan qurilgan kvadratur formulalarini qo'llash orqali algebraik tenglamalar sistemasiga keltirishga asoslangan.

Quyidagi

$$\varphi(P) = \int_{G_s} \prod_{j=1}^{s} ctg\pi\left(x_j - y_j\right) K(P, Q)\varphi(Q)dQ + f(P)$$

Gilbert yadroli ko'p o'lchovli singulyar integral

P = $(x_1, \dots, x_s)$, Q = $(y_1, \dots, y_s)$, dQ = $(dy_1, \dots, dy_s)$.

tenglamaning yechimi

$$\Phi(\mathrm{P}) = f(\mathrm{P}) + \frac{\lambda}{N}\sum_{k=1}^{N} K(P, M_{ks})\varphi(M_{ks})\phi_k(P) + O\left(N^{-a} log^{\alpha\beta} N\right)$$

ekanligi ko’rsatilgan

## XULOSA

Yuqorida ko‘rib chiqilganlarga asoslanib quyidagi xulosalarga kelishimiz mumkin:

[0; 2 ] segmentni ko‘rib chiqamiz va panjara $x_0 = 0; x_1=1; x_2 = 2$. To‘r bo‘limlari Biz shakldа kubik splaynni qidiramiz

$$S(x)=\begin{cases} S_1(x), & x\in[0;1] \\ S_2(x), & x\in[1;2] \end{cases} \tag{1}$$

(1) ga muvofiq

$$S_1(x)=a_1+b_1\,(x-1)+\frac{c_1}{2}(x-1)^2+\frac{d_1}{6}(x-1)^3 \tag{2}$$

$$S_2(x)=a_2+b_2(x-2)+\frac{c_2}{2}(x-2)^2+\frac{d_2}{6}(x-2)^3 \tag{3}$$

Interpolatsiya kubik splayn shartlarni hisobga olgan holda aniqlanishi kerak

$$\begin{gathered} S(x_i)=f_i, \quad i=0,1,2 \\ S''(0)=0; \quad S''(2)=12 \end{gathered} \tag{4}$$

Splayn koeffitsientlari (ularning 8 tasi bor), ya'ni: $a_i$, $b_i$, $c_i$, $d_i$, $i$ =1, 2, biz 1-teorema bo‘yicha topamiz, chunki taklif qilingan chegara shartlari ikkinchi hosilalar bo‘yicha shartlardir: $S''(0)=m_1=0$ ; $S''(2)=m_2=12$. Biz $c_0 = 0$ o‘zgaruvchini kiritamiz (chunki $m_1=0$ ). $c_i$, $i=1, 2$ ni topish uchun SLAE ni yozamiz

$$\begin{cases} c_0=0 \\ c_0\,h_1+2\,(h_1+h_2)\,c_1+c_2\,h_2=6\left(\dfrac{f_2-f_1}{h_2}-\dfrac{f_1-f_0}{h_1}\right) \\ c_2=12 \end{cases}$$

To‘r bir xil o‘rnatiladi: $h_1=1$ $h_2=2$

Interpolyatsiya qilinadigan f(x) funksiya qiymatlarni oladi $f_0=0$ $f_1=1$ $f_2=8$

SLAE tenglamaga kamayadi

$$c_0 + 4c_1 + c_2 = 6\left(\frac{8-1}{1} - \frac{1-0}{1}\right) = 36$$

qayerdan kelib chiqadi $0+4c_1+12=36$ $c_1=6$ $d_i$ , $i = 1, 2$ ni topish uchun

$$d_i = \frac{c_i - c_{i-1}}{h_i}, i = 1, 2 \quad d_1 = \frac{6-0}{1} = 6 \quad d_2 = \frac{12-6}{1} = 6$$

$$b_i = \frac{f_i - f_{i-1}}{h_i} + c_i \frac{h_i}{3} + c_{i-1} \frac{h_i}{6}, \quad i = 1, 2$$

$$b_1 = \frac{1-0}{1} + \frac{6}{3} + \frac{0}{6} = 1 + 2 = 3 \qquad b_2 = \frac{8-1}{1} + \frac{12}{3} + \frac{6}{6} = 12$$

Interpolyatsiya shartlaridan quyidagi hosil bo`ladi $a_1=f_1=1$ $a_2=f_2=8$

*Javob*

Chegara sharoitlari bilan interpolyatsiya qiluvchi kub splayn

S''(0) = 0 ; S''(2) = 12 ko‘rinishga ega bo`lib

$$S(x) = \begin{cases} S_1(x) = 1 + 3(x-1) + \frac{6}{2}(x-1)^2 + \frac{6}{6}(x-1)^3 & x \in [0; 1] \\ S_2(x) = 8 + 12(x-2) + \frac{12}{2}(x-2)^2 + \frac{6}{6}(x-2)^3 & x \in [1; 2] \end{cases},$$

то есть

$$S(x) = \begin{cases} S_1(x) = 1 + 3(x-1) + 3(x-1)^2 + (x-1)^3 & x \in [0; 1] \\ S_2(x) = 8 + 12(x-2) + 6(x-2)^2 + (x-2)^3 & x \in [1; 2] \end{cases} \quad (5)$$

Muammo hal qilindi.

Bu misolda $f(x) = x^3$ test funksiyasi taklif qilingan. $f''(0) = 0$ ; $f''(2) = 12$ shartlarga mos keladi. Demak, uning o‘zi yagona interpolyatsiya splayn hisoblanadi.

Qabul qilingan javobni tekshirish uchun biz shunga o‘xshash shartlarni taqdim etamiz:

$$\begin{aligned} S_1(x) &= 1 + 3(x-1) + 3(x-1)^2 + (x-1)^3 = \\ &= 1 + (3x-3) + (3x^2 - 6\cdot x + 3) + (x^3 - 3x^2 + 3x - 1) = \\ &= x^3 \end{aligned}$$

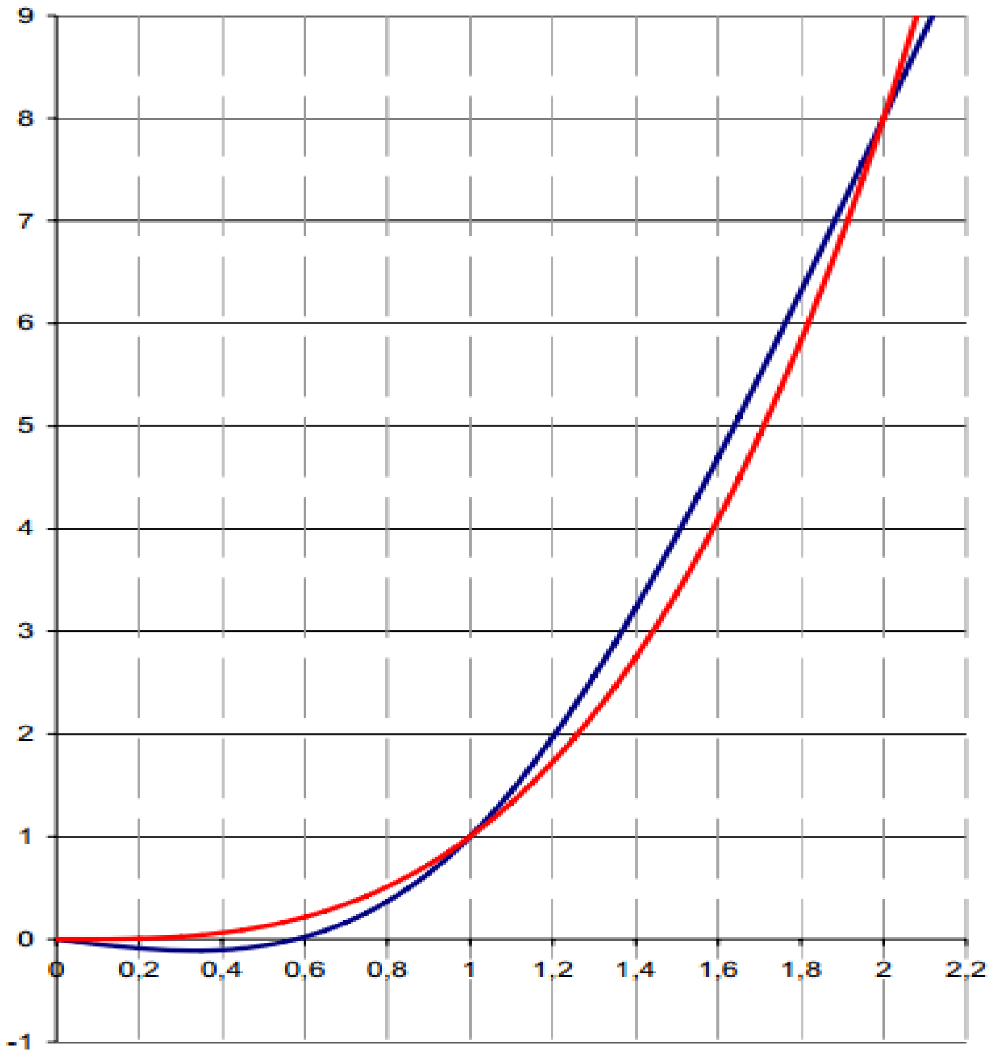
$$\begin{aligned} S_2(x) &= 8 + 12(x-2) + 6(x-2)^2 + (x-2)^3 = \\ &= 8 + (12x-24) + 6(x^2 - 4x + 4) + (x^3 - 2x^2 + 4x - 8) = \\ &= x^3 \end{aligned}$$

shundan kelib chiqadiki, yuqorida tuzilgan interpolyatsiya kubik splayn

$$S(x) = \begin{cases} S_1(x) = x^3, & x \in [\,0;1\,] \\ S_2(x) = x^3, & x \in [\,1;2\,] \end{cases}$$

chegara shartlari bilan S''(0) = 0 ; S''(2) =12

Yuqorida yechilgan masala to‘g‘ri yechilgan, grafik 4-rasmda keltirilgan.




Rasm- 4

Rasmda 0 to‘rda interpolyatsiya qiluvchi ikkita kubik splayn ko‘rsatilgan; bitta; 2 bir xil jadval funksiyasi $f(x) = x^3$ . 1-misolning yechimi qizil rangda, 2-misolning yechimi esa ko‘k rangda ko‘rsatilgan.Splaynlarning qaysi biri EHSga mos kelishini ko‘rish mumkin: interpolyatsiya segmentining uchlarida EHS bilan shplayn “ nol” botiqlik (qavariqlik).

Natija 2 (EGU bilan interpolyatsiya kub splayn) Griddagi funktsiya berilgan

| $x_i$ | 0 | 1 | 2 |
|---|---|---|---|
| $f_i$ | 0 | 1 | 8 |

EGU dan interpolyatsiya qiluvchi kubik splayn kerak $S''(0) = 0$; $S''(2) = 0$

[0; 2] segmentni va panjara $x_0 = 0$; $x_1=1$; $x_2 = 2$ ko‘rib chiqing . Grid bo‘limlari oldingi misolda bo‘lgani kabi, biz shartlarni hisobga olgan holda (1), (2), (3) ko‘rinishdagi splaynni qidiramiz.

$$\begin{aligned} &S(x_i) = f_i, \quad i = 0, 1, 2 \\ &S''(0) = 0\,; \quad S''(2) = 0 \end{aligned} \tag{6}$$

Splayn koeffitsientlari (ularning 8 tasi bor), ya'ni: $a_i$, $b_i$, $c_i$, $d_i$, i =1, 2, ni topamiz.

1-teorema, chunki EHUlar ikkinchi shartlarning alohida holatidir hosilalari: $S''(0)=m_1=0$; $S''(2)=m_2=0$.

Biz $c_0 = 0$ qo‘g‘irchoq o‘zgaruvchini kiritamiz. $c_i$, i =1, 2 ni topish uchun SLAE ni yozamiz.

$$\begin{cases} c_0 = 0 \\ c_0\, h_1 + 2\,(h_1 + h_2)\, c_1 + c_2\, h_2 = 6\left(\dfrac{f_2 - f_1}{h_2} - \dfrac{f_1 - f_0}{h_1}\right) \\ c_2 = 0 \end{cases}$$

To‘r bir xil o‘rnatiladi: $h_1 = 1$ $h_2 = 2$

Interpolyatsiya qilinadigan $f(x)$ funksiya qiymatlarni olinadi $f_0=0$ $f_1=1$ $f_2=8$

Tenglama quyidagicha bo`ladi

$$c_0 + 4\,c_1 + c_2 = 6\left(\frac{8-1}{1} - \frac{1-0}{1}\right) = 36$$

$$c_0 = 0,\; c_2 = 0$$

$$0 + 4\,c_1 + 0 = 36 \quad \Rightarrow \quad c_1 = 9.$$

$d_i$, i =1, 2 ni topish uchun quyidagini yozamiz

$$d_i = \frac{c_i - c_{i-1}}{h_i}, i = 1, 2 \quad d_1 = \frac{9-0}{1} = 9 \quad d_2 = \frac{0-9}{1} = -9.$$

$b_i$ ,i = 1, 2 ni topish uchun

$$b_i = \frac{f_i - f_{i-1}}{h_i} + c_i \frac{h_i}{3} + c_{i-1} \frac{h_i}{6}, \quad i = 1, 2$$

$$b_1 = \frac{1-0}{1} + \frac{9}{3} + \frac{0}{6} = 1 + 3 = 4 \qquad b_2 = \frac{8-1}{1} + \frac{0}{3} + \frac{9}{6} = 8.5$$

$$a_1 = f_1 = 1 \qquad a_2 = f_2 = 8$$

Javob Tabiiy chegara sharoitlari bilan interpolyatsiya qiluvchi kub splayn

S"(0) = 0 ; S"(2) = 0 ko‘rinishga ega

$$S(x) = \begin{cases} S_1(x) = 1 + 4(x-1) + \frac{9}{2}(x-1)^2 + \frac{9}{6}(x-1)^3 & x \in [0;1] \\ S_2(x) = 8 + 8.5(x-2) + \frac{0}{2} \cdot (x-2)^2 - \frac{9}{6}(x-2)^3 & x \in [1;2] \end{cases}$$

$$S(x) = \begin{cases} S_1(x) = 1 + 4(x-1) + 4.5(x-1)^2 + 1.5(x-1)^3 & x \in [0;1] \\ S_2(x) = 8 + 8.5(x-2) - 1.5(x-2)^3 & x \in [1;2] \end{cases}$$

Muammo hal qilindi.

Tasdiqlash (ta'rif bo‘yicha)

Bu misolda f(x) = x3 test funksiyasi taklif qilingan. U f "(0) = 0 ; f" (2) = 0 shartlariga javob bermaydi. Bu uning o‘zi splayn emasligini bildiradi. Keling, splayn qanday ko‘rinishini bilib olaylik:

$$S_1(x) = 1 + 4\,(x-1) + 4.5\,(x-1)^2 + \; 1.5\,(x-1)^3 =$$
$$= 1 + (4x-4) + (4.5x^2 - 9\cdot x + 4.5) + 1.5\,(x^3 - 3x^2 + 3x - 1) =$$
$$= 1.5\,x^3 - 0.5\,x$$

$$S_2(x) = 8 + 8.5\,(x-2) - 1.5\,(x-2)^3 =$$
$$= 8 + (8.5x - 17) - 1.5\,(x^3 - 3\cdot 2x^2 + 3\cdot 4x - 8) =$$
$$= -1.5\,x^3 + 9x^2 - 9.5x + 3$$

$$S(x) = \begin{cases} S_1(x) = 1.5x^3 - 0.5x, & x \in [\,0; 1\,] \\ S_2(x) = -1.5x^3 + 9x^2 - 9.5x + 3, & x \in [\,1; 2\,] \end{cases}$$

Keling, splayn uchun barcha talablarning bajarilishini tekshiramiz.

x ∈[0; 1] segmentda formula $S_1(x) = 1{,}5x^3 - 0{,}5\,x$

$S_1(0) = 0$ $S_1(1) = 1.5 - 0.5 = 1$ (interpolyatsiya shartlari sayt chegarasida bajariladi)

$S'_1\,(x) = 4.5\,x^2 - 0.5$ $S'_1\,(1) = 4.5 - 0.5 = 4$ (birinchi hosila maydonning o‘ng chegarasida hisoblanadi)

$S''1(x) = 9\,x$ $S''1(1) = 9$ (ikkinchi maydonning o‘ng chegarasida hisoblanadi)

$S''1(0) = 0$ (segmentning chap chegarasida tabiiy chegara sharti bajariladi)

Maydonda x∈[1; 2 ] formula $S_2(x) = -1{,}5\,x^3 + 9x^2 - 9{,}5x + 3$

$S_2(1) = -1{,}5+9-9{,}5+3 = 12-11 = 1$ $S_2(2) = -1{,}5\cdot 8+9\cdot 4-9{,}5\cdot 2+3 = 39-2-9 = 8$

(interpolyatsiya shartlari maydon chegaralarida bajariladi)

$S'_2(x) = -4{,}5x^2+18x-9{,}5$ $S'_2(1) = -4{,}5+18-9{,}5 = 18-14 = 4 = S'_1\,(1)$

(birinchi hosila bo‘limning chap chegarasida hisoblanadi, uzluksizligi tekshirildi)

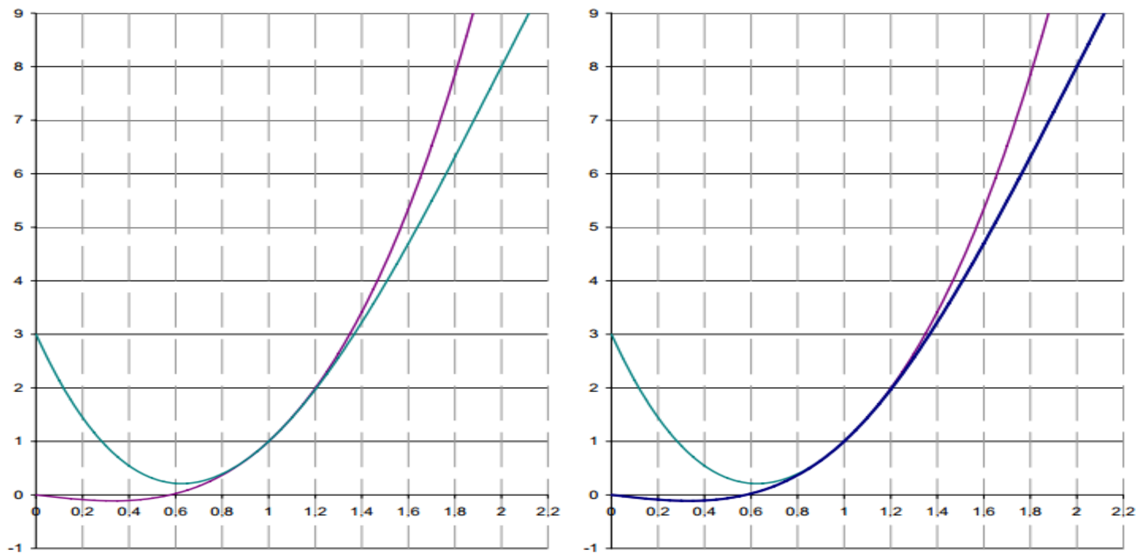
$S''2(x)=-9x+18 \quad S''2(1) = -9 +18 = 9 = S''1(1)$

(ikkinchi hosila bo‘limning chap chegarasida hisoblanadi, uzluksizligi tekshirildi)

$S''_2(2) = -9 \cdot 2 +18 =18 -18 = 0$ (segmentning o‘ng chegarasida tabiiy chegara sharti bajariladi)

Yuqorida hal qilingan muammo to‘g‘ri hal qilingan. 4 va 5-rasmlardagi yechim grafigi. 4-rasmda siz 1-misol va 2-misol yechimini solishtirishingiz mumkin.

5-rasmda EHC interpolyatsiyasi $f(x) = x^3$ bo‘lgan kub splayn 0 to‘rida ko‘rsatilgan; bitta; 2 ikkita kubikli polinomdan tuzilgan. Birinchi polinomlar, keyin splayn va "ishlatilmagan" polinom qoldiqlari ko‘rsatilgan.




Rasm-5

Rasmda $S_1(x) =1,5x^3 - 0,5x$, $S^2(x) = -1,5x^3 + 9x^2 - 9,5x + 3$ ko‘phadlari va

2-misolning “yopishtirilgan” yechimi YSU bilan interpolyatsiya kubik splayndir.

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